THERMALIZATION OF A COLOR GLASS CONDENSATE IN A PARTONIC CASCADE*

Andrej El, Zhe Xu, Carsten Greiner

Goethe-Universität
Senckenberganlage 31, 60325 Frankfurt am Main, Germany

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We investigate thermalization of a longitudinally expanding color glass condensate with Bjorken boost invariant geometry within microscopical parton cascade BAMPS. Our main focus lies on comparison of thermalization processes, observed in BAMPS with the picture suggested in the "Bottom–Up" scenario. Contrary to the "Bottom–Up" scenario, soft and hard gluons thermalize at the same time. No significant increase of the net gluon number is observed with both RHIC and LHC relevant initial conditions. The time scale of thermal equilibration in BAMPS calculations is of the order of $\alpha_{\rm s}^{-2}(\ln\alpha_{\rm s})^{-2}Q_{\rm s}^{-1}$. After this time the gluon system exhibits nearly hydrodynamical behavior.

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1. Introduction

Applicability of ideal hydrodynamics [1] to explanation of large values of the elliptic flow measured in Au+Au collisions at RHIC [2, 3] implies that a thermal equilibration is achieved on a short time scale and the shear viscosity to entropy density ratio of the produced medium is small. Moreover, the thermal state is maintained until hadronisation. A study of the mechanisms which drive the quark-gluonic system to equilibrium is thus of great interest. Coherent quantum effects like color instabilities [4] may play an important role at the very early stage when the system is super dense. As the system becomes more dilute due to the strong longitudinal expansion, perturbative QCD (pQCD) bremsstrahlung processes become essential for momentum isotropisation of quark–gluon matter [5, 6] and are responsible for the low value of the shear viscosity to entropy density ratio [8, 9].

As a possible initial state of the quark–gluon matter produced in high energy heavy ion collisions Color Glass Condensate (CGC) has been proposed [10, 11] and numerous studies of its evolution have been done. Ther-

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malization of an idealized form of color glass condensate [12] was studied in [13] by solving the Landau transport equation. Same topic has been investigated in [14] using the relaxation time approximation to simplify the collision term in the Boltzmann equation. A conclusion from these two studies is that pQCD $gg \to gg$ collisions are not sufficient to achieve thermal equilibrium. The importance of inelastic $gg \leftrightarrow ggg$ processes has been emphasized in the so called "Bottom–Up" scenario [15] of thermalization. The main aspect of the "Bottom–Up" scenario is the difference between the timescales of thermalisation of 'soft' and 'hard' gluons. In the "Bottom–Up" scenario 'soft', *i.e.* gluons with transverse momenta $p_{\mathrm{T}} < Q_{\mathrm{S}}$, where Q_{S} is the characteristic scale of the CGC, are produced in pQCD $gg \to ggg$ processes and thermalize first. "Bottom–Up" scenario predicts a strong parametric enhancement of 'soft' gluon number. Thermalization of 'hard' gluons proceeds on a longer time scale as they loose their energy to the thermal bath built up by 'soft' particles. A parametric time scale for overall thermalization is given by $\tau_{\mathrm{th}} \sim \alpha_{\mathrm{s}}^{-13/5} Q_{\mathrm{s}}^{-1}$ [15].

In this work we present our results on thermalization of both simple CGC proposed in [12] and already discussed by us in [7] and the so called KLN form of CGC [16,17]. We will show that using both forms of a CGC [12,16,17] with the BAMPS partonic cascade [5, 6] a thermalization on a short time scale (1 fm/c) is observed. The η/s ratio, extracted from BAMPS calculations with a CGC initial condition is small, which explains the observed quasi ideal hydrodynamic behavior at later (around 1 fm/c) times.

2. Parton cascade BAMPS

We will use the on-shell partonic cascade BAMPS presented in [5, 6] to solve the Boltzmann Equation for gluons. BAMPS has been recently used to study various topics on gluonic matter produced in Au+Au collisions at RHIC energy: thermalization has been studied in [5–7], elliptic flow v_2 in [18, 19], the energy loss in [20] and viscous shock waves are presented in [21].

In this work we investigate a Bjorken-type one-dimensional (0+1) expansion [22] in a tube with a radius of $R = 5$ fm. The transverse wall of the tube serves as a boundary to mimic one-dimensional (longitudinal) expansion. Gluons are simply reflected on the cylindrical wall. Longitudinally, space is divided in Δz bins, which have the same width in the space time rapidity $\eta = \frac{1}{2}\ln((t+z)/(t-z))$. $\Delta\eta = 0.2$ is set to be a constant for all Δz bins. The initial gluons are put into rapidity interval $[-3; 3]$. The results presented in this paper are extracted from the central rapidity bin $\eta \in [-0.1 : 0.1]$, which is the local rest frame of the medium.

Gluon interactions included in BAMPS are elastic pQCD $gg \to gg$ scatterings as well as pQCD inspired bremsstrahlung $gg \leftrightarrow ggg$. The details on the matrix elements applied to calculate differential cross sections and collision probabilities are given in [5]. The strong coupling α_s is a constant in our calculations.

3. CGC initial conditions

We apply the initial condition a gluon distribution of a color glass condensate [10]. The theory of a color glass condensate is given by the saturation picture, which assumes that the parton distribution in a hadron or nuclei saturates at high energies as a result of competition between QCD bremsstrahlung and annihilation processes. The initially present gluons are produced by the nonperturbative part of the nucleus–nucleus interaction. The CGC is characterized by the saturation momentum scale Q_S, on which the parton density is saturated, *i.e.* grows only slow (logarithmically). We will use two different types of CGC initial conditions, which will be discussed in the following sections.

3.1. Simplified Color Glass Condensate

This form of CGC has been used in our previous work [7] as well as in [13] and introduced in [12]. It is given by the boost-invariant form

$$f(x,p) = \frac{c}{\alpha_\mathrm{s} N_c} \frac{1}{\tau} \delta(y-\eta) \Theta\left(Q_\mathrm{s}^2 - p_\mathrm{T}^2\right) . \tag{1}$$

We take $N_c = 3$ for SU(3). The factor c in (1) is the "parton liberation coefficient" which accounts for the transformation of virtual partons in the initial state into on-shell partons in the final state, as introduced in [16]. The value of c for SU(3) gauge theory is $c \simeq 0.4$ [23, 24]. The initial gluons are produced at eigentime $\tau \sim \frac{1}{Q_\mathrm{S}}$. The initial particle density in this CGC approach is given by [12, 13]

$$\frac{1}{\pi R^2} \frac{dN}{d\eta} = c \frac{N_c^2 - 1}{4\pi^2 \alpha_\mathrm{s} N_c} Q_\mathrm{s}^2 . \tag{2}$$

For the application of the Boltzmann equation, we need the phase space density to be smaller than unity. If phase space density is high, Bose enhancement factors $(1 + f)$ should be considered in the collision integrals, which is not done in BAMPS model. Hence, as long as f is larger than 1 we underestimate the collision rates. With (1) initial transverse spectrum is a theta function, *i.e.* the momentum region $p_\mathrm{T} > Q_\mathrm{S}$ is empty and a sharp edge at $p_\mathrm{T} = Q_\mathrm{S}$ exists. This initial condition is clearly not realistic since

one would expect some smooth transition between the two momentum regions separated by Q_S. We will present results with $Q_S = 2, 3, 4$ GeV and different values of α_s. We will refer to this type of initial condition as CGC1.

3.2. KLN approach to Color Glass Condensate

Here we use the CGC initial condition of the form implemented in [17]. This form of CGC is based on the so called KLN approach [16]. The saturation scale Q_S is now dependent on the momentum fraction x of the gluons. For $p_T^2 < Q_S^2$ gluon distribution is saturated and increases only logarithmically for small p_T. For $p_T^2 > Q^2$ the distribution is given by a power law $f(p_T) \sim 1/p_T^4$. We will present results for Au+Au collision at 200 AGeV center of mass energy. We will refer to this type of initial condition as CGC2.

4. Results: Thermalization of a CGC

Here we discuss thermalization of CGC in our partonic cascade BAMPS and compare the results with the predictions of the "Bottom–Up" scenario. To estimate thermalization times we show the scaled temperature $T\,\tau^{1/3}$ in Fig. 1(a)–(b) (with $T = e/3n$, e denoting the energy and n the particle densities). If this value saturates, the system is (almost) thermalized, since $T \sim \tau^{-1/3}$ behavior is ideal hydrodynamic behavior and even dissipative systems relaxate at late times to ideal hydrodynamics. The thermalization times $t(Q_S)$ with CGC1 initial condition from Fig. 1(a) read as follows: $t_{th}(2\text{ GeV}) = 1.2$ fm/c, $t_{th}(3\text{ GeV}) = 0.75$ fm/c, $t_{th}(4\text{ GeV}) = 0.55$ fm/c. For the fixed $Q_S = 3$ GeV and $\alpha_s = 0.1$–0.3 (Fig. 1(b)) $t(\alpha_s)$ are: $t_{th}(0.1) = 1.75$ fm/c, $t_{th}(0.2) = 1$ fm/c, $t_{th}(0.3) = 0.75$ fm/c. From these thermalization times we can conclude, that the dependence of thermalization time on α_s $t_{th} \sim \alpha_s^{-13/5} Q_s^{-1}$ predicted by "Bottom–Up" scenario is wrong and reads in BAMPS $t_{th} \sim \alpha_s^{-2} (\ln \alpha_s)^{-2} Q_s^{-1}$. This is due to inelastic processes, as has been discussed in [6]. In Fig. 1(a) we observe, that CGC2 initial condition behaves similar to CGC1 with $Q_S \approx 3$ GeV and thermalization proceeds even faster (with $\alpha_s = 0.3$).

The evolution of the particle multiplicity $dN/d\eta$ is shown in Fig. 1(c). No increase of total particle number can be observed, in contrast to the "Bottom–Up" scenario. Instead, if using CGC1 initial condition, we observe particle annihilation at early times. During this time hard particles with $p_T > Q_S$ are produced, since they are missing in the initial distribution. Using CGC2 initial condition, where the hard sector is not empty initially, we observe only slight increase of particle number. The transverse spectra from simulations with CGC1 initial condition are shown in Fig. 1(d). The thermal fits are calculated using the distribution function

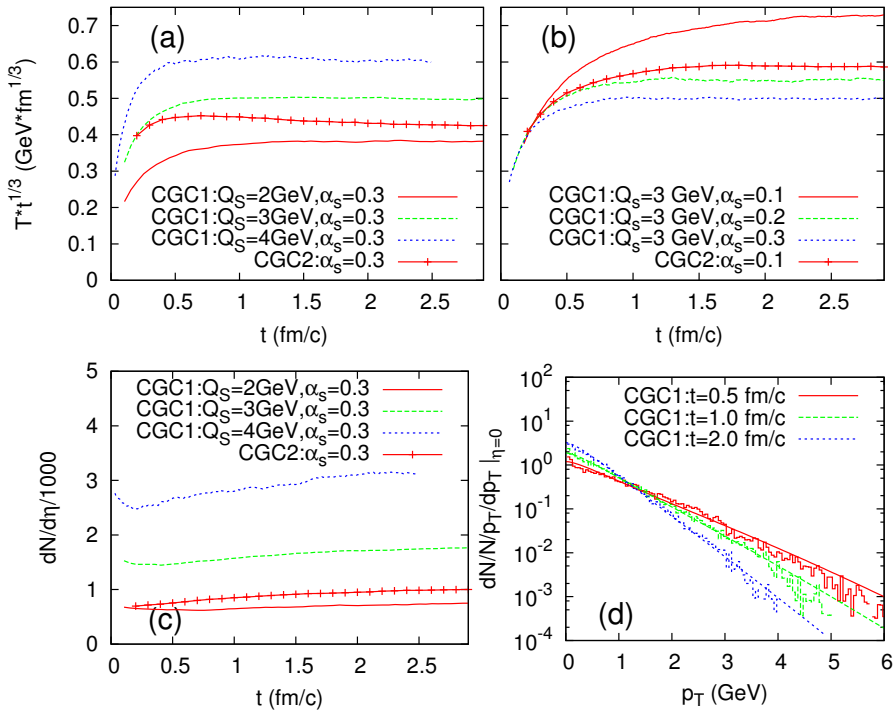

Fig. 1. (a) Temperature scaled by $\tau^{1/3}$ for $\alpha_s = 0.3$, $Q_S = 2$–4 GeV and AuAu@200GeV initial condition. (b) Temperature scaled by $\tau^{1/3}$ for $Q_S = 3$ GeV, AuAu@200GeV initial condition, $\alpha_s = 0.1$–0.3. (c) Particle multiplicity $dN/d\eta$ in central rapidity bin $\eta\ \epsilon[-0.1 : 0.1]$ for constant $\alpha_s = 0.3$. (d) Transverse spectra at different times and thermal fits for simulations with CGC1 initial condition, $Q_S = 3$ GeV, $\alpha_s = 0.3$.

$$f(x,p) = g\lambda \mathrm{e}^{-E/T}\left(1 + C_0\bar{\pi}\left(p_Z^2 - \frac{1}{2}p_\mathrm{T}^2\right)\right), \tag{3}$$

where $\lambda = n/n_\mathrm{eq}$ denotes the fugacity, $\bar{\pi} = T_{33} - T_{33}^\mathrm{eq}$ with $T_{33}^\mathrm{eq} = \frac{1}{3}e$ and $C_0 = \frac{1}{8nT^3}$. n, e, T_{33} are calculated by BAMPS. This form of distribution function models one dimensional hydrodynamic evolution of a viscous medium close to equilibrium, which has been studied recently in [9]. Fig. 1(d) shows a very good agreement between BAMPS results and fit already at $t = 0.5$ fm/c. Both 'hard' and 'soft' sectors are almost thermal at $t = 0.5$ fm/c, *i.e.* we do not observe any significant difference on thermalization time scales.

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