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BOUNDS ON THE HIGGS MASS FROM RENORMALIZATION GROUP IMPROVED POTENTIAL FOR GWS MODEL

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The exact value of Higgs boson mass cannot be determined theoretically due to lack of knowledge on the definite value of the Higgs self coupling constant λ. Following a result of P. Kielanowski and S.R. Juarez W. that for $0.369 \leq \lambda\,(m_{\rm t}) \leq 0.613$ the Standard Model is valid in the whole range $[m_{\rm t},\, E_{\rm GU} = 10^{14}\,{\rm GeV}]$, we obtain the bounds on the Higgs mass in the effective potential method from the renormalization group improved potential for Glashow–Weinberg–Salam (GWS) model. The above limits for $\lambda\,(m_{\rm t})$ corresponds to the following Higgs mass bounds $150.4\,{\rm GeV} \leq m_H \leq 195.9\,{\rm GeV}$ for the mass scale $M = 180\,{\rm GeV}$ and the bounds are almost independent of the mass scale (M).

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1. Introduction

The studies on Higgs mass in the Glashow–Weinberg–Salam model (GWS) [1] is very important for understanding the mechanism of spontaneous symmetry breaking for the origin of mass on the one hand and for testing the validity of the GWS model (also known as the standard model) on the other hand. Interestingly, the value of Higgs mass depends on the mechanism of symmetry breaking as well as the gauge model under consideration [2]. In a celebrated paper, Coleman and Weinberg [3] showed that in the case of field theory with more than one coupling constant, quantum (one-loop) corrections to the classical potential could induce symmetry breaking and using this mechanism they showed that Higgs mass could be predicted ($\approx 10.4\,{\rm GeV}$) for zero value of top quark mass. Subsequently, there have

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been other works giving actual value or constraints on the Higgs boson mass in the GWS model [4]. It may be mentioned that there have been intense theoretical activities in the area of research related to search for Higgs in the standard model. These works will have great importance to particle physics because the Higgs particle predicted by the standard model [5] is not yet discovered. The future experiments on LHC will most probably be able to test the standard model [6] completely.

The purpose of this paper is to obtain the bounds on Higgs mass on the basis of a renormalization group improved potential in GWS model [7,8], for bounds on $\lambda\,(m_{\rm t})$ given elsewhere [8]. In this model mass terms are included in the renormalization group equations, that relate various observables (*e.g.* coupling or masses) at different energies and allow the study of their asymptotic behaviour. There will be renormalization group equations for both ϕ^2 and ϕ^4 terms, occurring in the potential. The paper is organized as follows: The theory is given in Section 2. The discussions and conclusions are given in Section 3.

2. Theory

Considering the renormalization group improved potential in massive theories, we turn to the improved potential in the GWS model given by [8]

$$V_{\rm eff}(\phi) = \frac{1}{2}\mu^2(t)Z^2(t)\phi_c^2 + \frac{1}{8}\lambda(t)Z^4(t)\phi_c^4\,, \tag{1}$$

where

$$\begin{aligned} \frac{d\lambda}{dt} &= \beta_\lambda(g_i(t),\,\lambda(t))\,, && (2)\\ \frac{d\mu^2}{dt} &= \mu^2(t)\,\beta_{\mu^2}(g_i(t),\,\lambda(t))\,, && (3)\\ \beta_{\mu^2} &= 2\gamma + \frac{3\lambda}{4\pi^2}\,, && (4)\\ \frac{dg_i}{dt} &= \beta_{g_i}(g_i(t),\,\lambda(t)) && (5)\end{aligned}$$

and

$$Z(t) = \exp\left(-\int_0^t dt'\gamma\,(\,g_i(t'),\,\lambda(t'))\right) \tag{6}$$

with parameter $t = \ln(\phi/M), M$ being the mass scale. Other symbols have been explained elsewhere [7]. The minimum condition is given as

$$\left.\frac{\partial V_{\rm eff}}{\partial\phi}\right|_{\langle\phi\rangle=v} = 0\,. \tag{7}$$

The Higgs mass is given by

$$m_H^2 = \left.\frac{\partial^2 V_{\rm eff}}{\partial \phi^2}\right|_{\langle\phi\rangle = v} . \tag{8}$$

From Eq. (1) we get

$$\begin{aligned}\frac{dV_{\rm eff}}{d\phi} = \frac{1}{2}\left\{\frac{d}{dt}\left(\mu^2(t)\right)\right\} Z^2\left(t\right)\phi^2\frac{dt}{d\phi} + \mu^2\left(t\right)Z\left(t\right)\phi^2\frac{dZ}{dt}\frac{dt}{d\phi} + \mu^2\left(t\right)Z^2\left(t\right)\phi \\ + \frac{1}{8}Z^4(t)\,\phi^4\frac{d\lambda}{dt}\frac{dt}{d\phi} + \frac{1}{2}\lambda\left(t\right)Z^3(t)\,\phi^4\frac{dZ}{dt}\frac{dt}{d\phi} + \frac{1}{2}\lambda\left(t\right)Z^4(t)\,\phi^3 .\end{aligned} \tag{9}$$

Using Eq. (3) and (4) we obtain

$$\begin{aligned}\frac{dV_{\rm eff}}{d\phi} &= \frac{1}{2}\mu^2(t)\left\{2\gamma + \frac{3\lambda}{4\pi^2}\right\} Z^2\left(t\right)\phi + \mu^2\left(t\right)Z(t)\phi\frac{dZ}{dt} + \mu^2(t)Z^2(t)\phi \\ &\quad + \frac{1}{8}Z^4\left(t\right)\phi^3\frac{d\lambda}{dt} + \frac{1}{2}\lambda(t)\,Z^3(t)\phi^3\frac{dZ}{dt} + \frac{1}{2}\lambda(t)Z^4(t)\phi^3 .\end{aligned} \tag{10}$$

Use of Eq. (7) gives

$$\mu^2\left(t\right) = \frac{\left\{-\frac{1}{8}Z^4\left(t\right)\frac{d\lambda}{dt} - \frac{1}{2}\lambda\left(t\right)Z^3\left(t\right)\frac{dZ}{dt} - \frac{1}{2}\lambda\left(t\right)Z^4\left(t\right)\right\}v^2}{\frac{1}{2}Z^2\left(t\right)\left\{2\gamma + 3\lambda/(4\pi^2)\right\} + Z\left(t\right)\frac{dZ}{dt} + Z^2\left(t\right)} . \tag{11}$$

Hence, using Eq. (8) the value of Higgs mass is given by

$$\begin{aligned}m_H^2 &= \mu^2\left(t\right)\left[\left\{2\gamma + \frac{3\lambda}{4\pi^2}\right\}Z^2\gamma + Z^2\frac{d\gamma}{dt} + 2\gamma\,Z\,\frac{dZ}{dt} + Z^2\gamma + \frac{3Z^2\frac{d\lambda}{dt}}{8\pi^2}\right. \\ &\quad + \frac{3\lambda Z^2\left\{2\gamma + \frac{3\lambda}{4\pi^2}\right\}}{8\pi^2} + \frac{6\lambda Z\frac{dZ}{dt}}{8\pi^2} + \frac{3\lambda Z^2}{8\pi^2} + Z\frac{dZ}{dt} + Z\frac{dZ}{dt}\left\{2\gamma + \frac{3\lambda}{4\pi^2}\right\} \\ &\quad + \left.\left(\frac{dZ}{dt}\right)^2 + Z\frac{d^2Z}{dt^2} + Z^2 + Z^2\left\{2\gamma + \frac{3\lambda}{4\pi^2}\right\} + 2Z\frac{dZ}{dt}\right] + \frac{Z^4v^2\frac{d^2\lambda}{dt^2}}{8} \\ &\quad + Z^3v^2\frac{d\lambda}{dt}\frac{dZ}{dt} + \frac{3Z^4v^2\frac{d\lambda}{dt}}{8} + \frac{3}{2}Z^2\lambda v^2\left(\frac{dZ}{dt}\right)^2 + \frac{3}{2}\lambda Z^3v^2\frac{dZ}{dt} \\ &\quad + \frac{1}{2}\lambda Z^3v^2\frac{d^2Z}{dt^2} + \frac{1}{2}Z^4v^2\frac{d\lambda}{dt} + 2Z^3\lambda v^2\,\frac{dZ}{dt} + \frac{3}{2}\lambda Z^4v^2 .\end{aligned} \tag{12}$$

The expression for $d\lambda/dt$ is given by [7]

$$\frac{d\lambda}{dt} = 4\lambda\gamma + \frac{12\lambda^2 + B}{8\pi^2} , \tag{13}$$

where
$$B=\frac{3}{64\pi^2}\left[\frac{1}{16}\left(3g^4+2g^2g\prime^2+g\prime^2\right)-g_{\rm Y}^4\right], \tag{14}$$

with g, $g\prime$ and $g_{\rm Y}$ being SU(2), U(1) and top Yukawa couplings, respectively.

Now, the expression for γ is given by

$$\gamma=\frac{-9g^2-3g\prime^2+12g_Y^2}{64\pi^2}. \tag{15}$$

Since g, $g\prime$ and $g_{\rm Y}$ vary extremely slowly within the energy range of electroweak theory, we assume that γ is a constant. Hence $d\gamma/dt=0$.

Now, for the above approximation γ is a constant in the expression for $Z(t)$ (Eq. (6)) giving $Z(t)=\exp(-\gamma t)$, which is used in equation (12) to obtain the expression for the Higgs mass as

$$\begin{aligned}
m_H^2 \;=\; & \mu^2(t)\Bigg[\left\{2\gamma+\frac{3\lambda}{4\pi^2}\right\}\gamma\exp(-2\gamma t)+2\gamma\,\frac{dZ}{dt}\,\exp(-\gamma t)+\gamma\exp(-2\gamma t)\\
&+\frac{3\exp(-2\gamma t)\,\frac{d\lambda}{dt}}{8\pi^2}+\frac{3\lambda\exp(-2\gamma t)\left\{2\gamma+\frac{3\lambda}{4\pi^2}\right\}}{8\pi^2}+\frac{6\lambda\exp(-\gamma t)\,\frac{dZ}{dt}}{8\pi^2}\\
&+\frac{3\lambda\exp(-2\gamma t)}{8\pi^2}+\exp(-\gamma t)\,\frac{dZ}{dt}+\exp(-\gamma t)\,\frac{dZ}{dt}\left\{2\gamma+\frac{3\lambda}{4\pi^2}\right\}+\left(\frac{dZ}{dt}\right)^2\\
&+\exp(-\gamma t)\,\frac{d^2Z}{dt^2}+\exp(-2\gamma t)\left\{2\gamma+\frac{3\lambda}{4\pi^2}\right\}+2\exp(-\gamma t)\,\frac{dZ}{dt}\Bigg]\\
&+\frac{\exp(-4\gamma t)\,v^2\frac{d^2\lambda}{dt^2}}{8}+\exp(-3\gamma t)\,v^2\,\frac{d\lambda}{dt}\,\frac{dZ}{dt}+\frac{3\exp(-4\gamma t)\,v^2\frac{d\lambda}{dt}}{8}\\
&+\frac{3}{2}\exp(-2\gamma t)\,\lambda v^2\left(\frac{dZ}{dt}\right)^2+\frac{3}{2}\lambda\exp(-3\gamma t)\,v^2\frac{dZ}{dt}\\
&+\frac{1}{2}\lambda\exp(-3\gamma t)\,v^2\frac{d^2Z}{dt^2}+\frac{1}{2}\exp(-4\gamma t)\,v^2\frac{d\lambda}{dt}\\
&+2\exp(-3\gamma t)\,\lambda v^2\,\frac{dZ}{dt}+\frac{3}{2}\lambda\exp(-4\gamma t)\,v^2,
\end{aligned} \tag{16}$$

where $\mu^2(t)$ is given in equation (11).

3. Discussions and conclusions

The values of $g, g\prime$ and $g_{\rm Y}$ are obtained from the well known relations given by

$$g=\frac{2M_{\rm W}}{v},\qquad g\prime^2=\frac{4M_{\rm Z}^2}{v^2}-g^2,\qquad g_{\rm Y}=\frac{\sqrt{2}m_{\rm t}}{v}. \tag{17}$$

The values of the parameters used here for the calculation of upper bound on Higgs mass are as follows: $v = 246\,\mathrm{GeV}$, $M_{\mathrm{W}} = 80\,\mathrm{GeV}$, $M_{\mathrm{Z}} = 91\,\mathrm{GeV}$, $m_{\mathrm{t}} = 175\,\mathrm{GeV}$, $g = 0.6504$, $g\prime = 0.3525$, $g_{\mathrm{Y}} = 1.0060462, \gamma = 0.0126$, $B = -0.004634$, $\phi = v = 246\,\mathrm{GeV}$. Using the above parameters, we calculate the values of the Higgs mass (m_H) for different values of the Higgs self coupling constant λ using Eq. (16). The detailed variation of Higgs mass with λ for different M are given in Table I.

TABLE I

Higgs self coupling constant (λ)	Higgs mass (m_H)					
	160 GeV	180 GeV	200 GeV	220 GeV	240 GeV	250 GeV
0.369	149.981	150.427	150.827	151.189	151.521	151.677
0.380	152.479	152.932	153.339	153.707	154.045	154.203
0.392	154.941	155.402	155.815	156.190	156.533	156.694
0.405	157.370	157.838	158.257	158.638	158.986	159.150
0.417	159.766	160.241	160.667	161.053	161.407	161.573
0.429	162.131	162.613	163.046	163.438	163.796	163.965
0.441	164.467	164.956	165.394	165.792	166.156	166.327
0.453	166.774	167.270	167.715	168.118	168.487	168.660
0.465	169.054	169.557	170.008	170.417	170.791	170.966
0.477	171.309	171.818	172.275	172.689	173.068	173.246
0.489	173.538	174.053	174.516	174.936	175.320	175.500
0.500	175.743	176.265	176.734	177.159	177.547	177.730
0.513	177.924	178.453	178.928	179.358	179.752	179.937
0.525	180.084	180.619	181.099	181.535	181.933	182.120
0.537	182.222	182.763	183.249	183.690	184.093	184.282
0.549	184.339	184.886	185.378	185.824	186.232	186.423
0.561	186.435	186.990	187.487	187.938	188.350	188.544
0.573	188.513	189.073	189.576	190.032	190.449	190.645
0.585	190.572	191.138	191.646	192.107	192.529	192.727
0.597	192.612	193.185	193.698	194.164	194.590	194.791
0.609	194.635	195.214	195.733	196.203	196.634	196.836
0.613	195.309	195.892	196.411	196.882	197.315	197.517

From Table I, we find that the values of m_H is almost insensitive to the values of M, when M is varied from 160 GeV to 250 GeV (low energy) for the values of λ, lying within the bounds $0.369 \leq \lambda \leq 0.613$. We find that the Higgs mass lies within the range of $150.4\,\mathrm{GeV} \leq m_H \leq 195.9\,\mathrm{GeV}$. These results are very close to those of a previous work [8]. We hope that the above results would be useful to the LHC experimenters for the detection of Higgs particle of the Standard Model.

We now compare our results (*viz.* $150.4\,\mathrm{GeV} \leq m_H \leq 195.9\,\mathrm{GeV}$) with several other earlier predictions for the Higgs mass given by different authors, who obtained the Higgs mass bounds in the standard model by different approaches. For example, the current direct experimental bound based on the LEP2 data [9] is $m_H > 113.2\,\mathrm{GeV}$. Moreover, the upper limit based from the indirect precision electroweak data [10] is $m_H \leq 211\,\mathrm{GeV}$ at 95% C.L. Next, on the theoretical side, Ford *et al.* [11] applied the RGE to the effective potential of the standard model by imposing the condition $\lambda\,(t) \geq 0$, for all t, obtaining a lower bound on m_H given by $m_H \geq 152\,\mathrm{GeV}$ for $m_{\mathrm{t}} = 175\,\mathrm{GeV}$. Besides this, Pasupathy [12] has been able to fix the Higgs mass within the narrow limits at $m_H = 160\,\mathrm{GeV}$ using only the values of gauge couplings and top mass, under the assumption that the ratio of Higgs self-coupling to the square of its Yukawa coupling to the top is (almost) independent of the renormalization scale. Now, Pirogov and Zenin [13] investigated the two-loop renormalization group global profile of the Standard Model (SM) in its full parameter space and found that the cut-off equal to the Plank scale requires the Higgs mass bound to be $m_H \geq 140.7^{+10}_{-10}\,\mathrm{GeV}$ and under precision experiment restriction $m_H \leq 200\,\mathrm{GeV}$. Finally, Lykken [14] found that the physical Higgs boson mass should lie within the bound $130\,\mathrm{GeV} \leq m_H \leq 180\,\mathrm{GeV}$. Thus, the above results of different authors match well with the bounds on Higgs mass given in this paper.

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