

FEMTOSCOPY IN HADRON AND LEPTON  
COLLISIONS: WORLD SYSTEMATICS\*

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Femtoscopic measurements at a variety of facilities have established a clear dependence of spatial scales with event multiplicity and particle transverse mass ( $m_T$ ) in heavy ion collisions from  $\sqrt{s_{NN}} \sim 2\text{--}200$  GeV. The  $m_T$ -dependence is thought to arise from collective, explosive flow of the system, as probed by independent measurements, while the multiplicity dependence reflects the increased spatial extent of the final state with decreasing impact parameter. Qualitatively similar dependences have been reported from high energy hadron and lepton collisions, where the conceptual validity of an impact parameter or collective flow are less clear. We focus on results from elementary particle collisions, identify trends seen in the experimental data and compare them to those from heavy ion collisions.

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**1. Introduction**

The enhancement of the probability of having two bosons close in phase-space is a consequence of Bose–Einstein symmetrization. In astronomy, this effect was first observed by Hanbury Brown and Twiss [1] who measured the angular size of stars using photon intensity interference. For particle physicists, it all started about 50 years ago when Goldhaber *et al.* observed significant positive correlations between identical pions due to the Bose–Einstein effect [2]. It was almost two more decades before the availability of data with sufficient statistics and quality allowed pion interferometry to reliably extract spatial scales on the order of the interaction region,  $\sim 1$  fm. Later, with new facilities being built, the femtoscopic studies were extended to probe the sizes of the particle emitting sources as a function of the initial system size and the energy of the collisions as well as a function of kinematic

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variables like the rapidity and the transverse momentum of the particle pair. The interferometry method has been applied not just to pions but also to heavier particles. In principle, a systematic comparison of femtoscopic results from the elementary particle collisions (*e.g.*  $p+p$ ,  $p+\bar{p}$ ,  $e^+e^-$ ) through light nuclei (*e.g.*  $O+O$ ) and up to heavy ion collisions (*e.g.*  $Au+Au$ ) gives an opportunity to understand the physics of these collisions probed by two-particle interferometry. In practice, such a comparison has been hampered by two problems. Firstly, there have traditionally been a multitude of parameterizations of the Bose–Einstein effect, especially within the particle physics community. This, coupled with different analysis techniques, definitions of multiplicity, and acceptances, makes comparison of results between different experiments difficult. Secondly, communication between the high energy and heavy ion communities has unfortunately been rather limited; workshops such as this one should help rectify this situation.

This article is not a complete review of femtoscopic results from high energy collisions. Instead, we have collected a large fraction of the world dataset of such results and compare them in a common systematics. We focus on aspects of femtoscopy in elementary particle collisions that are important when comparing to heavy ion collisions. We also discuss some issues with results from  $e^+ + e^-$  collisions that complicate their interpretation.

The paper is organized as follows. In Sec. 2 we present different parameterizations of one- and three-dimensional correlation functions used to obtain results presented in this paper. A short summary of the most important observables in femtoscopic studies in heavy ion collisions is presented in Sec. 3. In Sec. 4 we attempt to present a consistent picture of the femtoscopic observables in the elementary particle collisions. In Sec. 5 we bring up a problem with mass dependence of HBT radii seen in  $e^+ + e^-$  collisions and comment on the Heisenberg uncertainty principle as a possible origin of both the mass ordering and the transverse mass dependence of  $R_z$ . Conclusions and discussion are presented in Sec. 6.

## 2. Definitions and parameterizations of Bose–Einstein effect

The correlation function is defined as

$$C(p_1, p_2) = \frac{P(p_1, p_2)}{P(p_1)P(p_2)}, \quad (1)$$

where  $P(p_1, p_2)$  is the probability of observing two particles with momenta  $p_1$  and  $p_2$ , while  $P(p_1)$  and  $P(p_2)$  denote single-particle probabilities.

Experimentally, the correlation function is defined as

$$C(Q) = \frac{A(Q)}{B(Q)}, \quad (2)$$

where  $Q$  is a difference between momenta of two particles.  $A(Q)$  represents a distribution of the pairs from the same event and  $B(Q)$  is the background, or reference, distribution that is supposed to include all physics effects as  $A(Q)$  except for femtoscopic correlations (quantum statistics, final state interactions including Coulomb and strong interaction, where applicable) between the pair being studied.

Femtoscopic correlations in  $\vec{Q}$ -space may be expressed (*e.g.* [3]) as a convolution of the pair spatial separation distribution with the two-particle wavefunction which includes quantum (anti-)symmetrization and final state interactions effects. At large  $|\vec{Q}|$ , these effects vanish, and femtoscopic correlation functions must assume a constant value independent of the direction of  $\vec{Q}$ . However, several experiments of collisions with low multiplicity (*e.g.* [4–8]) report correlation functions with large- $|\vec{Q}|$  structure which must be non-femtoscopic in origin. They may arise, for example, from jets or energy-momentum conservation.

Especially in  $e^+ + e^-$  experiments, there has been tremendous effort to remove these effects using different techniques to form the reference distribution,  $B(q)$  in Eq. (2) (*e.g.* [9]). For identical particle interferometry, these include using like-sign pion distributions or Monte Carlo simulations to generate the reference. We do not review them all here, simply noting that each technique has its advantages, but none completely removes non-femtoscopic structures. At the end, the correlation function is fit with a functional form which includes (or neglects) the non-femtoscopic structure. A plethora of forms has been used over the years in high-energy particle measurements. We discuss some here.

Usually, one assumes that the measured correlation function approximately factors into a femtoscopic ( $C_F(\vec{q})$ ) and a non-femtoscopic ( $\zeta(\vec{q})$ ) part

$$C(\vec{q}) = C_F(\vec{q}) \cdot \zeta(\vec{q}) . \quad (3)$$

### 2.1. Femtoscopic forms

Here, we list some fitting forms used in the high energy literature. If non-femtoscopic effects are ignored ( $\zeta = 1$ ), then these are the forms used to fit the measured correlation function.

The one-dimensional HBT radius can be obtained by fitting the correlation function (Eq. (2)) with an analytical parameterization. The most commonly used one, that assumes the Gaussian source distribution, is defined as

$$C_F(Q_{\text{inv}}) = 1 + \lambda e^{-Q_{\text{inv}}^2 R_{\text{inv}}^2} , \quad (4)$$

where  $Q_{\text{inv}} \equiv \sqrt{(\vec{p}_1 - \vec{p}_2)^2 - (E_1 - E_2)^2}$ .

Another parameterization, also assuming a Gaussian shape of the source distribution, but this with time  $Q$  is measured in the lab frame, is

$$C_F(q, q_0) = 1 + \lambda e^{-q^2 R_G^2 - q_0^2 \tau^2}, \quad (5)$$

where  $q = |p_1 - p_2|$ ,  $q_0 = E_1 - E_2$ ,  $R_G$ ,  $\tau$  and  $\lambda$  are the source size, lifetime and chaoticity parameter.

Kopylov and Podgoretsky [10] introduced an alternative parameterization

$$C_F(q_T, q_0) = 1 + \lambda \left[ \frac{2J_1(q_T R_B)}{q_T R_B} \right]^2 (1 + q_0^2 \tau^2)^{-1}, \quad (6)$$

where  $q_T$  is the transverse component of  $\vec{q} = \vec{p}_1 - \vec{p}_2$  with respect to  $\vec{p} = \vec{p}_1 + \vec{p}_2$ ,  $q_0 = E_1 - E_2$ ,  $R_B$  and  $\tau$  are the size and decay constants of a spherical emitting source, and  $J_1$  is the first order Bessel function.

Simple numerical studies show that  $R_G$  from Eq. (5) is approximately twice smaller than  $R_B$  obtained from Eq. (6) (*e.g.* [11, 12]).

With enough statistics, a femtoscopic analysis may be performed in 2- or 3-dimensions. Then, the correlation function is often expressed in the Bertsch–Pratt decomposition [13, 14]

$$C_F(q_o, q_s, q_l) = 1 + \lambda e^{-q_o^2 R_o^2 - q_s^2 R_s^2 - q_l^2 R_l^2}, \quad (7)$$

where,  $\vec{Q} = (q_o, q_s, q_l)$  is defined in the longitudinally co-moving frame,  $q_l$  is the component parallel to the beam axis or trust axis (in  $e^+ + e^-$ ),  $q_o$  is measured in transverse plane and points into the direction of outgoing pair and  $q_s$  is perpendicular to other two components. Analogously, the sizes of the source along these three directions are denoted as  $R_o$ ,  $R_s$  and  $R_l$ .

A similar form is used for two-dimensional correlation functions:

$$C_F(q_T, q_l) = 1 + \lambda e^{-q_T^2 R_T^2 - q_l^2 R_l^2}, \quad (8)$$

where,  $q_T = \sqrt{q_o^2 + q_s^2}$ .

Other groups have used double-Gaussian [15, 16] and exponential [17] fitting functions. A scan of the literature reveals even more. We do not wish to tabulate all the myriad forms used in high energy physics, but simply point out that this lack of a consistent analysis is rather a plague, greatly impeding any effort to make sense of the program as a whole.

## 2.2. Non-femtoscopic forms

Many experiments ignore any non-femtoscopic contributions to the measured correlation function, setting  $\zeta = 1$  (*cf.* Eq. (3)) and simply fitting it with one of the forms above. Of those that account for the long-range

structure, most simply choose a functional form that seems to describe the correlation at large  $Q_{\text{inv}}$ ,  $q$  or  $q_T$ , depending on the coordinate system used in the analysis, *assume* that it extrapolates smoothly into the femtoscopic region, and fit. These *ad hoc* forms have no real foundation in physics, but are simply guesses.

The most popular form, used *e.g.* in [18–22], is

$$\zeta(Q_{\text{inv}}) = 1 + \epsilon Q_{\text{inv}} + \delta Q_{\text{inv}}^2, \quad (9)$$

where  $\epsilon$  and  $\delta$  are free parameters. In some cases, this form was modified by setting  $\delta = 0$  [23–27] or  $\epsilon = 0$  [25].

An alternative one-parameter form was introduced in [7]

$$\zeta(Q_{\text{inv}}) = (1 + \delta Q_{\text{inv}}^2)^{-1}. \quad (10)$$

For simplicity, we expressed above forms as a function of  $Q_{\text{inv}}$  only, however, one can easily replace  $Q_{\text{inv}}$  with *e.g.*  $q$  or  $q_T$ .

While expressing it analytically here would require additional explanation, yet another *ad hoc* form was introduced by STAR [28] to describe the three-dimensional structure of the correlation function with two additional parameters.

Finally, a formulation has recently been proposed [29] which is not *ad hoc*, but provides a full three-dimensional analytic form for  $\zeta$ , assuming that the non-femtoscopic correlations arise from energy and momentum conservation. Recording the formula here would require extensive explanation, so we refer the reader to [29] for details. In this approach, the extra “parameters” are physical quantities like the total multiplicity, average energy, and so forth.

There are still other *ad hoc* forms not listed here. The reader should be impressed (or depressed) by the potential combinatorics of combining the  $\zeta$  terms from this section with the  $C_F$ ’s from Sec. 2.1.

### 3. Two scalings in femtoscopic results in heavy ion collisions

A systematic overview of femtoscopic studies in relativistic heavy ion collisions reveals a broad variety of interesting trends reflecting the underlying system dynamics [3]. Here, we mention two of the most important, shown in Fig. 1.

#### 3.1. Transverse mass dependence

The negative correlation between the femtoscopic sizes and the transverse mass of the particles is usually attributed to collective flow of a bulk system [31]. In such a scenario, approximately “universal”  $m_T$  dependence of femtoscopic radii applies not only to pions but to all particle types. This

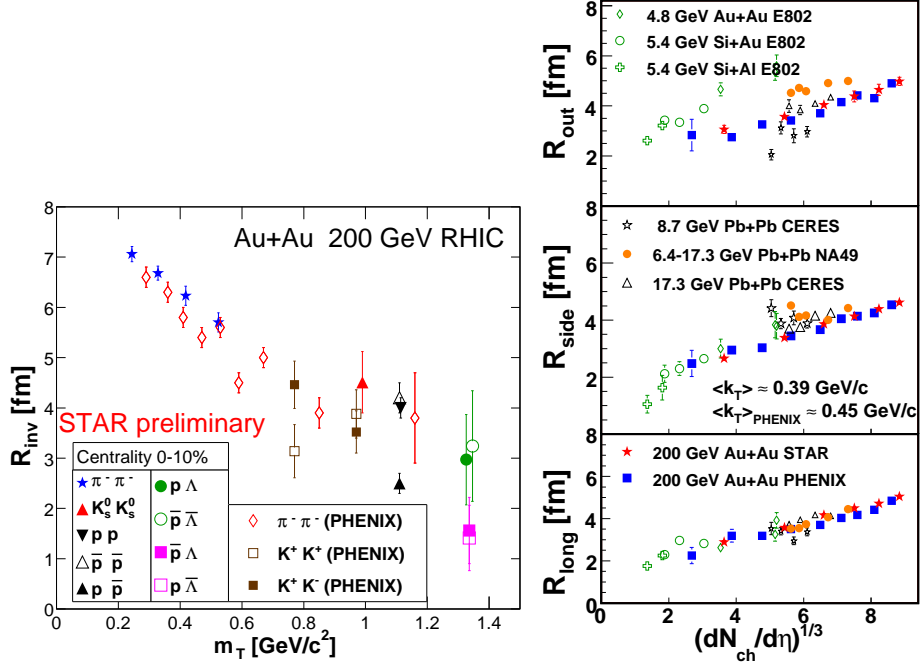


Fig. 1. Left:  $m_T$  dependence of  $R_{\text{inv}}$  for different particles. Figure taken from [28]. Right: Femtoscopic radii dependence on the number of charged particle. Figure taken from [30].

is in fact observed experimentally and results have been presented in the left panel of Fig. 1, in which one-dimensional radii from pion [32], charged kaon [32], neutral kaon [33], proton and anti-proton [34], and proton- $\Lambda$  [35] correlations are plotted. Characteristic signals of collective flow [36] are also observed in correlations between particles with very different masses.

### 3.2. Multiplicity scaling

The right panel of Fig. 1 presents AGS/SPS/RHIC systematics of HBT radii dependence on  $(dN_{\text{ch}}/d\eta)^{1/3}$  ( $N_{\text{ch}}$  — number of charged particles) for different colliding systems at different energies of the collisions. The main motivation for studying such a relation is its connection to the final state geometry through the particle density at freeze-out. As seen, all radii exhibit a scaling with  $(dN_{\text{ch}}/d\eta)^{1/3}$ . It is especially interesting that the radius parameters  $R_{\text{side}}$  and  $R_{\text{long}}$  follow the same trend for different collisions over a wide range of energies and given value of  $\langle k_T \rangle$ . It is a clear signature that the multiplicity is a scaling variable that drives these geometrical radius parameters. Since  $R_{\text{out}}$  mixes space and time information it is not clear whether one expects its simple scaling with the final state geometry [3].

#### 4. World systematics from elementary particle collisions

The Bose–Einstein effect has been studied in elementary particle collisions for decades by various experiments. We made attempt to collect experimental papers on two-particle correlations in small systems in Table I.

TABLE I

Collection of published experimental studies of two-particle correlations in small systems.

System	$\sqrt{s}$ [GeV]	Facility	Experiment	Refs.
$p + p$	1.9	LEAR	CLEAR	[37, 38]
	1.9	CERN	ABBCCLVW	[39]
	7.2	AGS	E766	[8]
	17	SPS	NA49	[40]
	26	SPS	NA23	[7]
	27.4	SPS	NA27	[25]
	31-62	ISR	AFS	[15, 41, 42]
	44,62	ISR	ABCDHW	[43]
	200	SPS	NA5	[44]
	200	RHIC	STAR	[45]
$p + \bar{p}$	53	ISR	AFS	[46]
	200	SPS	NA5	[44]
	200-900	SPS	UA1	[47]
	1800	Tevatron	E735	[12]
$h + p$	5.6	CERN	ABBCCLVW	[39]
	21.7	SPS	EHS/NA22	[5, 48]
$e^+ + e^-$	3-7,29	SLAC	Mark-II	[49]
	10	CESR	CLEO	[4]
	29	SLAC	TPC	[50]
	29-37	DESY-PETRA	TASSO	[51, 52]
	58	TRISTAN	AMY	[53]
	91	LEP	OPAL	[6, 18, 20, 21, 24, 54]
	91	LEP	L3	[24]
	91	LEP	DELPHI	[26, 27, 55–58]
	91	LEP	ALEPH	[22, 59–61]
$e^+p$	300	HERA	ZEUS	[62, 63]
	300	HERA	H1	[17]
$\mu p$	23	CERN	EMC-NA9	[64]
$\alpha + \alpha$	126	ISR	AFS	[15, 41, 42, 46]
$\mu N$	30	Tevatron	E665	[65]
$\nu N$	>10		BBNC	[66]

Here, we will focus on results from elementary particle collisions by looking at the multiplicity and transverse mass dependence of femtoscopic sizes measured in these collisions and we will compare them to those from heavy ion collisions (discussed in Sec. 3).

#### 4.1. Multiplicity dependence

Fig. 2 shows a collection of results from a number of experiments studying  $p + p$ ,  $p + \bar{p}$ ,  $e^+ + e^-$  and even  $\alpha - \alpha$  collisions plotted *versus* the number of charged particles per unit of pseudorapidity.  $R_{\text{inv}}$  (*cf.* Eq. (4)) is plotted on upper panel and  $R_G$  (*cf.* Eq. (5)) on the lower panel of this figure. Since  $R_B \approx 2R_G$  we were able to plot both radii together by dividing  $R_B$  (*cf.* Eq. (6)) radii by a factor of 2. All radii shown are from collisions with  $\sqrt{s} > 40$  GeV and increase with multiplicity. For collisions with  $\sqrt{s} < 40$  GeV, the multiplicity systematics are less clear, since some experiments report a clear relationship and some do not, as shown in detail in Fig. 3.

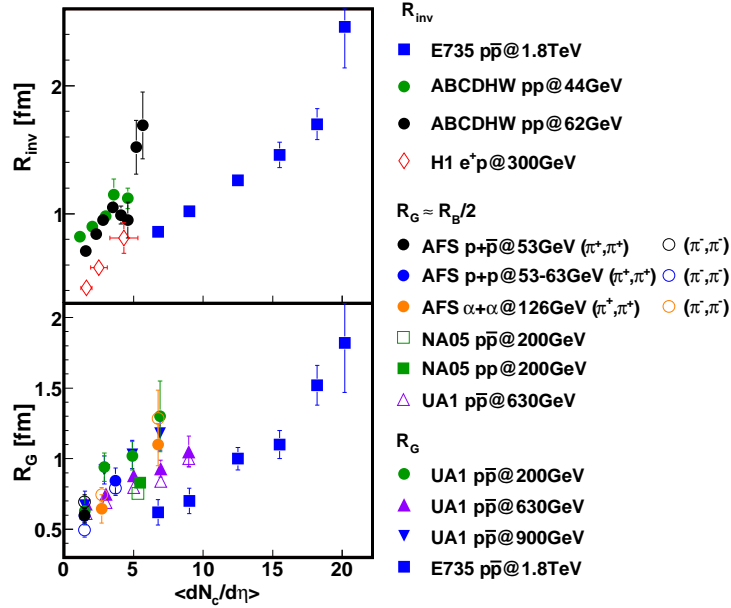


Fig. 2. The multiplicity dependence of the pion HBT radii. Compilation of results from various experiments. Only data from collisions at  $\sqrt{s} > 40$  GeV are shown.

While the multiplicity dependences from different experiments seen in Fig. 2 are qualitatively similar, we do not see the quantitatively universal dependence observed in heavy ion collisions (*cf.* right side of Fig. 1). This may indicate that, in fact, there is no such universal scaling in particle collisions.



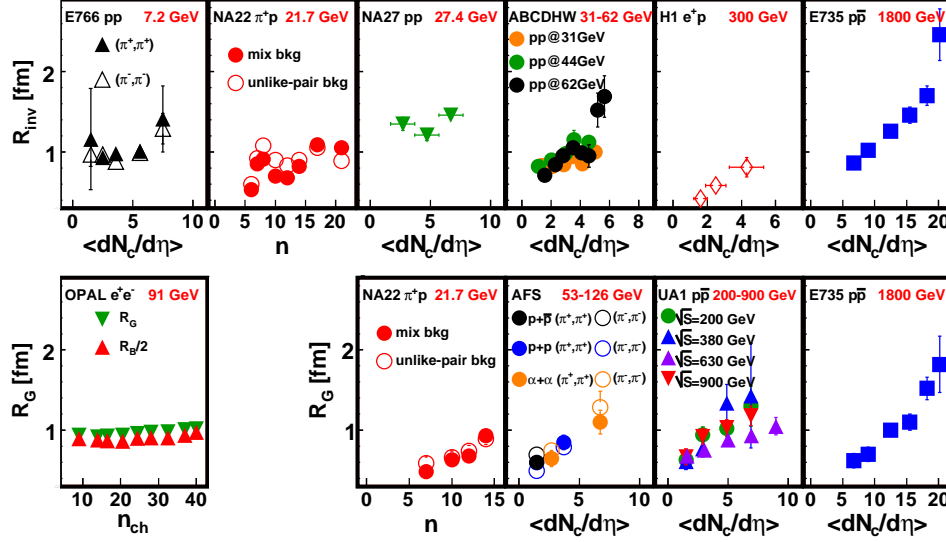


Fig. 3. Multiplicity dependence of pion HBT radii presented as a function of the energy of the collisions from the following experiments E735 [12], ABCDHW [43], UA1 [47], AFS [46], NA5 [44], E766 [8], NA22 [48], NA27 [25], H1 [17], OPAL [67].

However, even if there were a universal multiplicity dependence of femtoscopic scales, there are at least three other reasons for which such a scaling would not appear in Fig. 2.

Firstly, as discussed in Sec. 2, the various experiments use different fitting functions to extract radius parameters. Even when extracting the seemingly straightforward parameter  $R_{\text{inv}}$  with the form given by Eq. (4), different parameterizations of the non-femtoscopic term  $\zeta$  were used to fit the measured correlation function; *cf.* Eq. (3).

Secondly, as discussed below, the femtoscopic scales depend not only on multiplicity, but also kinematic quantities such as transverse momentum  $p_T$ . The various experiments had significantly different acceptances, for which it is hard to account after the fact; this will lead to systematic differences between one experiment's results and another's. For example, the Tevatron experiment E735 [12] was more biased towards high- $p_T$  particles than the other experiments. Thus, one expects E735's radii to be systematically lower (for fixed multiplicity) than the others; this is precisely what is seen in Fig. 2.

Thirdly, these experiments often use quite different definitions of “multiplicity” in their publications, making apples-to-apples comparisons more difficult. As best we could, we tried to compare results as a function of  $dN_{\text{ch}}/d\eta$ . For example, sometimes, the number of all particles per unit rapidity is reported; in such case we assumed that charged particles are two-third

of the total multiplicity. Some experiments provide the number of charged particles in some range of pseudorapidity (not always centered at  $\eta = 0$ ). In such cases we assumed a flat  $\eta$  distribution, and scaled. Additionally, we may expect some unknown bias between experiments in the method of extracting the number of (charged) particles, since these multiplicities were often “raw” numbers, not corrected for efficiency.

In general, one would like to assign systematic errors from all of these effects, to see whether the data are indeed consistent with a “universal” multiplicity dependence. However, beyond what is described above, we were unable to do this from the information published by the collaborations, so settle for a study of trends. We note that these complicating issues are not problems for the heavy ion data, in which the communication between different experiments is good, and apples-to-apples comparisons are seen as a high priority.

#### 4.2. The transverse mass/momentum dependence

In heavy ion collisions, the observed decrease of the HBT radii with increasing transverse mass has been associated with flow, as mentioned in Sec. 3. Whether or not it arises from the same physics, a similar  $m_T$  scaling is observed in small systems, as seen in Fig. 4. Recalling the discussion from

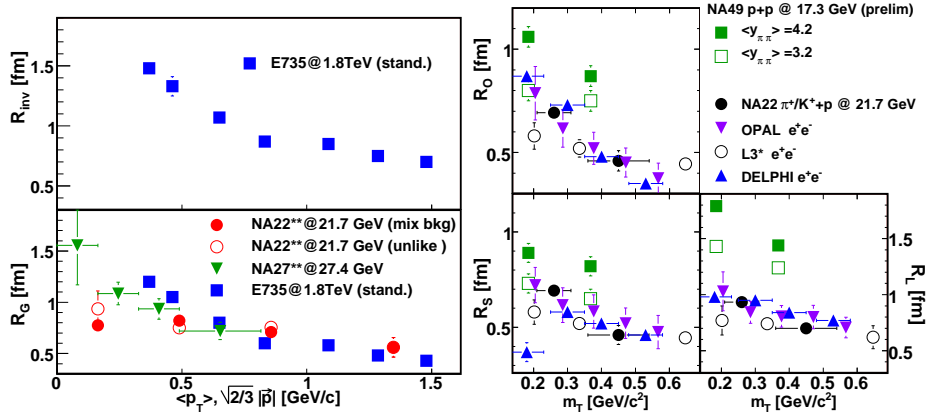


Fig. 4. The transverse mass dependence of  $R_{\text{inv}}$  (left panel) and the 3D HBT radii (right panel) from elementary particle collisions. Data from NA22 [5], NA49 preliminary [40], OPAL [6], L3 [24], DELPHI [55].

Sec. 4.1, we recall that these results come from different experiments, and that high energy particle measurements are hard to compare quantitatively. In particular, the average number of particles per unit of pseudorapidity is different in each experiment and as discussed above, the magnitude of the

HBT radii depends on this value. Additionally, as also discussed previously, somewhat different functional forms were used to extract the radii. However, despite these difficulties it is clear that pion HBT radii in elementary particle collisions show similar dependence on the transverse momentum as seen in heavy ion collisions, though it is not clear whether they have the same origin.

## 5. Some aspects of femtoscopy in elementary particle collisions

In this section, we will discuss two issues with femtoscopic results from elementary particle collisions. The first one is related to experimental data and the second one to the interpretation of these data.

### 5.1. Is there a mass dependence of HBT radii in $e^+ + e^-$ collisions?

As we showed in previous section, the pion HBT radii from small systems are decreasing with increasing transverse *momentum*, similar with the trend from heavy ion collisions. However, in heavy ion collisions, a scaling with transverse *mass* is seen for several particle types, not only pions. Thus, in this section, we would like to investigate whether there is mass dependence of femtoscopic radii in elementary particle collisions. As there is insufficient data available from hadron–hadron collisions we will focus on data from  $e^+ + e^-$  collisions only. There have been claims (*e.g.* [68]) of the mass ordering in these collisions, and these claims have been so often repeated so as to become “common knowledge.” However, we argue that a survey of  $e^+ + e^-$  publications reveals a much less clear picture. We found two main reasons that complicate the systematic comparison of the data. The first is the tendency of  $e^+ + e^-$  experiments to use very different fitting forms for the correlation function, even within one collaboration. We have discussed this extensively above in Sec. 2 and 4, so only mention it here. The second complication comes from the fact that experiments used various techniques to construct the reference distribution ( $B(q)$  from Eq. (2)).

Results from four  $e^+ + e^-$  experiments, ALEPH, DELPHI, OPAL and L3, are presented on Fig. 5. The upper panels show the mass dependence of HBT sizes measured using different techniques to construct the reference distribution from three experiments separately. The bottom panels present the same data but this time grouped by the technique of creating background of the correlation function. We make two interesting observations. Firstly, not a single experiment used the same method to construct the reference distribution for pion, kaon and proton correlation function. Instead, they used different techniques for different particles, and when we consider systematic errors that each technique introduces it is hard to make any quantitative statement about the mass dependence of HBT radii. Secondly, when we group the data by a given method of reference generation (lower panels of Fig. 5), identifying clear trends becomes more difficult.

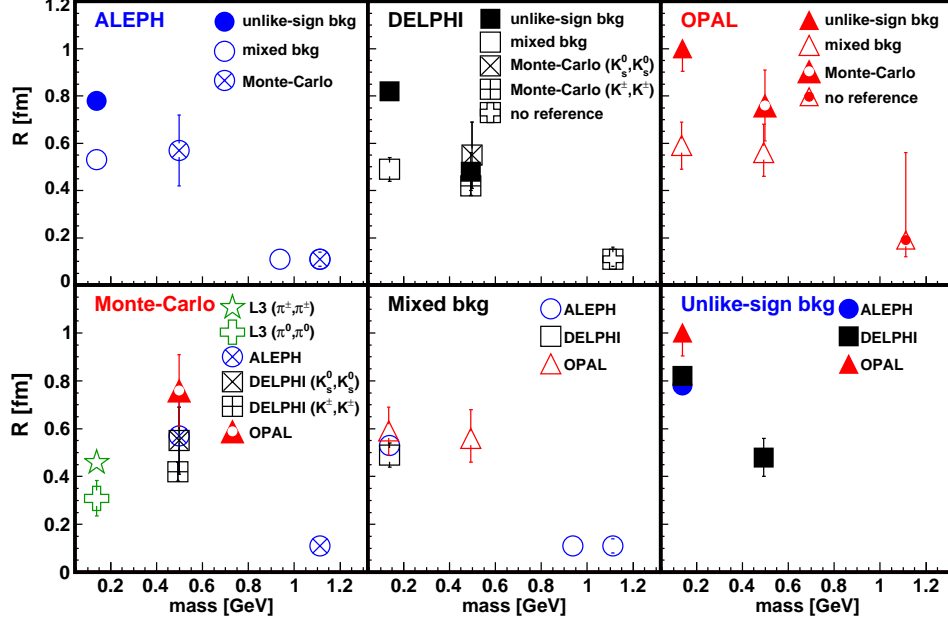


Fig. 5. Results from  $e^+ + e^-$  collisions. Data taken from the following publications: ALEPH — charged pions [60],  $(K_s^0, K_s^0)$ ,  $(p, p)$  and  $(p, \bar{p})$  [22],  $(\Lambda, \Lambda)$  and  $(\bar{\Lambda}, \bar{\Lambda})$  [61]; DELPHI — charged pions [26,27], charged kaons [56],  $(K_s^0, K_s^0)$  [57] and  $(\Lambda, \Lambda)$  [58]; OPAL — neutral pions [21],  $(\Lambda, \Lambda)$  [54], charged kaons [20],  $(K_s^0, K_s^0)$  [18] and L3 [24].

We believe that the only fair statement that we can make about the femtoscopic results from  $e^+ + e^-$  collisions is that the size of the source for mesons is larger than that for baryons. As Alexander pointed out [69], this alone is a big problem for the Lund string model.

### 5.2. Heisenberg uncertainty principle

The Heisenberg uncertainty principle was pointed by Alexander *et al.* [68] as a possible origin of the mass dependence of the HBT radii in  $e^+ + e^-$  collisions. Authors argue that the dependence of the one-dimensional radius  $R_{\text{inv}}$  on the hadron mass obtained at LEP is consistent with the formula derived from the Heisenberg uncertainty principle

$$R_{\text{inv}}(m) = \frac{c\sqrt{\hbar\Delta t}}{\sqrt{m}}, \quad (11)$$

where  $\Delta t$  was chosen to be  $10^{-24}$  s (0.3 fm).

However, the good agreement between the experimental data and the theoretical curve (Eq. (11)) presented by the authors [68] raises a concern especially in the light of what we just discussed in the previous section. In fact, when all results from  $e^+ + e^-$  experiments are plotted together as done on Fig. 6, we see that it is difficult to make any quantitative statement about the mass ordering of HBT radii due to a significant systematic bias on the radii coming from the different techniques to construct the background of the correlation function and the spread of results coming from different experiments.

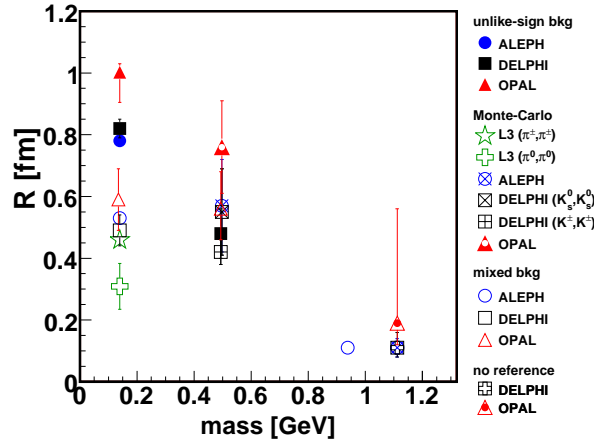


Fig. 6. Combined results from  $e^+ + e^-$  collisions. The same data as plotted on Fig. 5.

The one-dimensional radius provides limited information about the source that is in fact a three-dimensional distribution. Thus, to verify whether the Heisenberg uncertainty principle can really explain the  $m_T$  dependence of HBT radii one should look at three- (eventually two-) dimensional radii. The DELPHI experiment [55] report a decrease of three-dimensional radii with  $m_T$ ; *cf.* Sec. 4. Alexander argues [70] that the Heisenberg uncertainty principle can best be applied to the longitudinal component of the radius  $r_z$  (sometimes noted as  $R_L$ ) measured in LCMS frame using similar procedure as it was done for  $R_{inv}$  [68]. The final formula gives approximately the same dependence of the longitudinal femtoscopic size on  $m_T$  as in the one-dimensional case

$$R_L(m_T) \approx \frac{c\sqrt{\hbar\Delta t}}{\sqrt{m_T}}. \quad (12)$$

A very good agreement between the experimental data and the theoretical curve is observed. Two points, however, about this figure.

Firstly, we believe that such a good agreement is partially a result of a mistake that was made by Alexander. The author cites the following paper [71] as the source of the DELPHI results. However, we found that there is a discrepancy between the numbers presented in this paper and those plotted by Alexander. The main (and probably the only) difference seems to be in the last point for  $m_T = 0.81$  GeV where Alexander's point is significantly larger than DELPHI published value. The correct DELPHI data from [71] has been plotted on the right panel of Fig. 7 represented by red circles. For comparison, we plot the prediction from the Heisenberg uncertainty relations (Eq. (12) for  $\Delta t = 2.1 \times 10^{-24}$  s (0.63 fm) represented by the solid black line. This is the same curve plotted on the left panel of Fig. 7. We also performed a new fit to the correct DELPHI data (red circles) and found that the best  $\Delta t$  that described the results is  $1.9 \times 10^{-24}$  s (0.57 fm) (dash-dotted red line). This is not a serious mistake; however the agreement between Eq. (12) and experimental data is not as good as Alexander showed in his paper [70]. Thus, we think that this problem should be mentioned here since the figure plotted on the left panel of Fig. 7 was copiously cited in many other publications, including review articles (*e.g.* [9, 72]).

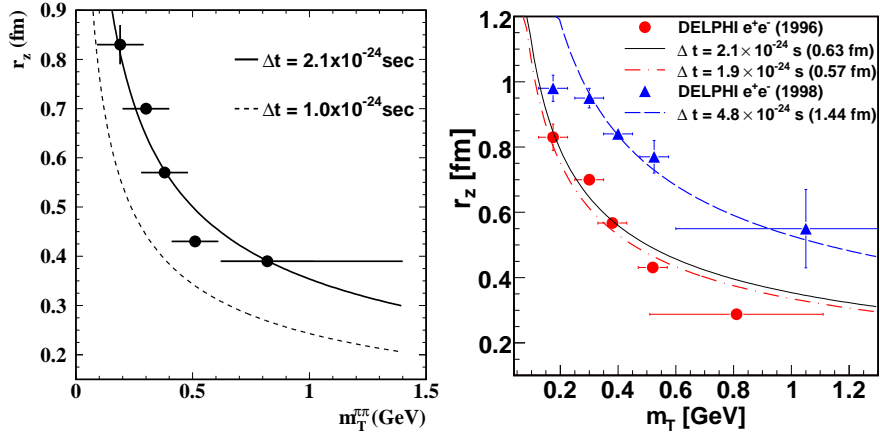


Fig. 7. The  $m_T$  dependence of the longitudinal component of the HBT source size for pions; Left panel: Figure taken from [70]; Right panel: Red circles represent DELPHI data taken from [71]. Blue triangles show data points from [55]. See text for explanation of the curves.

The second point is about the DELPHI data themselves. While Alexander refers to results presented in 1996 and published as proceedings in 1997 [71] it should be also mentioned that the same experiment published, also as conference proceedings, newer results in 1999 and presented in 1998 [55] that are different than the previous ones (see blue triangles on the right panel of Fig. 7). Presumably, the later DELPHI results represent

an improvement over the earlier ones, so it seems reasonable to use these. Until a final result is published in a refereed journal we have fitted newer DELPHI data [55] using Eq. (12) and found that the best value of the  $\Delta t$  that described the experimental data is  $4.8 \times 10^{-24}$  (1.44 fm).

In heavy ion collisions, it is primarily the *transverse* HBT radii which reflect collective flow. Thus, it is interesting to try to understand the  $m_T$ -dependence of femtoscopic parameters like  $R_T$ ,  $R_O$  and  $R_S$ . Any direct connection between these radii and the uncertainty principle is unclear.

## 6. Discussion and conclusions

Claims that we understand the physics of ultrarelativistic heavy ion collisions become much more compelling when a clear comparison of data from  $A + A$  and  $p + p$  collisions are made and the differences explained. The importance of such a comparison is obvious in studies of jet suppression from azimuthal correlations [73] and leading particle distributions [74, 75] at large  $p_T$ .

In this article, we presented a comprehensive review of the world systematics of the femtoscopic results from elementary particle collisions and identified trends seen in the data.

The transverse mass dependence of the femtoscopic results from heavy ion collisions at RHIC is an important evidence of flow [76]. Surprisingly, a similar dependence is observed in small systems, as presented in Sec. 4.2. It has been even shown that not only there is the  $m_T$  dependence of HBT radii in  $p + p$  collisions at RHIC but it is very similar to what is seen in  $d + \text{Au}$  and  $\text{Au} + \text{Au}$  collisions suggesting that the only scaling between small and big system is the size [45]. However, it is unclear whether the  $m_T$  dependence in small systems originates also from “flow”. Even though such explanation has been provided by Csörgő and collaborators [77] the literature includes rather alternative explanations. Among them is the Heisenberg uncertainty principle suggested by Alexander *et al.* [68, 70]. However, as demonstrated and discussed in Sec. 5, a more detailed study of the results from  $e^+ + e^-$  collisions complicates the quantitative comparisons of the data and thus the interpretation. Another physics process that could potentially generate the space-momentum correlations in small systems was the string fragmentation. However, as pointed out by Alexander [69], the mass dependence of the experimentally observed HBT radii (especially the difference between mesons and baryons) is a big problem for the Lund string model. Long-lived resonances (*e.g.*  $\omega$ ) may also generate a  $m_T$  dependence of femtoscopic radii [78]. However, since the resonance “halo” length scale is fixed (*e.g.*  $c\tau_\omega$ ) and the “core” scale varies by  $\approx 6x$  between  $\text{Au} + \text{Au}$  and  $p + p$ , one expects a different resonance-induced effect in the two systems. Preliminary studies [79] with

the THERMINATOR [80] model confirm this expectation. The different effect of resonances on the transverse mass dependence of the HBT radii on the system size has been demonstrated by THERMINATOR event generator [79]. Finally, Białas *et al.* provided a model [81] that assumes a proportionality between the four-momentum and the four-vector describing the particle space-time position at freeze-out and showed that it can explain the data from  $e^+ + e^-$  collisions.

Other interesting observations come from the multiplicity dependence of femtoscopic radii in elementary particle collisions. Surprisingly the results from various experiments show the increase of the femtoscopic sizes with the number of charged particles similarly as it has been observed in heavy ions collisions. These results should not be used as an evidence of flow but they strongly suggest that the only difference between the size of the source in small and big systems is the average multiplicity. Such observation in heavy ion collisions suggests that the system is entropy dominated [82].

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