

TEST OF BOWLER–SINYUKOV TREATMENT  
OF COULOMB INTERACTION\*RADOSŁAW MAJ<sup>a</sup>, STANISŁAW MRÓWCZYŃSKI<sup>a,b</sup><sup>a</sup>Institute of Physics, Jan Kochanowski University  
Świętokrzyska 15, 25-406 Kielce, Poland<sup>b</sup>Andrzej Sołtan Institute for Nuclear Studies  
Hoża 69, 00-681 Warsaw, Poland*(Received February 4, 2009)*

The Bowler–Sinyukov method to eliminate Coulomb interaction from a two-particle correlation function is discussed and tested.

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**1. Introduction**

The correlation functions of two particles with ‘small’ relative momenta provide information about space-time characteristics of particle’s sources in high-energy nucleus–nucleus collisions [1–3]. Within the standard ‘femtoscropy’ method, one obtains parameters of a particle’s source fitting experimental correlation functions with theoretical ones calculated in a given model. Since we usually deal with electrically charged particles, observed two-particle correlations are strongly influenced by the Coulomb interaction. The effect of the Coulomb force is eliminated from experimental data by means of the so-called Bowler–Sinyukov procedure [4, 5].

The femtoscopy was applied to a large volume of experimental data on nucleus–nucleus collisions at SPS energy [2]. The spatial size of particle’s sources appeared to be comparable to the expected size a fireball created in nucleus–nucleus collisions while the emission time of particles was significantly shorter. It was predicted that at RHIC energies the emission time would be significantly longer due to the long lasting hydrodynamic evolution of the system created at the early stage of nucleus–nucleus collisions [6, 7]. To a big surprise the experimental data obtained at RHIC [8, 9] show a very

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little, if any, change of the space-time characteristics of a fireball when compared to the SPS data. And in contradiction to hydrodynamic models the emission time of particles appeared to be as short as 1 fm/ $c$ . Because of this surprising result, which is known as the ‘HBT Puzzle’ [10, 11], a reliability of the femtoscopy method was questioned. Very recently it has been shown that the hydrodynamic calculations can be modified to give rather short emission time of produced particles [12, 13], and thus the ‘HBT Puzzle’ seems to be resolved. Nevertheless it is still of interest to quantitatively check the femtoscopy method.

Our aim here is to test the Bowler–Sinyukov correction procedure which is used to eliminate the Coulomb interaction from the experimental data. The procedure assumes that the Coulomb effects can be factorized out. The correction’s factor is calculated for a particle’s source which is spherically symmetric and has zero lifetime. We examine the procedure applying it to the computed Coulomb correlation functions of identical pions coming from anisotropic sources of finite lifetime. The effect of halo [17] is also studied. We treat the computed Coulomb correlation functions as experimentalists deal with the measured correlation functions. Thus, we extract the correlation function which is supposed to be free of the Coulomb interaction. However, in contrast to the situation of experimentalists we know actual parameters of particle sources which can be compared to the extracted ones. Our study is somewhat similar to that presented in [14].

We use the natural units, where  $c = \hbar = 1$ , and our metric convention is  $(+, -, -, -)$ .

## 2. Coulomb correlation function

We compute the correlation function using the well known Koonin formula [16]. Since the two particles of interest are described by means of nonrelativistic wave function, the computation is performed in the center-of-mass frame of the pair, as the pair motion can be treated as nonrelativistic in this frame. However, the source function, which gives a probability to emit two particles at a given space-time distance  $(t, \mathbf{r})$ , has to be transformed to the pair center-of-mass frame (where quantities are labeled with asterisks). The correlation function thus equals

$$C(\mathbf{q}_*) = \int d^3r_* D_r(\mathbf{r}_*) |\varphi_{\mathbf{q}_*}(\mathbf{r}_*)|^2, \quad (1)$$

where  $\varphi_{\mathbf{q}_*}(\mathbf{r}_*)$  is the non-relativistic wave function of relative motion and  $D_r(\mathbf{r}_*)$  is the effective source function

$$D_r(\mathbf{r}_*) \equiv \int dt_* D_r(t_*, \mathbf{r}_* - \mathbf{v}_* t_*), \quad (2)$$

where  $D_r(t_*, \mathbf{r}_*)$  is the ‘relative’ source function. Obviously, the velocity of the pair in its center-of-mass frame vanishes ( $\mathbf{v}_* = 0$ ). The ‘relative’ source function is defined through the single-particle source function as

$$D_r(\mathbf{r}, t) \equiv \int d^3R dT D(\mathbf{R} - \tfrac{1}{2}\mathbf{r}, T - \tfrac{1}{2}t) D(\mathbf{R} + \tfrac{1}{2}\mathbf{r}, T + \tfrac{1}{2}t). \quad (3)$$

As a probability density, the source function is normalized to unity

$$\int d^3r dt D(t, \mathbf{r}) = \int d^3r dt D_r(t, \mathbf{r}) = \int d^3r D_r(\mathbf{r}) = 1. \quad (4)$$

The Coulomb function of two non-identical particles interacting due to repulsive Coulomb force is well-known [15] to be

$$\varphi_{\mathbf{q}}(\mathbf{r}) = e^{-\frac{\pi\eta}{2q}} \Gamma\left(1 + i\frac{\eta}{q}\right) e^{i\mathbf{q}\mathbf{r}} F\left(-i\frac{\eta}{q}, 1, i(qr - \mathbf{q}\mathbf{r})\right), \quad (5)$$

where  $q \equiv |\mathbf{q}|$  and  $1/\eta$  is the Bohr radius of two-particle system which equals  $\eta_\pi^{-1} = 388$  fm for  $\pi\pi$ ;  $F$  denotes the hypergeometric confluent function. As we deal with pairs of identical bosons, the wave function  $\varphi_{\mathbf{q}}(\mathbf{r})$  is symmetrized.

We choose the Gaussian form of the single-particle source function  $D(t, \mathbf{r})$  but in order to easily transform it from the source rest frame to the center-of-mass frame of the pair, we write it down in the Lorentz covariant form

$$D(x) = \frac{\sqrt{\det\Lambda}}{4\pi^2} \exp\left[-\frac{1}{2}x_\mu \Lambda^{\mu\nu} x_\nu\right], \quad (6)$$

where  $x^\mu = (t, \mathbf{r})$  is the position four-vector and  $\Lambda^{\mu\nu}$  is the Lorentz tensor depending on the parameters  $\tau$ ,  $R_x$ ,  $R_y$  and  $R_z$ , which characterize the life-time and sizes of the source. In the source rest frame the matrix is diagonal with the  $\tau^{-2}$ ,  $R_x^{-2}$ ,  $R_y^{-2}$  and  $R_z^{-2}$  along the diagonal. The source function as written in Eq. (6) obeys the normalization condition (4) not only for the diagonal matrix  $\Lambda$  but for non-diagonal as well. The source function (6) is evidently the Lorentz scalar that is

$$D'(x') = \frac{\sqrt{\det\Lambda'}}{4\pi^2} \exp\left[-\frac{1}{2}x'_\mu \Lambda'^{\mu\nu} x'_\nu\right] = \frac{\sqrt{\det\Lambda}}{4\pi^2} \exp\left[-\frac{1}{2}x_\mu \Lambda^{\mu\nu} x_\nu\right] = D(x),$$

where  $x'_\mu = L_\mu^\nu x_\nu$  and  $\Lambda'^{\mu\nu} = L^\mu_\sigma \Lambda^{\sigma\rho} L_\rho^\nu$  with  $L^\mu_\sigma$  being the matrix of Lorentz transformation. We note that  $\det\Lambda' = \det L \det\Lambda \det L^{-1} = \det\Lambda$ .

The correlation function of two identical noninteracting bosons should equal 2 at vanishing momentum ( $C_{\text{free}}(\mathbf{q} = 0) = 2$ ) but free correlation

functions extracted from experimentally measured ones appear to be significantly smaller than 2 at  $\mathbf{q} = 0$ . There was introduced the idea of halo [17] to explain this fact. It is assumed that only a fraction  $f$  of particles contributing to the correlation function comes from the fireball while the remaining fraction  $(1-f)$  originates from long living resonances ( $0 \leq f \leq 1$ ). Then, we have two sources of the particles: the small one — the fireball and the big one corresponding to the long living resonances. The single-particle source function thus equals

$$D(t, \mathbf{r}) = f D_f(t, \mathbf{r}) + (1-f) D_h(t, \mathbf{r}), \quad (7)$$

where  $D_f(t, \mathbf{r})$  and  $D_h(t, \mathbf{r})$  represent the fireball and halo, respectively. If the halo radius  $R_h$  is so large that  $R_h^{-1}$  is below an experimental resolution of the relative momentum  $\mathbf{q}$ , the particles coming from halo do not contribute to the measured correlation function and one claims that  $C_{\text{free}}(\mathbf{q} = 0) = 1 + \lambda$ , where  $\lambda \equiv f^2 < 1$ .

We have computed the Coulomb correlation functions for anisotropic Gaussian sources of finite emission time. The halo has been also included. We use the Bertsch–Pratt coordinates [18, 19] *out*, *side*, *long*. These are the Cartesian coordinates, where the direction *long* is chosen along the beam axis ( $z$ ), the *out* is parallel to the component of the pair momentum which is transverse to the beam. The last direction — *side* — is along the vector product of the *out* and *long* versors. So, the vector  $\mathbf{q}$  is decomposed into the  $q_o$ ,  $q_s$ , and  $q_l$  components. If the particle's velocity is chosen along the axis  $x$ , the out direction coincides with the direction  $x$ , the side direction with  $y$  and the long direction with  $z$ .

### 3. The Bowler–Sinyukov procedure

The Coulomb effect is usually subtracted from the experimentally measured correlation functions by means of the Bowler–Sinyukov procedure. In the absence of halo, procedure assumes that the correlation function can be expressed as

$$C(\mathbf{q}) = K(q) C_{\text{free}}(\mathbf{q}), \quad (8)$$

where  $C_{\text{free}}(\mathbf{q})$  is the free correlation function and  $K(q)$  is the correction factor which depends only on  $q \equiv |\mathbf{q}|$ . The correction factor can be treated as the Coulomb correlation function of two nonidentical particles of equal masses and charges. The function is, however, rather unphysical as the pair velocity with respect to the source is assumed to vanish even so the calculation is performed in the rest frame of the source where the source is assumed to be symmetric and of zero lifetime. The correction factor  $K(q)$ , which is described in detail in the Appendix to the paper [14], is computed

as

$$K(q) = G(q) \int d^3r D_r(\mathbf{r}) \left| F \left( -\frac{i\eta}{q}, 1, i(qr - \mathbf{q}\mathbf{r}) \right) \right|^2, \quad (9)$$

where  $G(q)$  is the so-called Gamov factor equal

$$G(q) = \frac{2\pi\eta}{q} \frac{1}{\exp\left(\frac{2\pi\eta}{q}\right) - 1} \quad (10)$$

and  $D_r(\mathbf{r})$  describes the spherically symmetric Gaussian source of zero life-time and of the ‘effective’ radius  $R = \sqrt{(R_o^2 + R_s^2 + R_l)/3}$  where  $R_o$ ,  $R_s$  and  $R_l$  are the actual source radii.

To check the validity of Eq. (8), we have divided the computed Coulomb correlation function by the Correction factor  $K(q)$ . For the case of pion–pion correlations, the extracted free correlation function is almost identical with the actual correlation function of non-interaction particles. The procedure works very well even for strongly anisotropic sources.

The situation is more complex when the halo is taken into account. We test two versions of the Bowler–Sinyukov procedure: the dilution method and the proper Bowler–Sinyukov one. The experimentally measured correlation functions  $C(\mathbf{q})$  are fitted as

$$C(\mathbf{q}) = \begin{cases} (1 - \lambda + \lambda K(q))[1 + \lambda(C_{\text{free}}(\mathbf{q}) - 1)] & \text{for dilution,} \\ 1 - \lambda + \lambda K(q)C_{\text{free}}(\mathbf{q}) & \text{for Bowler–Sinyukov.} \end{cases} \quad (11)$$

The correlation function  $C_{\text{free}}(\mathbf{q})$  extracted by means of the dilution and Bowler–Sinyukov procedures are shown in Fig. 1. The expected free function is shown for comparison. The source parameters are given in the figures.

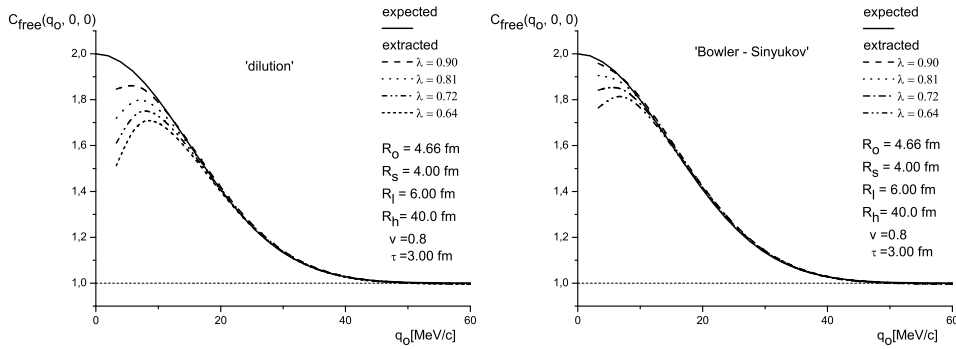


Fig.1. The free correlation function  $C_{\text{free}}(q_0, 0, 0)$  extracted from the Coulomb correlation function by means of the dilution (left) and Bowler–Sinyukov (right) procedure for various  $\lambda$ . The expected free correlation function is also shown.

The parameter  $\lambda$  is assumed to be known when  $C(\mathbf{q})$  are fitted. We show here only the function  $C_{\text{free}}(\mathbf{q})$  for  $\mathbf{q} = (q_o, 0, 0)$  which is crucial for the emission time determination. As seen, the extracted correlation function is distorted at small relative momenta but the width of the correlation function is unaltered and so are the source parameters. In our paper [20] we present a very detailed analysis of the Bowler–Sinyukov procedure. In particular, we show there that for the kaon–kaon correlations it works significantly worse than for the pion–pion ones.

We conclude our study as follows. In the absence of halo the Bowler–Sinyukov procedure works very well for  $\pi\pi$  correlations. When the halo is taken into account the extracted correlation functions are distorted at small relative momenta but the source parameters are still reproduced accurately.

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