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A SHELL MODEL CALCULATION FOR ^{52}Fe IN THE FULL fp SPACE

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We discuss a shell model calculation for ^{52}Fe in the full fp space using the GXPF1A interaction. Several energy levels for the same angular momentum are obtained. The results for the energy levels and transition rates are compared with previous calculations. Using collective models an attempt is made to classify the spectrum into bands.

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1. Introduction

Ideally we would like to have a description of collective properties of nuclei in terms of spherical shell model results. In practice this goal is hampered by the size of the Hilbert space which limits the potentials of the shell model approach to the description of nuclear energy levels, expectation values and transition rates. The traditional shell model approach (see for example Ref. [1,2] and references in there) consists first in the determination of the effective Hamiltonian, either by modifying renormalized nucleon–nucleon interactions, or simply by fitting all possible two-body matrix elements and single-particle energies to the available experimental data. Subsequently the Hamiltonian matrix, written in a selected single-particle space, is diagonalized. The most popular method used in the shell model approach is the Lanczos method and nowadays there are several computer codes that are used to describe the low-energy properties of nuclei.

It should be stressed that the shell model approach is not the only method that can be used in the description of nuclear spectra. Other methods such as The VAMPIR method and its extensions (Ref. [3]), the Quantum Monte Carlo Method (Ref. [4]) and the Hybrid Multi-determinant method (Ref. [5]) do not have the intrinsic limitations of the shell model diagonalization method, since they can approach the exact results as the number of trial wave functions is increased, and are being used and have been used in the description of nuclei especially in situations where the shell model diagonalization method is too demanding for the computers available today. The analysis of collective properties is usually carried out first diagonalizing the nuclear Hamiltonian and evaluating expectation values and B(E2) and then comparing the results with the predictions of collective models. In this way, for example, deformed bands (and a superdeformed band) have been identified in closed shells nuclei as ^{56}Ni (Ref. [6]) and ^{40}Ca (Ref. [7]).

In this work we perform a shell model calculation for ^{52}Fe using the full fp space and the modern GXPF1A interaction (Ref. [8]) for several states with spin $0, 2, 3, 4, 5, 6, 7, 8, 10$ and 12, altogether 34 levels, group these states according to the their B(E2) values into bands, and analize the results in terms of collective models. One of the motivations for calculating several states with the same spin and parity originally was whether this nucleus exibits states at high energy compatible with large deformations. The choice of the nucleus is motivated by the fact that the size of the Hilbert space is not prohibitively large and therefore a large number of levels can be computed without truncation of the Hilbert space, which can affect the results at large excitation energy. Moreover, it is interesting to see whether the shell model results are compatible with a stable triaxial deformation (this nucleus has an experimental and theoretical low energy 2^+_2 state, a typical signature of stable triaxial deformation). Previously this nucleus has been studied in the full fp space in Ref. [9] using the KB3 interaction for some even spins, and in Ref. [10] in a truncated Hilbert space using the KB3G interaction. The present study gives the prediction of the GXPF1A interaction for a large number of levels without truncation of the Hilbert space for even and odd spins. As an additional motivation, we would like to see whether the GXPF1A interaction predicts the experimental energy inversion between the isomeric 12^+_1 state and the the 10^+_1 state, since, as mentioned in Ref. [10], the GXPF1 interaction does not reproduce the experimental relative position of these two states.

2. Results and discussion

The shell model code used in this work was written in the same spirit of the code ANTOINE (Ref. [11]) and it is the same used in Ref. [12]. In Table I we give the theoretical energies and spectroscopic quadrupole moments of four $J^\pi = 0^+, 3^+, 4^+, 6^+$ states, five $J^\pi = 2^+$ states, three $J^\pi = 5^+, 7^+, 8^+, 10^+$ states and for the yrast 12^+ state. The experimental levels are taken from Ref. [13]. As in the case of the GXPF1 interaction reported in Ref. [10], the energy inversion between the 10^+ state and the 12^+ state is not reproduced, although the 10^+ state is lower than the 12^+ by only 4 keV.

TABLE I

Shell model energies in MeV and quadrupole moments in $e\text{fm}^2$ for ^{52}Fe. Experimental energies are from Ref. [13].

J^+_n	$E_{\rm th}$(MeV)	$Q_{\rm s}$($e$fm^2)	$E_{\rm exp}$(MeV)
0^+_1	0.0	0	0.0
2^+_1	0.884	−30.9	0.849
4^+_1	2.435	−38.5	2.385
2^+_2	2.669	33.9	2.760
3^+_1	3.198	−1.4	
4^+_2	3.344	−9.9	3.587
0^+_2	3.592	0	4.146
2^+_3	3.871	−25.6	4.456
6^+_1	4.206	−17.1	4.326
3^+_2	4.388	52.5	
4^+_3	4.429	28.0	
5^+_1	4.575	−19.3	
6^+_2	4.650	6.9	4.872
6^+_3	4.708	0.4	
5^+_2	4.768	−12.5	
0^+_3	4.916	0	
2^+_4	4.924	−16.8	
4^+_4	5.017	−14.2	
2^+_5	5.303	31.9	
3^+_3	5.394	9.5	
6^+_4	5.411	52.2	
3^+_4	5.559	−17.1	
5^+_3	5.617	15.0	
7^+_1	5.906	21.5	
7^+_2	5.996	−21.7	
8^+_1	5.997	−14.3	6.360
8^+_2	6.040	−22.8	6.493
0^+_4	6.056	0	
7^+_3	6.219	32.9	
8^+_3	6.661	1.0	
10^+_1	6.799	20.9	7.381
12^+_1	6.803	54.8	6.957
10^+_2	7.804	−5.8	
10^+_3	7.854		

The agreement with the experimental yrast states for $J^\pi = 2^+, 4^+, 6^+$ is good and the energy of the yrast 8^+ is only 363 keV below the experimental value, this discrepancy increases for the yrast 10^+ (which is predicted 482 keV lower the experimental value), while the 12^+ energy is in good agreement with the experimental value. The 4^+_2 and the 6^+_2 energies are also in good agreement with the data. The 0^+_2 is lower than the experimental value by 554 keV. In Ref. [10] it was found that the KB3G interaction predicts this level about 600 keV above the experimental value, in a truncated Hilbert space.

TABLE II

B(E2) in e^2fm^4 for selected transitions.

J^+_i	J^+_f	B(E2)	J^+_i	J^+_f	B(E2)	J^+_i	J^+_f	B(E2)	J^+_i	J^+_f	B(E2)
2^+_1	0^+_1	216.1	5^+_1	3^+_1	226.0	2^+_4	0^+_2	129.1	3^+_3	3^+_1	0.5
4^+_1	2^+_1	283.6		3^+_2	35.8		0^+_3	25.2		3^+_2	3.6
2^+_2	0^+_1	57.1		4^+_1	47.6		2^+_3	70.8		4^+_1	1.6
	2^+_1	11.7		4^+_2	67.5		3^+_2	4.6		4^+_2	0.3
	4^+_1	0.1		4^+_3	131.2		4^+_3	0.9		4^+_3	23.0
3^+_1	2^+_1	74.6		6^+_1	0.0	2^+_5	0^+_1	0.5		4^+_4	3.0
	2^+_2	406.4	6^+_2	6^+_1	99.3		0^+_2	2.0		5^+_1	4.4
	4^+_1	1.0		4^+_1	97.9		0^+_3	3.5		5^+_2	3.3
4^+_2	2^+_1	65.1		4^+_2	197.3		2^+_1	2.5	3^+_4	2^+_1	0.4
	2^+_2	214.3		4^+_3	7.6		2^+_2	0.2		2^+_2	5.2
	3^+_1	352.3		5^+_1	115.0		2^+_3	1.2		2^+_3	54.9
	4^+_1	4.2	6^+_3	6^+_1	119.5		2^+_4	1.2		2^+_4	36.7
0^+_2	2^+_1	41.6		4^+_1	0.0		4^+_1	0.2		2^+_5	6.2
	2^+_2	324.7		6^+_2	72.8		4^+_2	0.0		3^+_1	14.0
2^+_3	0^+_1	6.9		5^+_1	123.7		4^+_3	0.0		3^+_2	4.4
	0^+_2	125.7		4^+_2	197.6		4^+_4	0.0		3^+_3	3.4
	2^+_1	42.2		4^+_3	12.0	4^+_4	2^+_1	7.7		4^+_1	53.6
	2^+_2	32.4	5^+_2	3^+_1	29.3		2^+_2	3.6		4^+_2	0.0
	3^+_1	181.0		3^+_2	58.8		2^+_3	86.3		4^+_3	0.0
	4^+_1	0.5		4^+_1	29.8		2^+_4	12.2		4^+_4	78.7
	4^+_2	110.9		4^+_2	63.2		3^+_1	35.7		5^+_1	2.7
6^+_1	4^+_1	197.5		4^+_3	303.9		3^+_2	6.6		5^+_2	3.4
	4^+_2	0.0		5^+_1	272.7		4^+_1	73.8	6^+_4	4^+_1	0.1
3^+_2	2^+_1	0.0		6^+_1	2.9		4^+_2	0.3		4^+_2	0.0
	2^+_2	0.1		6^+_2	5.4		4^+_3	0.1		4^+_3	0.0
	2^+_3	1.4		6^+_3	0.4		5^+_1	96.6		4^+_4	0.2
	4^+_1	0.0	0^+_3	2^+_1	58.9		5^+_2	25.4		5^+_1	0.1
	4^+_2	4.7		2^+_2	0.5		6^+_1	8.6		5^+_2	0.1
	3^+_1	0.1		2^+_3	8.1		6^+_2	4.8		6^+_1	0.0
4^+_3	2^+_1	0.5	2^+_4	0^+_1	3.0		6^+_3	12.0		6^+_2	0.5
	2^+_2	16.8		2^+_1	7.0	3^+_3	2^+_1	1.4		6^+_3	0.2
	2^+_3	0.7		4^+_1	18.2		2^+_2	0.2	5^+_3	3^+_1	21.0
	3^+_1	19.3		2^+_2	37.1		2^+_3	0.0		3^+_2	29.0
	3^+_2	352.2		3^+_1	7.4		2^+_4	4.6		3^+_3	0.0
	6^+_1	0.1		4^+_2	7.5		2^+_5	413.5		3^+_4	0.5

In Table II and Table III we show the shell model B(E2) values. We have used effective charges $1.5e$ and $0.5e$ for protons and neutrons respectively, a common choice in this mass region.

TABLE III

The same as TABLE II.

J_i^+	J_f^+	B(E2)	J_i^+	J_f^+	B(E2)	J_i^+	J_f^+	B(E2)	J_i^+	J_f^+	B(E2)
5_3^+	4_1^+	0.2	6_4^+	6_3^+	0.2	7_3^+	6_1^+	2.3	8_2^+	7_1^+	25.5
	4_2^+	8.8	7_1^+	5_1^+	20.7		6_2^+	1.4		7_2^+	151.7
	4_3^+	18.2		5_2^+	9.5		6_3^+	1.7	8_3^+	6_1^+	5.4
	4_4^+	0.6		5_3^+	12.0		6_4^+	132.5		6_2^+	0.6
	5_1^+	24.0		6_1^+	17.6		7_1^+	0.1		6_3^+	22.9
	5_2^+	8.8		6_2^+	72.5		7_2^+	0.7		6_4^+	0.3
	6_1^+	1.8		6_3^+	39.5		8_1^+	0.3		7_1^+	72.1
	6_2^+	1.5		6_4^+	3.4		8_2^+	0.0		7_2^+	4.6
	6_3^+	3.0	7_2^+	5_1^+	217.7	8_1^+	6_1^+	129.8		7_1^+	1.0
	6_4^+	1.0		5_2^+	24.8		6_2^+	1.9		8_1^+	215.0
6_4^+	4_1^+	0.1		5_3^+	4.7		6_3^+	21.3		8_2^+	2.8
	4_2^+	0.0		6_1^+	1.5		6_4^+	0.6	10_1^+	8_1^+	85.3
	4_3^+	0.0		6_2^+	0.4		7_1^+	41.4		8_2^+	0.0
	4_4^+	0.2		6_3^+	59.5		7_2^+	0.0		8_3^+	0.3
	5_1^+	0.1		6_4^+	0.3	8_2^+	6_1^+	29.0	10_2^+	8_1^+	0.1
	5_2^+	0.1	7_3^+	5_1^+	0.0		6_2^+	251.8		8_2^+	268.3
	6_1^+	0.0		5_2^+	0.0		6_3^+	156.4		8_3^+	5.6
	6_2^+	0.5		5_3^+	0.0		6_4^+	0.0			

In order to analyze the shell model results we use the following expressions for the spectroscopic quadrupole moment, in the case of the axial rotor model (Ref. [14])

$$Q_s = Q_0 \frac{3K^2 - J(J+1)}{(J+1)(2J+3)}, \tag{1}$$

Q_0 being the intrinsic quadrupole moment. For transitions between levels of the same K band, in this model the expression of the $B(\mathrm{E2}, J_i \to J_f)$ is

$$B(\mathrm{E2}, J_i \to J_f) = \frac{5}{16\pi} Q_t^2 \, \langle J_i \, K \, 2 \, 0 \, | J_f \, K \rangle^2 \tag{2}$$

as customarily we take $Q_t = Q_0$. For transition between levels belonging to a $K = 2$ band and a $K = 0$ band we take

$$B(\mathrm{E2}, J_i K \to J_f, 0) = M \, \langle J_i \, K \, 2 \, -2 | J_f \, 0 \rangle^2 \, , \tag{3}$$

where M is a constant, since intraband transition may not be sufficient to verify the interpretation of the shell model results in terms of collective models. Since the 2_2^+ level is low in energy, we consider also the rigid triaxial model (Ref. [15]), which might be useful in analyzing the low energy part of the spectrum. We use the tables of Ref. [16] which give the spectroscopic quadrupole moments in units of

$$Q_0' = \frac{3eZR^2\beta'}{\sqrt{5\pi}}\,, \tag{4}$$

where Z is the proton number, $R = 1.2A^{1/3}$fm is the nuclear radius and β' is the deformation variable pertinent to the triaxial case (the $'$ is needed in order to distinguish its value from the case of the axial rotor). The unit for the B(E2) values in the tables of Ref. [16] is ${Q_0'}^2/(16\pi)$. The deformation variable β relevant to the axial case can be extracted from $Q_{\rm s}(2_1^+)$ using the expression

$$Q_0 = \frac{3eZR^2\beta(1+0.36\beta)}{\sqrt{5\pi}}\,. \tag{5}$$

For the rigid triaxial model the γ deformation variable, which determines the deviation from axial symmetry, is determined from the ratio $E(2_2^+)/E(2_1^+)$ which in this model is given by the expression

$$\frac{E(2_2^+)}{E(2_0^+)} = \frac{3+x}{3-x}\,, \qquad x = \sqrt{9 - 8\sin^2(3\gamma)}\,. \tag{6}$$

The value of the deformation variable β' can conveniently be determined from the expression for $B(\mathrm{E2} - 2_1^+ \to 0_1^+)$ which is given by the expression

$$B(\mathrm{E2}, 2_1^+ \to 0_1^+) = \frac{{Q_0'}^2}{16\pi}\frac{1}{2}\left[1 + \frac{3 - 2\sin^2(3\gamma)}{x}\right] \tag{7}$$

since this quantity has only a mild dependence in the variable γ. Also, in the rigid triaxial model the position of the first 3^+ state (not known experimentally) is fixed by the position of the 2_1^+ and 2_2^+ states as $E(3_1^+) = E(2_1^+) + E(2_2^+)$. In the shell model calculation this relation is reproduced within 10% error. The analysis of the shell model results with these models can be helpful in identifying bands characterized by large intraband B(E2) values (in the case of the triaxial model some intraband B(E2) can be 0, *cf.* Ref. [16]).

The interpretation of the shell model results in terms of the $\gamma-$unstable model of Wilets and Jean (Ref. [17]), at least in its simplest version, can be ruled out by looking at allowed transition in the model, for example the $3_1^+ \to 4_1^+$. However, this transition is strongly suppressed in the shell model calculation. Large B(E2) values are typically of the order of 10^2 e^2fm^4

while $B(\mathrm{E2}, 3_1^+ \to 4_1^+) = 1.\ e^2\mathrm{fm}^4$ (the Weisskopf unit for this nucleus is $11.5\ e^2\mathrm{fm}^4$). Moreover, in the γ-unstable model, the transition $3_1^+ \to 2_1^+$ is forbidden. Instead this transition is reasonably large in the shell model calculation since $B(E2, 3_1^+ \to 2_1^+) = 74.6\ e^2\mathrm{fm}^4$.

Using the shell model value for $E(2_2^+)/E(2_0^+)$ ratio and Eq. (6), assuming rigid triaxiality, we have $\gamma = 22.24°$ and from the expression of $B(\mathrm{E2}, 2_1^+ \to 0_1^+)$ in Eq. (7) we have $\beta' = +0.27$ (since $Q_{\mathrm{s}}(2_1^+) < 0$). If we assume axial symmetry, the value of β is 0.24, slightly lower than the value obtained in Ref. [10] using the KB3G Hamiltonian. In Table III, we show a comparison between the intraband B(E2)'s for the gs band and the quasi-γ band (the $2_2^+, 3_1^+, 4_2^+, 5_1^+, 6_2^+, 7_1^+, 8_2^+$ states) given by the shell model, with the triaxial (we use the tables for $\gamma = 22.5°$, of Ref. [16]) and the axial rotor model and the experimental values reported in Ref. [9]. Although the agreement is satisfactory at low spins, the collective models predict too large B(E2) at high spin (we considered energy independent deformation variables) compared with both experimental and shell model values.

The interband quasi-γ to gs band B(E2)'s, shown in Table IV, are far more difficult to reproduce. The coefficient M in Eq. (3), which is needed for the interband B(E2) in the case of the axial rotor model, has been fixed using the $B(\mathrm{E2}, 2_2^+ \to 0_1^+)$ shell model value. Both models fail to reproduce the shell model values although the axial rotor model is in fair agreement with the scarce experimental data. Although it seems that the bands have a mixed character it does not seem to be the K-mixing of the triaxial model.

The spectroscopic quadrupole moments for the triaxial rotor, the axial rotor and the corresponding shell model values are shown in Table V. The intrinsic Q_0 was determined from the $B(\mathrm{E2}, 2_1^+ \to 0_1^+)$ shell model value. Overall it seems that there is a considerable K mixing and that both models are not fully adequate for this nucleus.

From the $B(\mathrm{E2})(2_3^+ \to 0_2^+)$ and $B(\mathrm{E2})(4_4^+ \to 2_3^+)$ values of Table III it is plausible to interpret the $0_2^+, 2_3^+, 4_4^+$ states as members of a "quasi-β" band, although the intraband B(E2) should increase with the angular momentum. The large B(E2) between 0_2^+ and 2_2^+ ($324\ e^2\mathrm{fm}^4$) point out to a very strong (and unexpected) coupling between this "quasi-β" band and the quasi-γ band. Also the 2_3^+ has a large B(E2) to the 3_1^+ and 4_2^+

The 3_2^+ state (*cf.* Table II) has no B(E2) to the lower levels and, if collective, are not part of any quasi-γ band, since both axial and triaxial rotor models predict $Q_{\mathrm{s}} = 0$ for such a state. On the other hand, $Q_{\mathrm{s}}(3_2^+)$ is large and it is tempting to classify this state as the band head of a $K = 3$ band. If we assign to this state a $K = 3$, then $Q_0 = 126\ e\mathrm{fm}^2$. Other possible members of this band (very likely of strongly mixed nature) are the the $4_3^+, 5_2^+, 6_3^+$ states (these last two states are nearly degenerate). The interpretation of these states as members of a $K = 3$ band is not entirely con-

TABLE IV

Shell model gs and γ intraband and interband B(E2) in e^2fm^4 and their corresponding triaxial rotor ($\gamma = 22.5°$) and axial rotor values. The experimental values are from Ref. [9].

$J_i^+ \to J_f^+$	B(E2)(SM)	B(E2)(triaxial)	B(E2)axial	Exp.
$2_1^+ \to 0_1^+$	216.1	216.5	216	
$4_1^+ \to 2_1^+$	283.6	315.5	309	300 ± 69
$6_1^+ \to 4_1^+$	197.5	385	340	124 ± 40
$8_1^+ \to 6_1^+$	129.8	428	356	74 ± 25
$4_2^+ \to 2_2^+$	214.3	103.6	129	
$5_1^+ \to 3_1^+$	226	207	206	
$6_2^+ \to 4_2^+$	197.3	193	254	
$7_1^+ \to 5_1^+$	20.7	293	285	
$8_2^+ \to 6_2^+$	251.8	233	306	
$3_1^+ \to 2_2^+$	406.4	387	386	
$4_2^+ \to 3_1^+$	352.3	157	288	
$5_1^+ \to 4_2^+$	67.5	221	206	
$6_2^+ \to 5_1^+$	115.	32	152	
$7_1^+ \to 6_2^+$	72.5	143	116	
$8_2^+ \to 7_1^+$	25.5	5	91	
$2_2^+ \to 0_1^+$	57.1	14	57.1*	
$2_2^+ \to 2_1^+$	11.7	129	82	
$2_2^+ \to 4_1^+$	0.1		4	
$3_1^+ \to 2_1^+$	74.6	25	102	
$3_1^+ \to 4_1^+$	1.0		41	
$4_2^+ \to 2_1^+$	65.1	2	34	
$4_2^+ \to 4_1^+$	4.2	78	100	
$5_1^+ \to 4_1^+$	47.6	2	91	
$5_1^+ \to 6_1^+$	0.0		52	
$6_2^+ \to 4_1^+$	97.9	2	28	29 ± 14
$6_2^+ \to 6_1^+$	99.3	32	104	

* This value was used to determine the coefficient M.

sistent, since the ratios $Q_s(4_3^+)/Q_s(3_2^+), Q_s(5_2^+)/Q_s(3_2^+)$ and $Q_s(6_3^+)/Q_s(3_2^+)$ obtained from Eq. (3) are not consistent with the corresponding shell model ratios. The B(E2)'s of these states to all others lying below in energy shows that this "band" is of mixed nature.

The 2_5^+ state has negligible B(E2)'s to all states lying below. On the other hand, the 3_3^+ state has a rather large B(E2) to 2_5^+ (413.5 e^2fm^4), a small quadrupole moment, $Q(3_3^+) = 9.5$ $e$fm^2, and very small B(E2) to all other lower states. Using Eq. (1), with the assumption of $K = 2$, we obtain

TABLE V

Shell model, triaxial rotor and axial rotor spectroscopic quadrupole moments for the gs and γ bands in units $e$fm^2. The triaxial rotor values correspond to $\gamma = 22.5°$.

J_n^+	Q_s(Shell model)	Q_s(triaxial)	Q_s(axial)
$2+_1$	-30.86	-23.7	-29.8
$4+_1$	-38.5	-19.4	-37.9
$6+_1$	-17.1	-16	-41.7
$8+_1$	-14.31	-16	-43.9
$10+_1$	20.9	-15	-45.3
$2+_2$	33.91	23.7	29.8
$3+_1$	-1.4	0.0	0.0
$4+_2$	-9.9	-30	-15
$5+_1$	-19.3	-19	-24
$6+_2$	6.9	-44	-30
$7+_1$	21.5	-23	-34
$8+_2$	-22.8	-41	-36

an intrinsic quadrupole moment for the 2_5^+ state $Q_0 = 111.65\,e\text{fm}^2$ and using Eq. (2) for the intraband $K = 2$ B(E2) values, we obtain $B(\text{E2})(3_3^+ \to 2_5^+) = 443$ e^2fm^4, to be compared with the shell model value of 413.5 e^2fm^4. We conclude that these two states are members of a quasi-γ band distinct from the one previously discussed. These band was not seen in previous shell model studies.

In Ref. [10], the 6_2^+ state was interpreted as the band head of a $K = 6$ band. With the GXPF1A interaction such a band has the 6_4^+ and 7_3^+ states as members. In fact, the 6_4^+ state has no appreciable B(E2) to any state lying below and the 7_3^+ has large B(E2) only to 6_4^+ (132.5 e^2fm^4), all other B(E2) are smaller than 2.3 e^2fm^4. As a test we can extract the intrinsic quadrupole moment from $Q_s(6_4^+)$ using Eq. (1) and evaluate the axial rotor prediction for $Q_s(7_3^+)$ and for the intraband B(E2) value. The intrinsic Q_0 for the 6_4^+ state is 83.05 $e$fm^2 and Eq. (1) gives $Q_s(7_3^+) = 31.75$ $e$fm^2 in good agreement with the shell model value 32.9 $e$fm^2. The B(E2) value from Eq. (2) for the transition $7_3^+ \to 6_4^+$ is 191 e^2fm^4 to be compared with the shell model value 132.5 e^2fm^4.

Concluding, we have compared shell model transition rates and quadrupole moments with the triaxial and axial rotor models in their simplest versions. Although too simple to reproduce the low energy part of the spectrum, the axial rotor model has been useful to analyze other bands (a strongly mixed $K = 0$ band, another $K = 2$, possibly a $K = 3$ band and a $K = 6$ band).

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