Vol. 40 (2009) ACTA PHYSICA POLONICA B No 4

IS EARLY THERMALIZATION REALLY NEEDED IN $A+A$ COLLISIONS?*

Yu.M. Sinyukov, A.V. Nazarenko, Iu.A. Karpenko

Bogolyubov Institute for Theoretical Physics
Metrolohichna 14b, 03680 Kiev-143, Ukraine

(Received February 11, 2009)

In this note we review our ideas, first published in year 2006, and corresponding results, including the new ones, which show that whereas the assumption of (partial) thermalization in relativistic $A+A$ collisions is really crucial to explain soft physics observables, the hypotheses of *early* thermalization at times less than 1 fm/c is not necessary. The reason for the later conclusion is that the initial transverse flow in thermal matter as well as its anisotropy, leading to asymmetry of the transverse momentum spectra, could be developed at pre-thermal, either partonic or classical field — Glasma, stage with even more efficiency than in the case of very early perfect hydrodynamics. Such radial and elliptic flows develop no matter whether a pressure already established. The general reason for them is an essential finiteness of the system in transverse direction.

PACS numbers: 25.75.–q, 24.10.Nz

1. Introduction

The problem of thermalization of the matter in ultra-relativistic $A+A$ collisions is, certainly, one of the central in this field of physics. Even the answer to the basic question as for possible formation of new states of matter, such as the quark–gluon plasma (QGP), in these experiments depends on a clarification of this problem.

The experimental data support, in fact, an idea of thermalization. There are many dynamical models: hydrodynamic one [1], hybrid model, at first hydro evolution, then hadronic cascade [2], hydro-kinetic approach, coherent use of hydrodynamics and kinetics [3], that describe the soft physics observables in quite satisfactory way. All of them uses an assumption of thermalization: either complete (local equilibrium), or partial (with viscosity).

* Presented at the IV Workshop on Particle Correlations and Femtoscopy, Kraków, Poland, September 11–14, 2008.

The discovery and theoretical description of elliptic flows at RHIC [4] play especially intriguing role in the problem of thermalization. It was cleared up [5] that the anisotropy of transverse momentum spectra, expressed in terms of v_2 coefficients, can be explained and basically described within hydrodynamic model for perfect fluid. Since the geometry of initial state in non-central $A+A$ collision is ellipsoidal-like, where the biggest axis y is directed orthogonally to the reaction plane, then the collective velocities will be developed preferably along the smaller axis x — in reaction plane — since the gradient of pressure is larger in this direction. It leads to larger blue shift of quanta radiated from decaying fluid elements in x-direction at the freeze-out stage. This explains the anisotropy of the spectra, but to reach a quantitative agreement with the experimental data one needs to use very small initial time, $\tau \sim 0.5$ fm/c, to start the hydro-evolution. If it starts at later times, neither the collective velocities, nor their (and the spectra) anisotropy will be developed enough. These results, in fact, brought the two new ideas: first, that QGP, at least at the temperatures not much higher T_c, is the strongly coupled system (sQGP), and so is an almost perfect fluid, and, second, that thermalization happens at very early times of collisions.

A necessity for the thermal pressure to be formed as early as possible appears also in hydrodynamic description of the central collisions at RHIC. If one starts the hydrodynamic evolution in "conventional time" $\tau_i = 1$ fm/c *without* transverse flow, the latter will not be developed enough to describe simultaneously the pion, kaon and proton spectra.

The interesting observation related to the same topic was found in Ref. [6] concerning RHIC HBT puzzle. It was shown that to describe successfully pion, kaon and proton spectra as well as the interferometry radii R_i and explain the observed "puzzle", $R_{\rm out}/R_{\rm side} \approx 1$, one needs *(i)* initial transverse velocities developed by the time $\tau_i = 1$ fm/c and *(ii)* positive $r_{\rm T}$–t correlations between time t and absolute value of transverse radius $r_{\rm T}$ on the surface of the particle emission during the system evolution. In the case of positive correlations the contribution to $R^2_{\rm out}$ associated with the duration of emission squared can be compensated by that correlation term, which also contribute to $R^2_{\rm out}$, and, therefore, $R_{\rm out}/R_{\rm side}$ ratio can be fairly small even at protracted particle emission. In Ref. [7] the connection between these two conditions, *(i)* and *(ii)*, is established. Namely, in hydrodynamic models the positive $r_{\rm T}$–t correlations at SPS and RHIC energies can appear only if fairly developed transverse flow already exist at the time $\tau_i = 1$ fm/c.

At the first glance, all those results for central collisions just conform the conclusion coming from non-cental ones: the thermalization and pressure are established at times essentially smaller than 1 fm/c and because of this the radial/elliptic flows are already developed by this time. The crucial problem is, however, that even the most optimistic theoretical estimates give thermalization time 1–1.5 fm/c [8], typically, it is 2–3 fm/c. The discrepancy could be even more at LHC energies.

The way to solve the problem was proposed in year 2006, Ref. [7]. It was shown that the transverse collective velocities can be developed at the very early pre-thermal stage with even higher efficiency than in the locally equilibrated fluid. This result was developed and exploited in Refs. [9–11][1]. In what follows we stick to the main line developed in these papers.

2. Early developing of flow and its anisotropy

A developing of the initial transverse velocities at the pre-thermal stage in ultrarelativistic $A + A$ collisions is related to the complex problem of formation and evolution of chromo-electric and magnetic fields and their quanta — partons at very early stage of collision processes. In this note we analyze the developing of the transverse flows in the two opposite cases: evolution of non-interacting partons and fields and hydrodynamic expansion of strongly interacting system — perfect liquid.

2.1. Non-relativistic analytical results

We start from the non-relativistic analytical toy-models. Let us put the initial momentum distribution of particles with mass m to be spherically symmetric Gaussian with the width corresponding to the thermal Boltzmann distribution with uniform temperature T_0, and without collective flow: $\boldsymbol{v}(t = 0, \boldsymbol{r}) = 0$. Also the initial spatial distribution of particle density corresponds to the spherically symmetric Gaussian profile with radius R_0. Let particles just to stream freely. Then, according to [14], the collective velocities, which can be defined at any time t according to Eckart:

$$v^i = \int \frac{d^3p}{m^4} p^i f(t, \boldsymbol{x}; p)\,, \tag{1}$$

are

$$\boldsymbol{v}(t, \boldsymbol{r}) = \boldsymbol{r}\,\frac{tT_0}{mR_0^2 + T_0 t^2} \to \frac{t\boldsymbol{r}}{\lambda_{\text{hom}}^2} \quad \text{at} \quad t \to 0\,, \tag{2}$$

where r is the radial coordinate and the homogeneity length λ_{hom} is equal in this case to R_0. For the boost-invariant situation with initially elliptic transverse profile (corresponding Gaussian radii are $R_x = \lambda_{\text{hom},x}$ and $R_y = \lambda_{\text{hom},y}$) [14] the similar results for transverse velocities can be obtained at small t:

$$v_i(t, \boldsymbol{r}) \sim \frac{tr_i}{\lambda_{\text{hom},i}^2}\,. \tag{3}$$

So, the collective velocities, which are developed in some direction i, are inversely proportional to the corresponding homogeneity length squared,

[1] See also recent contribution to this topic [12, 13].

$\lambda^2_{\text{hom},i}$. The pressure plays no role in this collisionless process. The collective flow in the expanding system appears because of a deficit of particles moving inward as compared to the particles moving outward. It happens due to finiteness of the system: one has less particles at the periphery than in the central part (the corresponding deficit is determined by the homogeneity length), and the flow develops during the evolution since the momentum of the particles that move outward cannot be compensated by the momentum of the particles which move inward, even if at the initial time such a compensation takes place.

It is worth noting that, at least for fairly small time interval, the flows (2), (3) for collisionless expansion coincide with flows, which are developed in hydrodynamically expanding perfect fluid ([14]) at the same initial conditions.

2.2. Partonic free-streaming

Let us start from the initial conditions inspired by the Color Glass Condensate (CGC) approach. According to the results of Ref. [15] the time $\tau_0 \approx 3/\Lambda_{\text{s}}$ (where $\Lambda_{\text{s}} = g^2\mu \equiv 4\pi\alpha_s\mu$ and μ^2 is the variance of a Gaussian weight over the color charges of partons) is an appropriate scale controlling the formation of gluons with a physically well-defined energy. In Ref. [11] the results found in [15] for gluon momentum spectra in spatially transverse homogeneous case are transformed for the transversally finite systems and longitudinally smeared distribution instead of $\delta(y-\eta)$ ($\eta = \frac{1}{2}\ln(t+x_{\text{L}})/(t-x_{\text{L}}) \simeq 0, y = \frac{1}{2}\ln(p_0+p_{\text{L}})/(p_0-p_{\text{L}})$). Then the initial local boost-invariant phase-space density f at the hypersurface $(t^2-x_{\text{L}}^2)^{1/2} = \tau_0$ takes the form [11]

$$f(x,p)_{|\tau_0} \equiv \frac{dN}{d^3}xd^3p_{|\tau_0} = \frac{a_1(b)}{g^2}\left(\tau_0 m_{\text{T}}\cosh(y-\eta)\right)^{-1} \times \left(\exp\left(\sqrt{p_{\text{T}}^2+m_{\text{eff}}^2}/T_{\text{eff}}\right)-1\right)^{-1}\frac{\rho(b,\boldsymbol{r}_{\text{T}})}{\rho_0(b)}\,, \tag{4}$$

where $m_{\text{eff}} = a_2\Lambda_s$, $T_{\text{eff}} = a_3\Lambda_s$; $a_2 = 0.0358, a_3 = 0.465$. For finite systems we approximate the transverse profile of the gluon distribution by the ellipsoidal Gaussian $\rho(b,\boldsymbol{r}_{\text{T}}) = \rho_0(b)\exp(-r_x^2/R_x^2 - r_y^2/R_y^2)$ defined from the best fit of the participant number density in the collisions with the impact parameter $\boldsymbol{b} = (b,0)$. Constant $a_1(b)$ in Eq. (4) is found then by the comparison of the total gluon number with results of Ref. [15].

Let us suppose that an actual thermalization happens about the time $\tau_i = 1\,\text{fm}/c > \tau_0$ and partons just stream freely between τ_0 and τ_i (see details for the τ-evolution of the phase-space density in Refs. [7,9]), then the system is suddenly thermalized at τ_i with the pressure $P(\epsilon;\tau_i,x)$ corresponding to the lattice QCD results. Then for $\tau_0 = 0.3$ fm/c, that corresponds to $\Lambda_{\rm s} = 2\,{\rm GeV}$ [15], we find for the energy density profile: $\epsilon(\tau_0, b = 0; \boldsymbol{r} = 0) \approx 0.09\Lambda_{\rm s}^4/g^2$, Gaussian width $R(\tau_0, b = 0) \approx 5.33$ fm ($R_x = R_y = R$). To describe pion spectra and interferometry radii one should use [11] $\Lambda_{\rm s}^4/g^2 \approx 6\ {\rm GeV}^4$ and so, $\epsilon(\tau_0 = 0.3\ {\rm fm}/c,\ \boldsymbol{r} = 0) \equiv \epsilon_0 = 67\,{\rm GeV/fm}^3$. For non-central collisions with $b = 6.3\,{\rm fm}$: $R_x = 3.7\,{\rm fm}$, $R_y = 4.6\,{\rm fm}$.

The results for the collective flows are presented in Fig. 1 ($b = 0$ and $b = 6.3\,{\rm fm}$). We see qualitatively the same effect of the transverse flow and their anisotropy in non-cental collisions at pre-thermal stage as in the analytic non-relativistic examples.

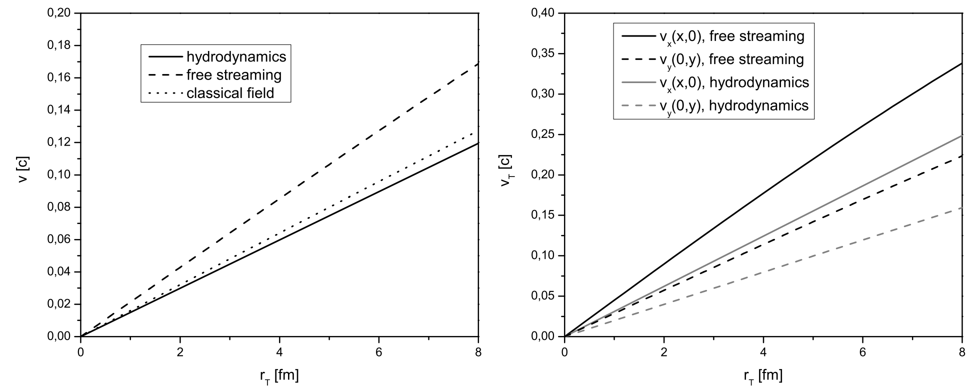

Fig. 1. Collective velocity developed at pre-thermal stage from proper time $\tau_0 = 0.3\,{\rm fm}/c$ by supposed thermalization time $\tau_i = 1$ fm/c for scenarios of partonic free streaming and free expansion of classical field. The results are compared with the hydrodynamic evolution of perfect fluid with hard equation of state $p = \frac{1}{3}\epsilon$ started at τ_0. Left: central ($b = 0$), right: non-central ($b = 6.3\,{\rm fm}$) collisions.

2.3. Field expansion

Let us describe now the flows at the early pre-thermal stage in Glasma approach [16] which deals with evolution/rearrangement of the classical gluonic field. Let us again simplify the situation and consider extreme case of the boost-invariant evolution of the free field after moment $\tau_0 \approx 3/\Lambda_{\rm s}$ [15]. Then we actually come to the Maxwell theory with 4-potential A_μ in space-time with the pseudo-cylindrical metric $ds^2 = d\tau^2 - \tau^2 d\eta^2 - dx^2 - dy^2$.

Imposing the gauge $A_\tau = 0$ used within the CGC concept, the dynamical variables of the model are $\Phi \equiv A_\eta$, A_i $(i = x, y)$. We will assume that these potentials and all the observables do not depend on space-time rapidity η. It turns out that A_x and A_y are related by condition $\partial_i A_i = 0$ following from the gauge-fixing.

The space-time evolution of the potentials is determined by the Maxwell equations which read

$$\partial_\tau^2 A_i + \frac{1}{\tau}\partial_\tau A_i - \Delta_{\rm T} A_i \; = \; 0\,, \tag{5}$$

$$\partial_\tau^2 \Phi - \frac{1}{\tau}\partial_\tau \Phi - \Delta_{\rm T} \Phi \; = \; 0\,, \tag{6}$$

where $\Delta_{\rm T} = \partial_i\partial_i$.

The components of the energy-momentum tensor, which are needed for finding collective velocity, are defined by the well-known formula:

$$T^{\mu\nu} = -F^{\mu\lambda}F^{\nu}{}_{\lambda} + \frac{1}{4}g^{\mu\nu}F^{\lambda\sigma}F_{\lambda\sigma}\,, \tag{7}$$

where $F_{\lambda\sigma} = \partial_\lambda A_\sigma - \partial_\sigma A_\lambda$.

If $T^{\mu\nu}$ are known, the collective velocity is determined in accordance with the Landau–Lifshitz definition from equations:

$$u^\mu = \frac{T^{\mu\nu}u_\nu}{u^\lambda T_{\lambda\sigma}u^\sigma}\,, \qquad u^2 = 1\,. \tag{8}$$

Now let us focus on the initial conditions for field equations. We apply the Glasma-like ones [16]:

$$\partial_\tau A_i|_{\tau_0} \; = \; 0\,, \qquad \Phi|_{\tau_0} = 0\,, \tag{9}$$

$$E_z \sim \left.\frac{\partial_\tau \Phi}{\tau}\right|_{\tau_0} \; \neq \; 0\,, \qquad H_z \sim F_{xy}|_{\tau_0} \neq 0\,. \tag{10}$$

These equations mean that, at the initial moment τ_0, the electric and magnetic fields are longitudinal, while the transverse components vanish. So there is no field flow at τ_0.

With those initial conditions we get (at $\eta = 0$)

$$T_{tt}|_{\tau_0} \; \equiv \; \varepsilon(r_{\rm T}) = \frac{1}{2}\left(\left.\frac{\partial_{r_{\rm T}}\Psi}{r_{\rm T}}\right|_{\tau_0}\right)^2 + \frac{1}{2}\left(\left.\frac{\partial_\tau \Phi}{\tau}\right|_{\tau_0}\right)^2\,, \tag{11}$$

$$T_{tx}|_{\tau_0} \; = \; 0\,. \tag{12}$$

Let us choose the initial energy density profile $\varepsilon(r_{\rm T}) = \epsilon_0 \exp\left(-\frac{r_{\rm T}^2}{2R^2}\right)$ to be the same as in the previous case of partonic system for $b = 0$.

To provide this we use, accounting for (11), the following transverse profile for potentials at the initial proper time τ_0:

$$\partial_{r_{\rm T}}\Psi|_{\tau_0} = \sqrt{\alpha}\, r_{\rm T} f(r_{\rm T})\,, \qquad \partial_\tau \Phi|_{\tau_0} = \sqrt{1-\alpha}\, \tau_0 f(r_{\rm T})\,, \tag{13}$$

where $f(r_{\rm T}) \equiv \sqrt{2\varepsilon(r_{\rm T})}$ and α is a constant. Since the potentials Ψ, Φ are real, α bounds are $0 \le \alpha \le 1$. The observable quantities $T_{tt}(\tau, r_{\rm T}, \varphi = 0, \eta{=}0)$, $T_{tx}(\tau, r_{\rm T}, \varphi = 0, \eta{=}0)$, and $v(\tau, r_{\rm T})$ are independent on parameter α.

Further, we will treat these relations (together with $\partial_\tau \Psi|_{\tau_0} = 0$ and $\Phi|_{\tau_0} = 0$) as the initial conditions for our problem.

The solution for the field is expressed through the field amplitude at the initial moment τ_0:

$$\Psi_0(k_{\rm T}) = \sqrt{\alpha}\ \tilde{f}(k_{\rm T})\,, \qquad \Phi_0(k_{\rm T}) = \sqrt{1-\alpha}\ \tilde{f}(k_{\rm T})\,, \tag{14}$$

here

$$\tilde{f}(k_{\rm T}) = \int\limits_0^\infty f(r_{\rm T}) J_0(k_{\rm T} r_{\rm T}) r_{\rm T} dr_{\rm T}\,. \tag{15}$$

Then the time-dependence of amplitudes takes the form

$$\tilde{\Psi}(\tau, k_{\rm T}) = \Psi_0(k_{\rm T}) \frac{\pi k_{\rm T}\tau_0}{2} [J_1(k_{\rm T}\tau_0)Y_0(k_{\rm T}\tau) - J_0(k_{\rm T}\tau)Y_1(k_{\rm T}\tau_0)]\,, \tag{16}$$

$$\tilde{\Phi}(\tau, k_{\rm T}) = \Phi_0(k_{\rm T}) \frac{\pi k_{\rm T}\tau}{2} [J_1(k_{\rm T}\tau_0)Y_1(k_{\rm T}\tau) - J_1(k_{\rm T}\tau)Y_1(k_{\rm T}\tau_0)]\,. \tag{17}$$

To restore the spatial dependence, the Bessel–Fourier transform should be applied:

$$\Psi(\tau, r_{\rm T}) = r_{\rm T} \int\limits_0^\infty \tilde{\Psi}(\tau, k_{\rm T}) J_1(k_{\rm T} r_{\rm T})\, dk_{\rm T}\,, \tag{18}$$

$$\Phi(\tau, r_{\rm T}) = \int\limits_0^\infty \tilde{\Phi}(\tau, k_{\rm T}) J_0(k_{\rm T} r_{\rm T})\, dk_{\rm T}\,, \tag{19}$$

where an integration over the angle has been already carried out. It allows one to calculate the evolution of the momentum–energy tensor (7) and then find the transverse flows according to Eq. (8). The results are presented at the Fig. 1. One can see that they are similar to ones obtained for partonic free streaming.

2.4. Hydrodynamic evolution

We compare the developing of the transverse collective velocities at prethermal stage by the time $\tau_i = 1\,{\rm fm}/c$ with the hydrodynamic evolution of perfect fluid having the hard equation of state (EoS) $p = 1/3\,\epsilon$. The initial conditions for hydro expansion: the time τ_0 and the energy density profile are supposed to be the same as in previous cases (Subsections 2 and 3). One can see from Fig. 1 that the hydrodynamic expansion is relatively less effective for producing transverse flows and their anisotropy than the non-interacting evolution.

3. Conclusion

We considered a possible developing of the transverse flows in different extreme cases: the evolution of non-relativistic free gas, partonic free streaming, classical field expansion and hydrodynamic expansion of the perfect fluid with the hard equation of state. All the results are similar, though in relativistic situation the transverse flows, as well as their anisotropy, are developed at pre-thermal, either partonic or classical field, stage with even more efficiency than in the case of very early perfect hydrodynamics with hard EoS. So, one can conclude that the transverse flows and their anisotropy in non-central $A + A$ collisions, in fact, do not depend on the nature of the systems formed and complicated kinetics of non-equilibrium processes. They are defined mostly by the homogeneity lengths in the initial system. However, that is crucially important, the transformation of these radial and elliptic flows into observed high effective temperatures of proton and kaon spectra and anisotropy of pion spectra becomes possible *only if (local) thermalization happens*. Otherwise, *e.g.*, at a free streaming, the initial partonic momentum distribution and its symmetry properties are preserved despite the flow and its asymmetry in non-central collisions are developed. So, the thermalization is certainly required for description of the hadronic spectra and v_2 coefficients but it does not matter whether thermalization is early or not.

This work was supported in part by the Ukrainian–French grant (programme DNIPRO), Agreement with MESU No. M/4-2009; the Bilateral award DLR (Germany)–MESU (Ukraine) for the UKR 06/008 Project, Agreement No. M/26-2008; and the Program "Fundamental Properties of Physical Systems under Extreme Conditions" of the Bureau of the Section of Physics and Astronomy of NAS of Ukraine. The research was carried out within the scope of the ERG (GDRE): Heavy ions at ultrarelativistic energies — a European Research Group comprising IN2P3/CNRS, Ecole des Mines de Nantes, Université de Nantes, Warsaw University of Technology, JINR Dubna, ITEP Moscow, and Bogolyubov Institute for Theoretical Physics, NAS of Ukraine.

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