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PROBING THE SCHWARZSCHILD HORIZON TEMPERATURE

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In this paper using a Gedanken experiment we have measured the black hole horizon temperature. In this process, the total thermal uncertainty is calculated.

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1. Introduction

It is interesting that there is a relationship between Schwarzschild solutions with laws of thermodynamics [1–6]. This picture was completed when the black hole evaporation was discovered [7] and the relation between the horizon and temperature was obtained [8–10]. It has been pointed out that the temperature of black hole is usually regarded as a kinetic effect, depending on the coordinate chart used by a class of observers such as free falling and fiducial observers, but not a property of the space-time geometry in general [14–20]. On the other hand, as pointed out in [13], if the horizon links the aspects of microscopic physics with the bulk dynamics, it can provide a link between the statistical mechanics and dynamics of a solid. In this picture, it is interesting that one can connect the field equations of describing the dynamics of gravity with the horizon thermodynamics. So there is an intriguing analogy between the gravitational dynamics and thermodynamics of horizons. This idea was further developed when the thermodynamics interpretation of Einstein equations was obtained [12, 13]. As shown by Kothawala *et al.* [12], it is possible to interpret the field equations near any spherically symmetric horizon as thermodynamics identity, $T\,dS = dE + PdV$. So the thermodynamical interpretation of gravitational dynamics is not restricted to only the spherically symmetric or static horizons but is quite generic in nature and indicates a deeper connection between gravity and thermodynamics. In this paper, using a Gedanken experiment we have studied the horizon temperature. In this study, a generalised form of thermal uncertainty is obtained. The organization of the paper is as follows: in Section 2, we will consider the standard thermal uncertainty relation in the Schwarzschild space time. In Section 3, using a Gedanken experiment we will obtain a generalised thermal uncertainty relation. Final remarks are presented in Section 4.

2. Standard thermal uncertainty relation

In the Schwarzschild black hole, a fiducial observer measures an effective temperature at distance r as (for example see [31–36],

$$T = \left(1 - \frac{2GM}{r}\right)^{-1/2} \frac{\hbar}{8\pi GM}\,, \tag{1}$$

where $\frac{\hbar}{8\pi GM}$ is the Hawking temperature. Climbing-out of gravitational potential well, the radiation is gravitationally red-shifted by the factor $\left(1 - \frac{2GM}{r}\right)$. It could be detected by an observer at infinity as Hawking temperature.

Instead, at horizon $r \sim 2GM$ the temperature as measured by a fiducial observer diverges. In viewpoint of fiducial observer, this temperature is certainly a real effect. Since, in the point of view of a free falling observer the horizon is no special place, after passing the horizon, the free falling observer cannot communicate with fiducial observer, and no immediate logical contradiction arises. In classical general relativity an observer has no direct access to the apparent horizon; no signal is emitted from black hole. If an observer tries to obtain the temperature of the apparent horizon, it should measure the mass (and charge in the Reissner–Nordström case) of the black hole from the motion of test particles at infinity and then resort to the theory which predicts Eq. (1) for T as function of M. Alternatively, one can perform a scattering experiment and again resort to the theory which predicts relation between the measured cross-section and T. For instance, for ultra-relativistic particles impinging on a Schwarzschild black hole, general relativity predicts a lower capture cross-section. Instead, it is not possible to measure the temperature of horizon directly, that is, using an apparatus which records photons emitted by the horizon itself, and without resorting to relationship predicted by the theory. In this sense, from an operational view in classical general relativity, relation (1) must be considered as definition of T, rather than a prediction which can be tested experimentally. In a quantum theory of gravitation, however, the emission of Hawking radiation allows an observer at infinity to receive a signal coming from apparent horizon and to perform a direct measurement of its temperature at least at level of a *gedanken* experiment. The temperature of the horizon and mass and charge (in the RN case) of black hole becomes three quantities subject to independent experimental determination. It makes sense to ask whether the relation given in (1) is satisfied. In following, we examine if there is an intrinsic bound to precision of experimental determination of T. First, let us better specify the setting of our gedanken experiment. In a Schwarzschild black hole, the Hawking radiation is emitted spontaneously as photons with energy E. It might even consist of a single photon, again of wavelength $\lambda = hc/E$. For definiteness, we indeed consider the situation in which a single photon is emitted. At 90 degree with incoming photon, detector containing a thermometer can detect the temperature of photons emitted. Recording the experiment, we obtain a "thermal images" of black hole.

Together with a measurement of temperature T of black hole from thermometer, this provides the measurement of T_{h} (temperature of the horizon). One can measure the direction of propagation of photon emitted at different angles and trace them back in order to determine the temperature of center of black hole horizon. This experimental precision could be obtained in the measurement process of horizon temperature. However, a photon might carry information on a more detailed scale than λ itself [21]. As classical thermal Heisenberg uncertainty, the resolving power of thermometer gives a minimum error $\Delta(T_1)$ on T_{h},

$$\Delta(T_1) \sim \frac{hc}{k}\frac{\sin\theta}{\lambda}\,, \tag{2}$$

where θ, is the angle and is defined in the range of 0–90 degrees. Since the error of final energy of black hole is $\Delta U \sim hc\sin\theta/\lambda$, this gives a standard thermal uncertainty relation as [22–30],

$$\Delta\left(\frac{1}{T_1}\right) \sim \frac{k}{\Delta U}\,. \tag{3}$$

3. Generalized thermal uncertainty relation

More precisely from infinity we actually observe the projection of temperature of black hole on a (x, y) plane parallel to the thermometer. So $\Delta(T_1)$ is the error of temperature of horizon measured along this direction and ΔU is the error of final black hole energy. The black hole mass could be decreased by Hawking radiance very slowly; in fact the radiated energy is only some fraction of m, and $E \sim m$. So the temperature of black hole changes accordingly. In addition, we neglect the energy losses by the gravitational radiation, known to be $O(E_2/M)$, as well as losses to Hawking radiation. The internal energy of black hole, when it emits a quantum of light, the quantity that we are measuring, changes discontinuously. It does not make sense to ask whether the information carried by the outgoing photon refers to black hole immediately before emission, or immediately after, or to something in between. The corresponding error must be considered as intrinsic to measurement. This gives a second source of error on T which for a Schwarzschild black hole is, $\Delta(T_2) \sim hc/2kG\Delta M$. Since $\Delta M = h/\lambda$ we obtain an uncertainty

$$\Delta\left(\frac{1}{T_2}\right) \sim \frac{2hc^4}{k\lambda} \cdot \frac{1}{T_{\mathrm{P}}^2}\,, \tag{4}$$

where T_{P} is the Planck temperature. Note that $\Delta(1/T_2)$ is only a lower bound in uncertainty and only a Schwarzschild black hole can saturate it. If we combine $\Delta(1/T_1)$ and $\Delta(1/T_2)$, using the trivial inequality

$$\frac{\lambda}{\sin\theta} \geq \lambda\,, \tag{5}$$

we obtain

$$\Delta\left(\frac{1}{T}\right) \geq \frac{k\lambda}{hc} + \frac{hc^4}{k\lambda}\,\frac{1}{T_{\mathrm{P}}^2}\,. \tag{6}$$

In our approach, the relative numerical constant between two terms could not be predicted. Indeed, in order to write a relation like (6) which does not depend on features of the apparatus we have eliminated θ using the trivial inequality (5), the price we pay is that we cannot consistently estimate the value of this constant. Relation (6) implies that there exists a minimum error $\Delta(1/T) \sim 1/T_{\mathrm{P}}$, therefore the temperature of horizon is equal to the right hand side of Eq. (1) and has no operational meaning. If we aim to obtain a precision better than $1/T_{\mathrm{P}}$, it is also suggestive to set a lower bound $\Delta(1/T)$ in terms of $\Delta U \sim hc\sin\theta/\lambda$. Remember that Eq. (4) is due to the discontinuous change of horizon during the measurement process and it is not directly related to the uncertainty of black hole internal energy after the measurement. In fact, it is present even when the latter is zero ($\theta \to 0$). However, we can use the trivial inequality $\Delta U/\sin\theta \geq \Delta U$ to obtain

$$\Delta\left(\frac{1}{T}\right) \geq \frac{k}{\Delta U} + c^{-3}\frac{1}{T_{\mathrm{P}}^2}\,\frac{\Delta U}{k}\,. \tag{7}$$

It is natural to investigate whether the relation given in (7) that is obtained considering only a very specific measurement, has a more general validity in quantum gravity. A second conclusion that we find is an example of a more general situation, and relation (7) is indeed a generalized thermodynamics uncertainty relation, which governs all measurement processes in quantum gravity. Relation (7) is quite generic and has both quantum mechanical and quantum gravity limits. The quantum mechanical limit is obtained when the second term in the right hand side of relation (7) is negligible,

$$c^{-3}\frac{1}{T_{\mathrm{P}}^2}\frac{\Delta U}{k} \ll 1 \tag{8}$$

and the quantum gravity limit is obtained when

$$c^{-3}\frac{1}{T_{\mathrm{P}}^2}\frac{\Delta U}{k} \approx 1\,. \tag{9}$$

The lower bound on uncertainty of temperature is

$$\Delta\left(\frac{1}{T}\right) \geq \frac{1}{T_{\min}}\,. \tag{10}$$

4. Final remarks

The standard thermodynamics uncertainty is obtained when $1/T_{\min}$ is negligible. For semiclassical black hole where $\Delta(1/T) \ll 1/T_{\min}$ then the generalized thermodynamics uncertainty applies. Both thermodynamics limits of the Schwarzschild black hole (quantum mechanics and quantum gravity) follow from the two-limit relation between inverse temperature and energy.

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