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VISCOSITY AND DISSIPATION — EARLY STAGES* **

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A very early start up time of the hydrodynamic evolution is needed in order to reproduce observations from relativistic heavy-ion collisions experiments. At such early times the systems is still not locally equilibrated. Another source of deviations from local equilibrium is the viscosity of the fluid. We study these effects at very early times to obtain a dynamical prescription for the transition from an early 2-dimensional expansion to a nearly equilibrated 3-dimensional expansion at latter stages. The role of viscosity at latter stages of the evolution is also illustrated.

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1. Introduction

Recent hydrodynamic calculations modelling heavy-ion collisions can reproduce experimentally measured soft observables: transverse momentum spectra, collective elliptic flow and Hanbury-Brown–Twiss correlation radii [1, 2] if the initial time of the collective expansion is pushed down to $\tau_0 = 0.25$ fm/c. This raises the question about the applicability of perfect fluid hydrodynamics at such small proper times. The mechanism of the formation of the dense matter in the fireball is not understood up to now. However, in all imaginable scenarios some time is required for the formation of the matter constituents and for their subsequent equilibration. In hydrodynamics, which is a coarse-grained description, the dynamics is defined by the local thermodynamical quantities, such as the energy density and pressure. The details of the underlying microscopic degrees of freedom are irrelevant. Although formally, perfect fluid thermodynamics requires that local thermal equilibrium is maintained, phenomenological applicability of

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the hydrodynamics in the description of heavy-ion collisions starts as soon as the pressure becomes approximately isotropic. The dense matter in the fireball can be described by the hydrodynamic model after the time when the effective pressure in the system is similar in the longitudinal and transverse directions. Complete kinetic equilibrium is not required, since the model has other sources limiting the robustness of its predictions, such as the uncertainties in the high temperature equation of state, in the initial density, and in the freeze-out procedure.

When the deviations of the energy momentum tensor $T^{\mu\nu}$ from its form in a perfect fluid $T_0^{\mu\nu}$

$$T^{\mu\nu} = T_0^{\mu\nu} + \pi^{\mu\nu} \tag{1}$$

is small the evolution can be formulated as the hydrodynamics of a viscous fluid [3–8]. But, in the very early evolution the initial anisotropy of the pressure is the main contribution that makes the matter to evolve differently from the perfect fluid [9]. These early dissipative effects are strong, since the initial pressure anisotropy is large.

2. Early dissipation

The initial anisotropy of the pressure and its relaxation towards the perfect fluid value cannot be reliably described with the second order Israel–Stewart relativistic viscous fluid formalism [10]. The applicability of the viscous fluid equations requires $\pi^{\mu\nu}\pi_{\mu\nu} \ll p^2$, where $\pi^{\mu\nu}$ is the stress tensor. Instead, we propose an effective description of the transition from the anisotropic system with a two-dimensional pressure to the three-dimensional hydrodynamics [9]. The energy momentum tensor is the sum of the perfect fluid energy momentum tensor and a stress correction

$$T^{\mu\nu} = \begin{pmatrix} \epsilon & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \pi/2 & 0 & 0 \\ 0 & 0 & \pi/2 & 0 \\ 0 & 0 & 0 & -\pi \end{pmatrix} . \tag{2}$$

The dissipative correction π quantifies the pressure anisotropy in the transverse and longitudinal directions. A similar form of the stress tensor appears in the hydrodynamics with shear viscosity for the case of the Bjorken flow [4]. For large stress corrections the second order viscous hydrodynamics equations for π cannot be reliably applied. Instead an effective equation describing the relaxation of the pressure asymmetry is used. Neglecting the shear viscosity we take

$$\pi(\tau) = \pi(\tau_0) e^{-(\tau-\tau_o)/\tau_\pi} , \tag{3}$$

where τ_π is a phenomenological parameter, in principle unrelated to the relaxation time in the Israel–Stewart equation for the stress-tensor.

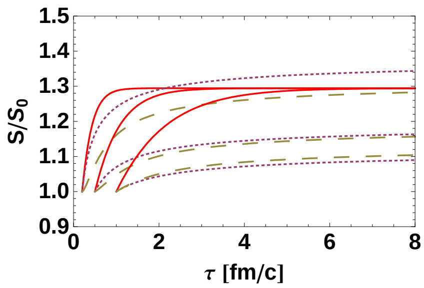


Fig. 1. Relative increase of the entropy from dissipative processes in the early stage of the collision for several initial times τ_0 of the evolution. The dotted line represents the entropy production from the Navier–Stokes shear viscosity tensor with $\eta = 0.1$ s, the dashed line represents the increase of the entropy obtained from the second order viscous hydrodynamic equation with $\eta = 0.1$ s, $\tau_\pi = 6\eta/T$ s, and $\Pi(\tau_0) = \frac{4\eta}{3\tau_0}$, and the solid represents the relative entropy production due to the stress tensor term of the form $\Pi(\tau) = p(\tau_0)\exp(-(\tau-\tau_0)/\tau_0)$ [9].

The dynamics is followed using a numerical solution of the relativistic hydrodynamic equations

$$\partial_\mu T^{\mu\nu} = 0 \tag{4}$$

with some assumed symmetry of the fireball. Entropy production from the dissipative relaxation of the pressure can be estimated in the Bjorken solution. Depending on the ratio τ_π/τ_0, up to 30% increase of the entropy is possible in the early phase. This additional entropy forces a retuning of the initial conditions of the evolution to reproduce final particle multiplicities. After this retuning is taken into account, most of the effect of the early dissipation on final observables is cancelled. However, we note that the transverse momentum spectra of final particles are harder if the early dissipative phase is present.

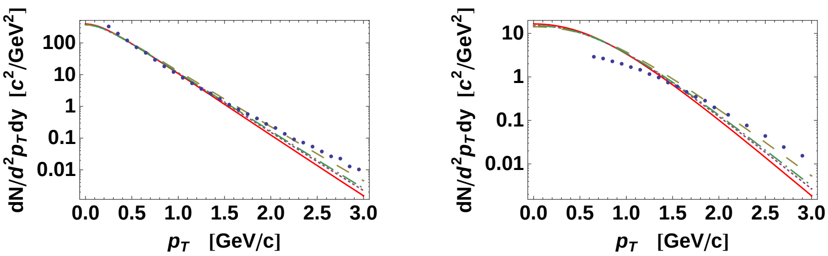


Fig. 2. π^+ (left) and proton (right) spectra from hydrodynamic calculations (solid and dashed-dotted line are for the ideal hydrodynamics starting at $\tau_0 = 1$ fm/c and $\tau_0 = 0.5$ fm/c, respectively. The dotted and dashed lines are for the dissipative evolution corresponding to $\tau_0 = 1$ fm/c and $\tau_0 = 0.5$ fm/c.). Data are from the PHENIX Collaboration [11] for most central events (0–5%) [9].

3. Dissipation and viscosity

We use relativistic hydrodynamics with viscosity [10]. The stress tensor $\pi^{\mu\nu}$ is the solution of a dynamical equation

$$\tau_\pi \Delta^\mu_\alpha \Delta^\nu_\beta \pi^{\alpha\beta} + \pi^{\mu\nu} = \eta \left\langle \nabla^\mu u^\nu \right\rangle - \frac{\eta T}{2\tau_\pi} \pi^{\mu\nu} \partial_\beta \left(\frac{\tau_\pi u^\beta}{\eta T} \right) , \tag{5}$$

where

$$\langle \nabla_\mu u_\nu \rangle = \nabla_\mu u_\nu + \nabla_\nu u_\mu - \tfrac{2}{3} \Delta_{\mu\nu} \nabla_\alpha u^\alpha \, , \tag{6}$$

$$\nabla^\mu = \Delta^{\mu\nu} \partial_\nu \tag{7}$$

with u^μ the fluid velocity, $\Delta_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu$, η the shear viscosity, τ_π the relaxation time. We solve the equations numerically in a boost-invariant geometry with an azimuthally asymmetric expansion in the transverse directions. We use $\eta/s = 1/4\pi$, $\tau_0 = 0.25$ fm$/c$ and $\pi^{zz}(\tau_0)/2 = \pi^{xx}(\tau_0) = \pi^{yy}(\tau_0) = p/2$. Compared to other calculations of the hydrodynamic model with viscosity, we use a small initial time and a large value of the initial stress correction $\pi(\tau_0)$. The model encompasses both the relaxation of the initial pressure anisotropy, and the latter interplay of the relaxation and velocity gradients. To compare with perfect fluid results again a retuning of the initial energy density is necessary to reproduce the final multiplicity. The additional transverse push is strong, it has a contribution from the initial stage of large pressure anisotropy and another one due to the viscosity driven stress corrections. As a consequence of the prolongated transverse push, the transverse momentum spectra get even harder for the case when shear viscosity and initial anisotropy are combined than for the case with only initial dissipation. A similar effect is observed for the elliptic flow. The reduction of the azimuthal asymmetry is the strongest for the viscosity + dissipation scenario of the fluid evolution.

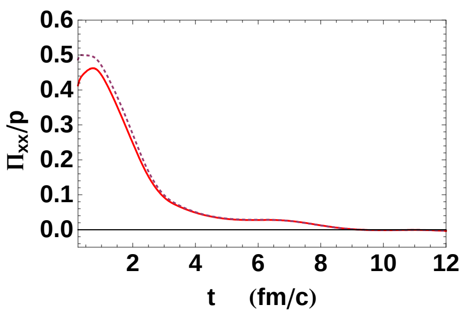
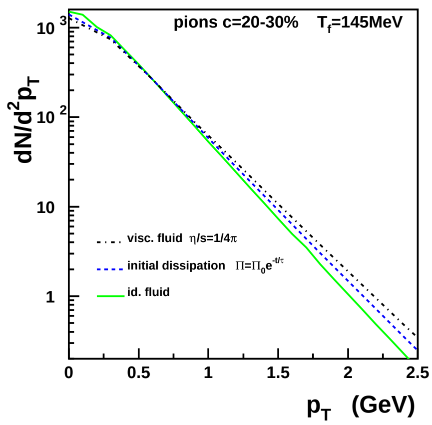


Fig. 3. Ratio of the stress correction to the pressure at the center of the fireball for two initial conditions for π^{xx}.



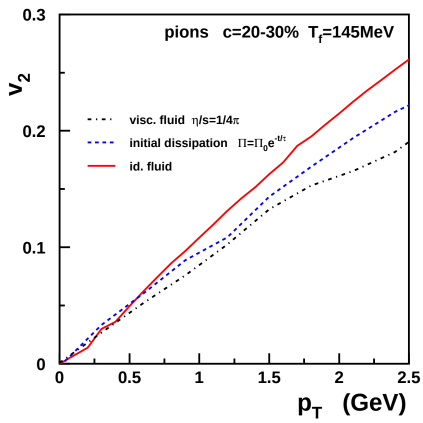


Fig. 4. Transverse momentum spectra (left) and elliptic flow coefficient (right) for π^+ for the perfect fluid (solid line), for the perfect fluid with initial pressure anisotropy (dashed line) and for the viscous fluid (dashed-dotted line).

4. Summary

We discuss dissipative effects in the very early phase of the collective development of the fireball created in relativistic heavy-ion collisions. The initial anisotropy of the effective fluid pressure must dissipate. In the process entropy is produced. After the retuning of the initial conditions to accommodate for this additional entropy, the effect of the early dissipation is most pronounced in the transverse momentum spectra of emitted particles. The initial dissipation of the pressure can be taken together with the effect of the shear viscosity at latter stages. These corrections to the energy momentum-tensor combine to increase the transverse push in the collective flow and cause a significant reduction of the elliptic flow.

REFERENCES

[1] M. Chojnacki *et al.*, *Phys. Rev.* **C78**, 014905 (2008) `[arXiv:0712.0947 [nucl-th]]`.

[2] W. Broniowski *et al.*, *Phys. Rev. Lett.* **101**, 022301 (2008) `[arXiv:0801.4361 [nucl-th]]`.

[3] H. Song, U.W. Heinz, *Phys. Lett.* **B658**, 279 (2008) `[arXiv:0709.0742 [nucl-th]]`.

[4] D. Teaney, *Phys. Rev.* **C68**, 034913 (2003) `[nucl-th/0301099]`.

[5] R. Baier, P. Romatschke, U.A. Wiedemann, *Nucl. Phys.* **A782**, 313 (2007) [nucl-th/0604006].

[6] R. Baier, P. Romatschke, *Eur. Phys. J.* **C51**, 677 (2007) [nucl-th/0610108].

[7] A.K. Chaudhuri, *Phys. Rev.* **C74**, 044904 (2006) [nucl-th/0604014].

[8] A. Muronga, D.H. Rischke, nucl-th/0407114.

[9] P. Bozek, *Acta Phys. Pol. B* **39**, 1375 (2008) [arXiv:0711.2889 [nucl-th]].

[10] W. Israel, J. Stewart, *Ann. Phys.* **118**, 341 (1979).

[11] S.S. Adler *et al.* [PHENIX Collaboration], *Phys. Rev.* **C69**, 034909 (2004) [nucl-ex/0307022].