Vol. 40 (2009) *ACTA PHYSICA POLONICA B* No 4

MODELING OF RELATIVISTIC HEAVY-ION COLLISIONS WITH 3+1D HYDRODYNAMIC AND HYBRID MODELS*

Steffen A. Bass

Department of Physics, Duke University
Durham, NC, 27708-0305 USA

(Received February 4, 2009)

The current status of the modeling of relativistic heavy-ion collisions with 3+1D hydrodynamic and hybrid models is reviewed. Particular emphasis is placed in the use of hydrodynamics as a "standard medium", enabling the calculation of rare probe — medium interactions in a way consistent with the bulk evolution of the medium.

PACS numbers: 12.38.Mh, 11.10.Wx, 24.10.Nz, 25.75.Dw

1. Introduction

A major goal of colliding heavy-ions at relativistic energies is to heat up a small region of space-time to temperatures as high as are thought to have occurred during the early evolution of the Universe, a few microseconds after the Big Bang. In ultra-relativistic heavy-ion collisions, such as are currently being explored at the Relativistic Heavy-Ion Collider (RHIC), the four-volume of hot and dense matter, with temperatures above ~ 150 MeV, is on the order of $\sim (10\ \text{fm})^4$. The state of strongly interacting matter at such high temperatures (or density of quanta) is usually called quark–gluon plasma (QGP).

The first five years of RHIC operations at $\sqrt{s_{\rm NN}} = 130$ GeV and $\sqrt{s_{\rm NN}} =$ 200 GeV have yielded a vast amount of interesting and sometimes surprising results [1–4], many of which have not yet been fully evaluated or understood by theory. There exists mounting evidence that RHIC has created a hot and dense state of deconfined QCD matter with properties similar to that of an ideal fluid [5, 6] — this state of matter has been termed the *strongly interacting Quark–Gluon-Plasma* (sQGP).

* Presented at the IV Workshop on Particle Correlations and Femtoscopy, Kraków, Poland, September 11–14, 2008.

Heavy-Ion collisions at RHIC involve several distinct reaction stages, starting from the two initial ground states of the colliding nuclei, followed by the high density phase in which a sQGP is formed up to the final freezeout of hadrons. Relativistic Fluid Dynamics (RFD, see *e.g.* [7–9]) is ideally suited for the *QGP and hydrodynamic expansion* reaction phase, but breaks down in the later, dilute, stages of the reaction when the mean free paths of the hadrons become large and flavor degrees of freedom are important. The most important advantage of RFD is that it directly incorporates an equation of state as input and thus is so far the only dynamical model in which a phase transition can explicitly be incorporated. In the ideal fluid approximation (*i.e.* neglecting off-equilibrium effects) — and once an initial condition has been specified — the EoS is the *only* input to the equations of motion and relates directly to properties of the matter under consideration. The hydrodynamic description has been very successful [10–12] in describing the collective behavior of soft particle production at RHIC.

Conventional RFD calculations need to assume a *freezeout* temperature at which the hydrodynamic evolution is terminated and a transition from the zero mean-free-path approximation of a hydrodynamic approach to the infinite mean-free-path of free streaming particles takes place. The freezeout temperature usually is a free parameter which (within reasonable constraints) can be fitted to measured hadron spectra.

The reach of RFD can be extended and the problem of having to terminate the calculation at a fixed freezeout temperature can be overcome by combining the RFD calculation with a microscopic hadronic cascade model — this kind of hybrid approach (dubbed *hydro plus micro*) was pioneered in [13] and has been now also taken up by other groups [14–16]. Its key advantages are that the freezeout now occurs naturally as a result of the microscopic evolution and that flavor degrees of freedom are treated explicitly through the hadronic cross sections of the microscopic transport. This has been in particular important for the description of the dynamics and spectra of multi-strange baryons and the ϕ-meson, which decouple early on from the hadronic evolution [16–19]. Due to the Boltzmann equation being the basis of the microscopic calculation in the hadronic phase, viscous corrections for the hadronic phase are by default included in the approach.

2. Model description

In hydrodynamic models, the starting point is the relativistic hydrodynamic equation

$$\partial_\mu T^{\mu\nu} = 0\,, \tag{1}$$

where $T^{\mu\nu}$ is the energy momentum tensor which is given by

$$T^{\mu\nu} = (\varepsilon + p)U^\mu U^\nu - p g^{\mu\nu}\,. \tag{2}$$

Here ε, p, U and $g^{\mu\nu}$ are energy density, pressure, four velocity and metric tensor, respectively. We solve the relativistic hydrodynamic equation Eq. (1) numerically with baryon number n_B conservation

$$\partial_\mu(n_B(T,\mu)U^\mu) = 0\,. \tag{3}$$

In order to optimize our hydrodynamic calculations for ultra-relativistic heavy-ion collisions, the calculation is usually carried out in light-cone coordinates (τ, x, y, η) $(\tau = \sqrt{t^2 - z^2})$. Of the currently available two implementations of 3D-RFD, the one by Hirano *et al.* [20] utilizes a fixed Eulerian grid for the calculation and the one by Nonaka *et al.* [16] a co-moving Lagrangian grid.

The initial time for the hydrodynamic expansion at RHIC is usually set to $\tau_0 = 0.6$ fm. Initial energy density and baryon number density are parameterized, *e.g.* in [16] via

$$\begin{aligned} \varepsilon(x,y,\eta) &= \varepsilon_{\rm max} W(x,y;b)H(\eta)\,, \\ n_B(x,y,\eta) &= n_{B\rm max} W(x,y;b)H(\eta)\,, \end{aligned} \tag{4}$$

where b and $\varepsilon_{\rm max}$ ($n_{B\rm max}$) are the impact parameter and the maximum value of energy density (baryon number density), respectively. $W(x,y;b)$ is given by a combination of the wounded nucleon and binary collision model [21] and $H(\eta)$ is given by

$$H(\eta) = \exp\left[-(|\eta| - \eta_0)^2/2\sigma_\eta^2\,\theta(|\eta| - \eta_0)\right]\,. \tag{5}$$

The parameters $\varepsilon_{\rm max}$, $n_{B\rm max}$, η_0 and σ_η are adjusted to data [16] and the initial flow in the longitudinal direction is set to $v_{\rm L} = \eta$ (Bjorken's solution) and $v_{\rm T} = 0$ in the transverse plane (note that this is part of the initial condition and as such the dependency of the results of the calculation on this assumption should be investigated). Currently all 3D-RFD approaches use equation of state with a 1st order phase transition, namely a Bag model EoS with and excluded volume correction [22, 23]. However, the incorporation of more realistic equations of state motivated by recent Lattice QCD data is already in progress. Standard RFD calculations for RHIC usually set the thermal freezeout temperature to 110 MeV. For hybrid hydro+micro approaches, however, the transition from the hydrodynamic evolution to the microscopic evolution occurs at a far higher temperature, just below $T_{\rm C}$ at around 155 MeV. The phase-space distribution of particles of species i on the switching hypersurface $\sigma^\mu_{\rm switch}$ is then given by the Cooper–Frye formula:

$$E_i \frac{dN_i}{d^3p} = \int d\sigma p\, f(p\cdot u)\,, \tag{6}$$

where u^μ is the four-velocity of the local rest-frame. Once an ensemble of hadrons has been created in that fashion, the the semi-classical evolution of the distribution function in the forward light-cone is described by means of a so-called *transport* equation, *e.g.* the Boltzmann equation:

$$p\,\partial f_i(x^\mu, p^\nu) = \mathcal{C}_i\,. \tag{7}$$

$\mathcal{C}_i$ is the collision kernel, describing gain or loss of quanta (particles) of species i in the phase-space cell around (x^μ, p^ν) due to *collisions*. Note that possible classical background fields have been neglected in Eq. (7).

3. Bulk evolution

As mentioned previously in the introduction, RFD (already in its 2+1D incarnations) has been extremely successful in reproducing hadron spectra and collective flow observables at RHIC [10–12, 16]. The same holds true for hybrid hydro+micro approaches in 1+1D [13] and 2+1D [14]. The big advance in 3+1D RFD and hybrid models is their capability of describing the collision dynamics away from mid-rapidity (see *e.g.* the charged particle pseudo-rapidity distribution in the left frame of figure 1) and dropping the assumption of boost-invariance, which may play a significant effect in the description of HBT source radii [24].

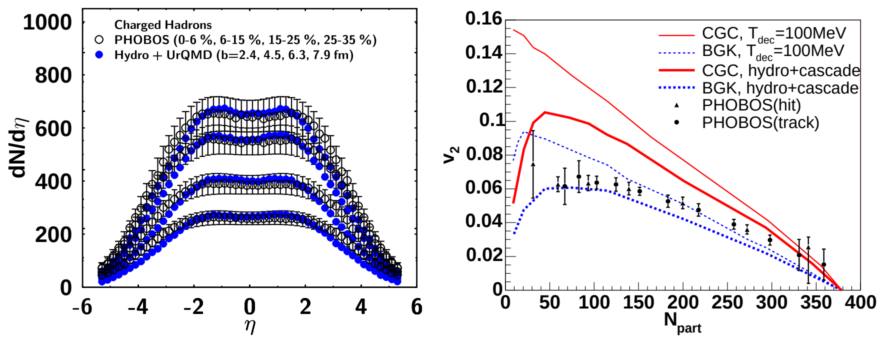

Fig. 1. Left: Centrality-dependence of charged particle pseudo-rapidity distributions in the Hydro+UrQMD approach (figure taken from [16]). Right: Centrality dependence of the elliptic flow coefficient v_2 for different initial conditions and treatment of the hadronic phase (figure taken from [15]).

However, quite a number of innovative and important studies have been made in the context of 3+1D hydro+micro models for observables at mid-rapidity. Among them is the investigation of the initial and final state dependence of elliptic flow: the right frame of figure 1 shows the centrality dependence of the elliptic flow coefficient v_2 for two different initial conditions. The *standard* initial condition, denoted BGK, is based on a superposition of binary collision and wounded nucleon scaling [25], whereas the Color-Glass-Condensate (CGC) initial condition is based on saturation physics [26–28], considered to be more realistic for conditions at RHIC. However, one can clearly see that the CGC initial condition combined with an ideal RFD evolution yields values of v_2 significantly larger than seen by experiment. The introduction of a dissipative hadronic phase via the Hydro+Cascade model [15] reduces this overestimate. However, a good description of the data while using the CGC initial condition will first be obtained with the introduction of a viscous RFD approach for the deconfined phase, coupled with a dissipative hadronic phase [29–31].

The importance of the dissipative hadronic phase is demonstrated in the left frame of figure 2, which shows the transverse momentum dependence of v_2 for protons and pions in the hydro+cascade model [19]: the calculation clearly shows how the mass-splitting between proton and pion elliptic flow (a key feature for determining the success of fluid dynamics at RHIC) develops in the later hadronic stage of the reaction. One therefore finds a clear separation of time-scales: the magnitude of the elliptic flow develops early on during the deconfined phase [23], whereas the mass-splitting is necessarily rooted in the hadronic phase of the reaction, when hadrons are the relevant degrees of freedom.

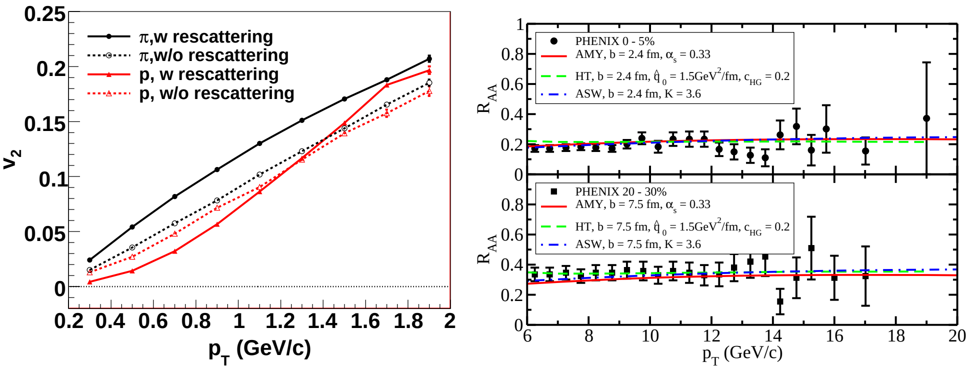

Fig. 2. Left: transverse momentum dependence of v_2 for protons and pions in the hydro+cascade model (figure taken from [19]). The hadronic phase is responsible for the mass-splitting between protons and pions. Right: Nuclear modification factor R_{AA} in Au–Au collisions at 0–5% (top) and 20–30% (bottom) centrality calculated in the ASW, HT and AMY approaches compared to data from PHENIX [44] (figure taken from [43]).

4. Jet-medium interactions

Experiments at the Relativistic Heavy Ion Collider (RHIC) have established a significant suppression of high-p_{T} hadrons produced in central $A+A$ collisions compared to those produced in peripheral $A+A$ or binary scaled $p+p$ reactions, indicating a strong nuclear medium effect [32,33], commonly referred to as *jet-quenching*. Within the framework of perturbative QCD, the leading process of energy loss of a fast parton is gluon radiation induced by multiple soft collisions of the leading parton or the radiated gluon with color charges in the quasi-thermal medium [34–36].

Over the past two years, a large amount of jet-quenching related experimental data has become available, including but not limited to the nuclear modification factor R_{AA}, the elliptic flow v_2 at high p_{T} (as a measure of the azimuthal anisotropy of the jet cross section) and a whole array of high p_{T} hadron–hadron correlations. Computations of such jet modifications have acquired a certain level of sophistication regarding the incorporation of the partonic processes involved. However, most of these calculations have been utilizing simplified models for the underlying soft medium, *e.g.* assuming a simple density distribution and its variation with time. Even in more elaborate setups, most jet quenching calculations assume merely a one- or two-dimensional Bjorken expansion.

The availability of a three-dimensional hydrodynamic evolution code [16,37] and related hybrid approaches allow for a much more detailed study of jet interactions in a longitudinally and transversely expanding medium. The variation of the gluon density in these approaches is very different from that in a simple Bjorken expansion. The first calculation in this direction [37,38] estimated the effects of 3-D expansion on the R_{AA} within a simplified version of the GLV approach [39].

More recently, the Duke group has utilized its 3-D hydrodynamic model to provide the time-evolution of the medium produced at RHIC for jet energy-loss calculations performed in the BDMPS/ASW [40], Higher Twist [41] and AMY [42] approaches. In each of the three efforts, the inclusive as well as the azimuthally differential nuclear suppression factor R_{AA} of pions was studied as a function of their transverse momentum p_{T}. In addition, the influence of collective flow, variations in rapidity, and energy-loss in the hadronic phase were addressed for the selected approaches. For details regarding the implementation of the energy-loss schemes and their interface to the hydrodynamic medium, we refer the reader to the publications cited above and to [43]. The most noteworthy feature of this work is that it allows for a systematic comparison between the three aforementioned jet energy-loss approaches, utilizing the same hydrodynamic medium evolution as well as the same structure and fragmentation functions for calculating the initial state and final high-p_{T} hadron distributions.

The left frame of figure 2 shows the nuclear modification factor R_{AA} in Au–Au collisions at 0–5% (top) and 20–30% (bottom) centrality calculated in the ASW, HT and AMY approaches compared to data from PHENIX [44]. As can be seen, the parameters for all three approaches (initial maximal value for the transport coefficient $\hat{q}_0$ or coupling constant $\alpha_{\rm s}$ in the AMY case) can be adjusted such that the approaches are able to describe the centrality dependence of the nuclear modification factor reasonably well. When using the same temperature scaling law to couple to the medium [43], the values are $\hat{q}_0 \approx 2.3$ GeV^2/fm for the HT approach, $\hat{q}_0 \approx 10.0$ GeV^2/fm for the ASW formalism and $\alpha_{\rm S} \approx 0.33$ for the AMY approach, which can be converted into a value of $\hat{q}_0 \approx 4.1$ GeV^2/fm. Note that the ASW value for $\hat{q}_0$ at $\tau = 0.6$ fm/c and $\varepsilon_0 = 55$ GeV/fm^3 lies significantly higher than the Baier estimate for an ideal QGP, $\hat{q} \approx 2\,\varepsilon^{3/4}$ [45], while the HT and AMY values lie significantly below that estimate. The large difference in $\hat{q}_0$ values between HT and ASW has been pointed out previously. However, our calculation shows for the first time that this difference is not due to a different treatment of the medium or initial state.

Furthermore, it was found that slight differences appear between the approaches when R_{AA} is studied as a function of azimuthal angle. This can be seen in the left frame of figure 3 where R_{AA} is plotted as a function of azimuthal angle at $p_{\rm T} = 10$ GeV/c (solid line) and $p_{\rm T} = 15$ GeV/c (dashed line) for all three approaches in the 20–30% centrality bin.

Overall, the systematic comparison shows that under identical conditions (*i.e.* same medium evolution, same choice of parton distribution functions, scale *etc.*) all three jet energy-loss schemes yield very similar results. This finding is very encouraging since it indicates that the technical aspects of the formalisms are well under control. However, there still exists a *puzzle* regarding the extracted value for the transport coefficient $\hat{q}_0$. While this discrepancy among these approaches is not new, the comparison in [43] has for the first time been able to rule out differences in the medium evolution or initial setup as cause for the differing values of $\hat{q}$.

5. Heavy-quark flow and charmonium suppression

The above section has established how useful a *standard medium*, as provided by a 3+1D hydro+micro model, can be for the exploration of hard probe medium interactions. Note that hard probes in this context are by no means restricted to high-$p_{\rm T}$ partons, but can include heavy quarks or quarkonium states as well. For the dynamics of heavy quarks, collisional energy-loss is regarded as driving the dynamics — such a system can be described with a Langevin equation for the evolution of the heavy quarks propagating through a hydrodynamic medium [46,47]. A first calculation in a 3+1D hydrodynamic medium has been performed in [48] — the biggest advantage of this new calculation over the previous work being the use of the *standard medium*, which should help to improve the analytic power with respect to determining the drag coefficient of heavy quarks in medium.

Regarding the suppression of charmonium, the first calculation utilizing a realistic hydrodynamic medium has been published in [49]. This calculation is quite innovative, since previous attempts at understanding the suppression of charmonium in heavy-ion collisions have not utilized any realistic hydrodynamic medium which is simultaneously capable of describing the bulk properties of matter, such as spectra and collective flow. The right frame of figure 3 shows the J/ψ survival probability as function of $N_{\rm part}$ for different J/ψ dissociation temperatures. In principal, these dissociation temperatures should be calculable via charmonium spectral functions on the lattice. However, current calculations still suffer from large error-bars and do not account for the finite momentum of the charmonium state. Regarding the results of [49], it is quite remarkable how sensitive the survival probability actually is on the dissociation temperature in this simple model approach for J/ψ dissociation.

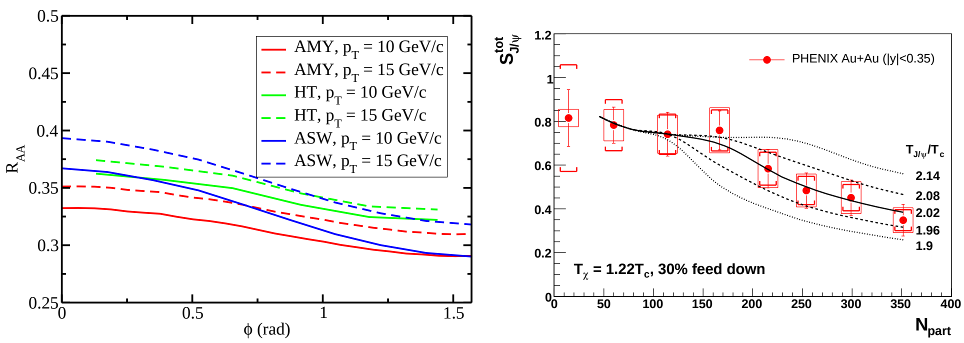

Fig. 3. Left: R_{AA} as a function of azimuthal angle at $p_{\rm T} = 10$ GeV/c (solid line) and $p_{\rm T} = 15$ GeV/c (dashed line) for all three approaches in the 20–30% centrality bin (figure taken from [43]). Right: J/ψ survival probability as function of $N_{\rm part}$ for different J/ψ dissociation temperatures (figure taken from [49]).

6. Summary and outlook

3+1D Hydrodynamic and hybrid hydro+micro models have proven to be among the most successful for describing the evolution of bulk QCD matter created in relativistic heavy-ion collisions at RHIC. They also provide a *standard medium*, which can be utilized for the calculation of medium effects on hard probes (jet energy-loss, heavy quark diffusion, charmonium absorption) in a consistent and comprehensive fashion. In 3+1 dimensions, these models currently rely on ideal RFD to model the deconfined phase. However, considerable progress has already been made in developing 2+1D viscous RFD models. In the foreseeable future, 3+1D hybrid models based on viscous RFD coupled with a microscopic hadronic transport model for the hadronic breakup stage will surely be developed. Such models will enjoy a broad range of applicability in incident beam energy from FAIR to LHC and will provide a reliable baseline for the investigation of novel phenomena expected in future experiments at these facilities.

This work was supported by the U.S. Department of Energy (grant DE-FG02-05ER41367). S.A.B. is grateful for many helpful discussions with G.-Y. Quin, U. Heinz, T. Hirano, A. Majumder, C. Nonaka, T. Renk and J. Ruppert.

REFERENCES

[1] K. Adcox *et al.* [PHENIX Collaboration], *Nucl. Phys.* **A757**, 184 (2005) [nucl-ex/0410003].

[2] I. Arsene *et al.* [BRAHMS Collaboration], *Nucl. Phys.* **A757**, 1 (2005) [nucl-ex/0410020].

[3] B.B. Back *et al.*, *Nucl. Phys.* **A757**, 28 (2005) [nucl-ex/0410022].

[4] J. Adams *et al.* [STAR Collaboration], *Nucl. Phys.* **A757**, 102 (2005) [nucl-ex/0501009].

[5] T. Ludlam, *Nucl. Phys.* **A750**, 9 (2005).

[6] M. Gyulassy, L. McLerran, *Nucl. Phys.* **A750**, 30 (2005) [nucl-th/0405013].

[7] J.D. Bjorken, *Phys. Rev.* **D27**, 140 (1983).

[8] R.B. Clare, D. Strottman, *Phys. Rep.* **141**, 177 (1986).

[9] A. Dumitru, D.H. Rischke, *Phys. Rev.* **C59**, 354 (1999), [nucl-th/9806003].

[10] P.F. Kolb, U.W. Heinz, nucl-th/0305084.

[11] P. Huovinen, nucl-th/0305064.

[12] T. Hirano, K. Tsuda, *Phys. Rev.* **C66**, 054905 (2002) [nucl-th/0205043].

[13] S.A. Bass, A. Dumitru, *Phys. Rev.* **C61**, 064909 (2000) [nucl-th/0001033].

[14] D. Teaney, J. Lauret, E.V. Shuryak, nucl-th/0110037.

[15] T. Hirano, U.W. Heinz, D. Kharzeev, R. Lacey, Y. Nara, *Phys. Lett.* **B636**, 299 (2006) [nucl-th/0511046].

[16] C. Nonaka, S.A. Bass, *Phys. Rev.* **C75**, 014902 (2007) [nucl-th/0607018].

[17] H. van Hecke, H. Sorge, N. Xu, *Phys. Rev. Lett.* **81**, 5764 (1998), [nucl-th/9804035].

[18] A. Dumitru, S.A. Bass, M. Bleicher, H. Stoecker, W. Greiner, *Phys. Lett.* **B460**, 411 (1999) [nucl-th/9901046].

[19] T. Hirano, U.W. Heinz, D. Kharzeev, R. Lacey, Y. Nara, *Phys. Rev.* **C77**, 044909 (2008) [arXiv:0710.5795[nucl-th]].

[20] T. Hirano, K. Tsuda, *Nucl. Phys.* **A715**, 821 (2003) [nucl-th/0208068].

[21] P.F. Kolb, U.W. Heinz, P. Huovinen, K.J. Eskola, K. Tuominen, *Nucl. Phys.* **A696**, 197 (2001) [hep-ph/0103234].

[22] D.H. Rischke, M.I. Gorenstein, H. Stoecker, W. Greiner, *Z. Phys.* **C51**, 485 (1991).

[23] P.F. Kolb, J. Sollfrank, U.W. Heinz, *Phys. Rev.* **C62**, 054909 (2000) [hep-ph/0006129].

[24] S. Pratt, arXiv:0811.3363 [nucl-th].

[25] S.J. Brodsky, J.F. Gunion, J.H. Kuhn, *Phys. Rev. Lett.* **39**, 1120 (1977).

[26] L.D. McLerran, R. Venugopalan, *Phys. Rev.* **D49**, 3352 (1994) [hep-ph/9311205].

[27] L.D. McLerran, R. Venugopalan, *Phys. Rev.* **D49**, 2233 (1994) [hep-ph/9309289].

[28] D. Kharzeev, E. Levin, M. Nardi, *Nucl. Phys.* **A747**, 609 (2005) [hep-ph/0408050].

[29] H. Song, U.W. Heinz, *Phys. Lett.* **B658**, 279 (2008) [arXiv:0709.0742[nucl-th]].

[30] M. Luzum, P. Romatschke, *Phys. Rev.* **C78**, 034915 (2008) [arXiv:0804.4015 [nucl-th]].

[31] H. Song, U.W. Heinz, arXiv:0812.4274 [nucl-th].

[32] K. Adcox *et al.* [PHENIX Collaboration], *Phys. Rev. Lett.* **88**, 022301 (2002) [nucl-ex/0109003].

[33] C. Adler *et al.* [STAR Collaboration], *Phys. Rev. Lett.* **89**, 202301 (2002) [nucl-ex/0206011].

[34] M. Gyulassy, X.-n. Wang, *Nucl. Phys.* **B420**, 583 (1994) [nucl-th/9306003].

[35] R. Baier, Y.L. Dokshitzer, A.H. Mueller, S. Peigne, D. Schiff, *Nucl. Phys.* **B483**, 291 (1997) [hep-ph/9607355].

[36] B.G. Zakharov, *JETP Lett.* **65**, 615 (1997) [hep-ph/9704255].

[37] T. Hirano, Y. Nara, *Phys. Rev.* **C66**, 041901 (2002) [hep-ph/0208029].

[38] T. Hirano, Y. Nara, *Phys. Rev.* **C69**, 034908 (2004) [nucl-th/0307015].

[39] M. Gyulassy, P. Levai, I. Vitev, *Phys. Rev. Lett.* **85**, 5535 (2000) [nucl-th/0005032].

[40] T. Renk, J. Ruppert, C. Nonaka, S.A. Bass, *Phys. Rev.* **C75**, 031902 (2007) [nucl-th/0611027].

[41] A. Majumder, C. Nonaka, S.A. Bass, *Phys. Rev.* **C76**, 041902 (2007) [nucl-th/0703019].

[42] G.-Y. Qin *et al.*, *Phys. Rev.* **C76**, 064907 (2007) [arXiv:0705.2575 [hep-ph]].

[43] S.A. Bass *et al.*, arXiv:0808.0908[nucl-th].
[44] M. Shimomura [PHENIX Collaboration], *Nucl. Phys.* **A774**, 457 (2006), [nucl-ex/0510023].
[45] R. Baier, *Nucl. Phys.* **A715**, 209 (2003) [hep-ph/0209038].
[46] G.D. Moore, D. Teaney, *Phys. Rev.* **C71**, 064904 (2005) [hep-ph/0412346].
[47] H. van Hees, R. Rapp, *Phys. Rev.* **C71**, 034907 (2005) [nucl-th/0412015].
[48] Y. Akamatsu, T. Hatsuda, T. Hirano, arXiv:0809.1499[hep-ph].
[49] T. Gunji, H. Hamagaki, T. Hatsuda, T. Hirano, *Phys. Rev.* **C76**, 051901 (2007) [hep-ph/0703061].