

SOFT GLUON EFFECTS IN SUPERSYMMETRIC
PARTICLE PRODUCTION AT THE LHC*

A. KULESZA

Institut für Theoretische Physik E, RWTH Aachen
52056 Aachen, Germany
`Anna.Kulesza@physik.rwth-aachen.de`

L. MOTYKA

II Institute for Theoretical Physics, University of Hamburg
Luruper Chaussee 149, 22761, Germany
and
Institute of Physics, Jagellonian University
Reymonta 4, 30-059 Kraków, Poland
`leszek.motyka@desy.de`

(Received May 4, 2009)

We report on recent results A. Kulesza, L. Motyka, *Phys. Rev. Lett.* **102**, 111802 (2009) on soft gluon effects in the production of squark–antisquark and gluino–gluino pairs at the LHC. The soft gluon corrections are resummed at the next-to-leading logarithmic (NLL) accuracy and matched with the known next-to-leading (NLO) order estimates of the cross-sections. The one-loop soft anomalous dimension matrices controlling the colour evolution of the gluino pair production are presented. We show that the resummation of soft gluon effects reduces substantially the theoretical uncertainty for gluino pair-production at the LHC.

PACS numbers: 12.38.Bz, 12.60.Jv

1. Introduction

The Large Hadron Collider opens possibilities to search for new physics effects and new particles, beyond the spectrum described by the Standard Model (SM). Among the most important ones are extensions of the Standard Model that incorporate supersymmetry (SUSY). There exists a variety of supersymmetric models. The natural starting point for phenomenological considerations is supersymmetric model with the minimal content of supersymmetric particles (sparticles) — the Minimal Supersymmetric Standard

* Presented by L. Motyka at the Cracow Epiphany Conference on Hadron Interactions at the Dawn of the LHC, Cracow, Poland, January 5–7, 2009.

Model (MSSM) [2]. Even with the minimal extension of the SM given by MSSM, one still finds many free parameters, in particular the masses of sparticles are free to vary in a wide range.

Within MSSM, however, some of the couplings of the sparticles and the SM particles are related to the gauge couplings of the corresponding SM particles. In consequence, in typical scenarios, the cross-sections for hadroproduction of sparticles at the LHC are largest for final states containing strongly interacting sparticles [3]. These are gluinos and squarks, the superpartners of gluons and quarks, respectively. Due to the R -parity conservation assumed in the MSSM, the sparticles must be produced in pairs. Thus, within the MSSM, the pair-production of gluinos and squark–antisquark production are among the most important channels of the sparticle production. The discovery of squarks and gluinos at the LHC should be possible for masses of up to 2 TeV [4].

The masses of sparticles and their couplings are difficult to measure in hadron colliders. The sparticles tend to decay in long decay cascades, in which heavy sparticles are produced, that interact weakly and cannot be detected. Therefore, some of the kinematic information is lost in the sparticle production events and the produced sparticle masses cannot be determined in the direct way. One of the possible indirect determination of the sparticle masses can be performed using the measurements of the total cross-sections for sparticle production. It follows from the fact, that the total cross-sections depend strongly on the masses of the produced sparticles. Therefore, using suitable theoretical mass-dependent predictions of the total cross-sections, one can extract the values of sparticle masses from the measured cross-sections, see *e.g.* [5]. The accuracy of this determination crucially depends both on the accuracy of the theoretical calculation and on the experimental errors.

The predictions for the total cross-sections are known for all hadroproduction processes of pairs of squarks and gluinos at the leading order (LO) [6] since a long time. The corresponding next-to-leading order (NLO) SUSY-QCD corrections were also calculated [7, 8]. The NLO effects have been found to be positive and large. Among the pair-production processes of coloured sparticles at the LHC, the gluino-pair ($\tilde{g}\tilde{g}$) production receives the largest NLO SUSY-QCD correction [8], that may reach 100% for gluino mass $m_{\tilde{g}} = 1$ TeV. The corrections to the squark–antisquark ($\tilde{q}\tilde{q}$) total cross-section can be also sizable, of order of 30% for the squark mass $m_{\tilde{q}} = 1$ TeV, and are the second largest in a certain range of mass parameters. The occurrence of large corrections indicates that computation of higher order terms of the perturbative expansion is necessary in order to achieve precise theoretical predictions.

The NLO corrections to the total cross-section for the hadroproduction of heavy coloured particles receive a strong contribution from the region of parton collision energy near the kinematic threshold [8]. In more detail, the threshold region is reached when the square of the partonic center-of-mass (c.o.m.) energy, $\hat{s}$, approaches $4m^2$, where m is the average particle mass in the produced pair. The velocity of the produced heavy particles in the partonic c.o.m. system $\beta \equiv \sqrt{1 - 4m^2/\hat{s}}$ is then small, $\beta \ll 1$. In this region, the radiative gluon corrections are enhanced by powers of large logarithms of β . In particular, at the NLO, one finds the relative corrections $\sim \alpha_s \log^2(\beta^2)$ and $\sim \alpha_s \log(\beta^2)$ [8]. These terms emerge as a consequence of large virtual gluon corrections uncompensated by the real corrections. Specifically, the integrals over quantum loops with a virtual gluon are characterised by the upper energy and momentum scale m . The real gluon emission is kinematically constrained by the energy scale $m\beta^2$. In effect, the NLO corrections coming from the real gluon emission cancel the virtual corrections only up to the energy scale $m\beta^2$, and the loop integration beyond the latter scale give rise to the NLO corrections enhanced by the logarithms of the ratio of scales m and $m\beta^2$, that is by $\log^2(\beta^2)$ and by $\log(\beta^2)$. According to the same mechanism, at n -th order of the perturbative expansion in α_s , corrections emerge proportional to $\alpha_s^n \log^k(\beta^2)$ where $k = 2n, \dots, 0$. The logarithmically enhanced contributions can be taken into account to all orders in α_s by means of the threshold resummation.

2. Threshold resummation

In [1] we considered the total hadronic cross-sections for gluino-pair and squark-antisquark production, $pp \rightarrow \tilde{g}\tilde{g}$ and $pp \rightarrow \tilde{q}\tilde{q}$. For these processes we performed the soft gluon resummation at the level of next-to-leading logarithms (NLL).

In order to study the soft gluon effects at the NLL accuracy, one has to take into account the colour flow in partonic processes. Thus, at leading order, two partonic channels contribute to the $\tilde{g}\tilde{g}$ production:

$$q(p_i, \alpha_i) \bar{q}(p_j, \alpha_j) \rightarrow \tilde{g}(p_k, a_k) \tilde{g}(p_l, a_l), \quad (1)$$

and

$$g(p_i, a_i) g(p_j, a_j) \rightarrow \tilde{g}(p_k, a_k) \tilde{g}(p_l, a_l), \quad (2)$$

where p are particle four-momenta and α and a are color indices in the fundamental and adjoint representation of SU(3), correspondingly.

For process (1) the colour basis is given by three colour tensors, corresponding to $\{\mathbf{1}, \mathbf{8}_S, \mathbf{8}_A\}$ representations:

$$\begin{aligned}
c_1^{g,\bar{q}} &= \delta^{\alpha_i \alpha_j} \delta^{a_k a_l} , \\
c_8^{g,\bar{q}} &= T_{\alpha_j \alpha_i}^b d^{ba_k a_l} , \\
c_8^{g,\bar{q}} &= iT_{\alpha_j \alpha_i}^b f^{ba_k a_l} .
\end{aligned} \tag{3}$$

where T^b matrices are the $SU(3)$ generators.

For the $gg \rightarrow \tilde{g}\tilde{g}$ process there are eight independent colour tensors. Following [12] we choose an orthogonal basis, $\{c_I^{g,\tilde{g}}\}$, $I = 1, 2, \dots, 8$, consisting of five tensors $c_1^{g,\tilde{g}}$, $c_2^{g,\tilde{g}}$, $c_3^{g,\tilde{g}}$, $c_4^{g,\tilde{g}}$ and $c_5^{g,\tilde{g}}$ corresponding to the **1**, **8_S**, **8_A**, **10** $\oplus$ **10** and **27** representations in the s -channel, and three additional tensors, $c_6^{g,\tilde{g}}$, $c_7^{g,\tilde{g}}$, and $c_8^{g,\tilde{g}}$. The base tensors are

$$\begin{aligned}
c_1^{g,\tilde{g}} &= \frac{1}{8} \delta^{a_i a_j} \delta^{a_k a_l} , \\
c_2^{g,\tilde{g}} &= \frac{3}{5} d^{a_i a_j b} d^{ba_k a_l} , \\
c_3^{g,\tilde{g}} &= \frac{1}{3} f^{a_i a_j b} f^{ba_k a_l} , \\
c_4^{g,\tilde{g}} &= \frac{1}{2} (\delta^{a_i a_k} \delta^{a_j a_l} - \delta^{a_i a_l} \delta^{a_j a_k}) - \frac{1}{3} f^{a_i a_j b} f^{ba_k a_l} , \\
c_5^{g,\tilde{g}} &= \frac{1}{2} (\delta^{a_i a_k} \delta^{a_j a_l} + \delta^{a_i a_l} \delta^{a_j a_k}) - \frac{1}{8} \delta^{a_i a_j} \delta^{a_k a_l} - \frac{3}{5} d^{a_i a_j b} d^{ba_k a_l} , \\
c_6^{g,\tilde{g}} &= \frac{i}{4} \left(f^{a_i a_j b} d^{ba_k a_l} + d^{a_i a_j b} f^{ba_k a_l} \right) , \\
c_7^{g,\tilde{g}} &= \frac{i}{4} \left(f^{a_i a_j b} d^{ba_k a_l} - d^{a_i a_j b} f^{ba_k a_l} \right) \text{ and} \\
c_8^{g,\tilde{g}} &= \frac{i}{4} \left(d^{a_i a_k b} f^{ba_j a_l} + f^{a_i a_k b} d^{ba_j a_l} \right) .
\end{aligned}$$

For the $\tilde{q}\bar{\tilde{q}}$ production two partonic processes contribute at LO

$$q_i(p_i, \alpha_i) \bar{q}(p_j, \alpha_j) \rightarrow \tilde{q}(p_k, \alpha_k) \bar{\tilde{q}}(p_l, \alpha_l) \tag{4}$$

and

$$g(p_i, a_i) g(p_j, a_j) \rightarrow \tilde{q}(p_k, \alpha_k) \bar{\tilde{q}}(p_l, \alpha_l) . \tag{5}$$

In the quark-channel (4) we have only two possible colour exchanges: the singlet and the octet, **{1, 8}**, and the basis consists of two colour tensors

$$\begin{aligned}
c_1^{q,\bar{q}} &= \delta^{\alpha_i \alpha_j} \delta^{\alpha_k \alpha_l} , \\
c_8^{q,\bar{q}} &= -\frac{1}{6} \delta^{\alpha_i a_j} \delta^{\alpha_k \alpha_l} + \frac{1}{2} \delta^{\alpha_i \alpha_k} \delta^{\alpha_j \alpha_l} .
\end{aligned} \tag{6}$$

In the gluon channel, the basis is identical to the case of $q\bar{q} \rightarrow \tilde{g}\tilde{g}$, given by (3).

Due to the non-trivial colour flow, the soft gluon resummation for the considered processes is governed by the one-loop soft anomalous dimension matrices [9–13] in the bases of the colour tensors. It turns out that in the threshold limit all the soft anomalous dimension matrices for the considered partonic processes of sparticle production, tend to the diagonal form. Hence, in the threshold limit one has,

$$\begin{aligned}
\Gamma^{q\bar{q} \rightarrow \tilde{g}\tilde{g}} &\rightarrow \frac{\alpha_s}{2\pi} \text{diag}(0, -3, -3), \\
\Gamma^{gg \rightarrow \tilde{g}\tilde{g}} &\rightarrow \frac{\alpha_s}{2\pi} \text{diag}(0, -3, -3, -6 - 8; -3, -3, -6), \\
\Gamma^{q\bar{q} \rightarrow \tilde{q}\tilde{q}} &\rightarrow \frac{\alpha_s}{2\pi} \text{diag}(0, -3), \\
\Gamma^{gg \rightarrow \tilde{q}\tilde{q}} &\rightarrow \frac{\alpha_s}{2\pi} \text{diag}(0, -3, -3),
\end{aligned} \tag{7}$$

where the results for $\Gamma^{ij \rightarrow \tilde{g}\tilde{g}}$ were first obtained in Ref. [1].

The resummation of the soft gluon corrections was carried out in Mellin- N space in the variable $\rho = 4m^2/S$ with S being the square of the hadronic c.o.m. energy. In the Mellin space, the moments of the partonic cross-section $ij \rightarrow kl$ are given by

$$\hat{\sigma}_{ij \rightarrow kl, N}(m^2, \mu_F^2, \mu_R^2) \equiv \int_0^1 d\hat{\rho} \hat{\rho}^{N-1} \hat{\sigma}_{ij \rightarrow kl}(\hat{\rho}, m^2, \mu_F^2, \mu_R^2) \tag{8}$$

with $\hat{\rho} = 4m^2/\hat{s}$ and ij denoting the incoming partons and kl the produces sparticles.

In an orthogonal basis in the colour space for which the matrix $\Gamma^{ij \rightarrow kl}$ at threshold is diagonal, the NLL resummed cross-section in the N -space may be written as [10, 14]

$$\hat{\sigma}_{ij \rightarrow kl, N}^{(\text{res})} = \sum_I \hat{\sigma}_{ij \rightarrow kl, I, N}^{(0)} \Delta_{N+1}^i \Delta_{N+1}^j \Delta_{ij \rightarrow kl, I, N+1}^{(\text{int})}, \tag{9}$$

where the explicit dependence on the scales was suppressed. The index I in Eq. (9) distinguishes between contributions from different colour channels. The radiative factors Δ_N^i describe the effect of the soft gluon radiation collinear to the initial state partons and are universal. Soft, non-collinear gluon emission is accounted for by the factors $\Delta_{ij \rightarrow kl, I, N}^{(\text{int})}$ which depend on the partonic process and the colour configuration of the participating particles.

The expressions for the radiative factors in the $\overline{\text{MS}}$ factorisation scheme read (see *e.g.* [14])

$$\ln \Delta_N^i = \int_0^1 dz \frac{z^{N-1} - 1}{1 - z} \int_{\mu_F^2}^{4m^2(1-z)^2} \frac{dq^2}{q^2} A_i(\alpha_s(q^2)),$$

$$\ln \Delta_{ij \rightarrow kl, I, N}^{(\text{int})} = \int_0^1 dz \frac{z^{N-1} - 1}{1 - z} D_{ij \rightarrow kl, I}(\alpha_s (4m^2(1-z)^2)) .$$

The coefficients $\mathcal{F} = A_i$, $D_{ij \rightarrow kl, I}$ are power series in the coupling constant α_s , $\mathcal{F} = (\alpha_s/\pi) \mathcal{F}^{(1)} + (\alpha_s/\pi)^2 \mathcal{F}^{(2)} + \dots$. The universal LL and NLL coefficients $A_i^{(1)}$, $A_i^{(2)}$ are well known [15, 16] and given by

$$A_i^{(1)} = C_i, \quad A_i^{(2)} = \frac{1}{2} C_i \left(\left(\frac{67}{18} - \frac{\pi^2}{6} \right) C_A - \frac{5}{9} n_f \right) \quad (10)$$

with $C_g = C_A = 3$, and $C_q = C_F = 4/3$.

Let us introduce the notation for the eigenvalues of the Γ matrices at threshold, $\gamma_I^{ij \rightarrow kl}$, such that $\Gamma^{ij \rightarrow kl} = (\alpha_s/\pi) \text{diag}(\{\gamma_I^{ij \rightarrow kl}\})$. Then one has $D_{ij \rightarrow kl, I}^{(1)} = 2\text{Re}(\gamma_I^{ij \rightarrow kl})$ [10–12], and,

$$\begin{aligned} \{D_{q\bar{q} \rightarrow \tilde{g}\tilde{g}, I}^{(1)}\} &= \{0, -3, -3\}, \\ \{D_{g\bar{g} \rightarrow \tilde{g}\tilde{g}, I}^{(1)}\} &= \{0, -3, -3, -6 - 8; -3, -3, -6\}, \\ \{D_{q\bar{q} \rightarrow \tilde{q}\tilde{q}, I}^{(1)}\} &= \{0, -3\}, \\ \{D_{g\bar{g} \rightarrow \tilde{q}\tilde{q}, I}^{(1)}\} &= \{0, -3, -3\}. \end{aligned} \quad (11)$$

Note that the values of the $D^{(1)}$ -coefficients are the negative values of the quadratic Casimir operators for the SU(3) representations for the outgoing state. This agrees with the physical picture of the soft gluon radiation from the total colour charge of the heavy-particle pair produced at threshold [14].

3. Phenomenological results

In Ref. [1] we investigated in detail the effect of the soft gluon corrections on the cross-sections of the sparticle production processes at the LHC, $pp \rightarrow \tilde{g}\tilde{g}$ and $pp \rightarrow \tilde{q}\tilde{q}$, at $\sqrt{S} = 14$ TeV. We obtained the resummation improved total cross-sections. We studied also the uncertainty of these cross-sections due to the renormalisation and factorisation scale dependence.

In the phenomenological analysis we considered various choices of gluino and squark masses, assuming that left- and right-handed squarks of all flavours are mass degenerate. For the $\tilde{g}\tilde{g}$ production the gluino mass, $m_{\tilde{g}}$, was varied between 200 GeV and 2 TeV. Similarly, for the $\tilde{q}\tilde{q}$ production we took $200 \text{ GeV} < m_{\tilde{q}} < 2 \text{ TeV}$. We present the results for a fixed ratio of gluino and squark masses, $r = m_{\tilde{g}}/m_{\tilde{q}}$, taking $r = 0.5, 0.8, 1.2, 1.6, 2.0$. The $\tilde{q}\tilde{q}$

cross-section accounts for production of all $\tilde{q}\tilde{q}$ flavour combinations apart from the ones with scalar top particles.

The resummation-improved cross-sections are obtained through matching the NLL resummed expressions with the full NLO cross-sections,

$$\begin{aligned} \sigma_{pp \rightarrow kl}^{(\text{match})}(\rho) = & \sigma_{pp \rightarrow kl}^{(\text{NLO})}(\rho) + \sum_{i,j=q,\bar{q},g} \int_{\mathcal{C}} \frac{dN}{2\pi i} \rho^{-N} f_{i/p}^{(N+1)} f_{j/p}^{(N+1)} \\ & \times \left[\hat{\sigma}_{ij \rightarrow kl,N}^{(\text{res})} - \hat{\sigma}_{ij \rightarrow kl,N}^{(\text{res})} \Big|_{(\text{NLO})} \right], \end{aligned}$$

where the dependence on the renormalisation and factorisation scale is suppressed, $\hat{\sigma}_{ij \rightarrow kl,N}^{(\text{res})}$ is given in Eq. (9) and $\hat{\sigma}_{ij \rightarrow kl,N}^{(\text{res})} \Big|_{(\text{NLO})}$ represents its perturbative expansion truncated at NLO. The contour $\mathcal{C}$ of the inverse Mellin transform (12) in the complex- N plane was chosen according to the “Minimal Prescription” method developed in Ref. [17]. The integrals in (12) were performed numerically.

The NLO cross-sections were evaluated using PROSPINO [18], the numerical package based on calculations employing the $\overline{\text{MS}}$ renormalisation and factorisation schemes. We used the CTEQ6M [19] parameterization of parton distribution functions (pdfs). In the chosen set of pdfs a usual assumption of five massless quark flavours active at large scales is made. Consequently, in the NLO and NLL calculations we use the two-loop $\overline{\text{MS}}$ QCD running coupling constant α_s with $n_f = 5$. The effects due to virtual top quarks and virtual particles in the running of α_s and in the evolution of pdfs were not included in our predictions. However, the value of the top mass, $m_t = 175 \text{ GeV}$, enters the matched NLL cross-sections through the NLO corrections.

In Fig. 1, we present the relative enhancement of the NLO total cross-sections due to soft gluon resummation, $K_{\text{NLL}} \equiv \sigma^{(\text{match})}/\sigma^{\text{NLO}}$. The NLL K -factors are shown for $\tilde{g}\tilde{g}$ and $\tilde{q}\tilde{q}$ production cross-sections at the LHC, in Fig. 1a and Fig. 1b, respectively. In the plots we set the scales $\mu_F = \mu_R = \mu_0$, where $\mu_0 = m_{\tilde{g}}$ ($\mu_0 = m_{\tilde{q}}$) for the $\tilde{g}\tilde{g}$ production (the $\tilde{q}\tilde{q}$ production). K_{NLL} grows with the final-state mass and depends on the mass ratio r in a moderate way. The relative correction, $K_{\text{NLL}} - 1$, reaches 16% (8%) for the $\tilde{g}\tilde{g}$ production with $r = 1.2$ and $m_{\tilde{g}} = 2 \text{ TeV}$ (1 TeV), and 4% (2%) for the $\tilde{q}\tilde{q}$ production with $r = 2$ and $m_{\tilde{q}} = 2 \text{ TeV}$ (1 TeV). The stronger effect found in the $\tilde{g}\tilde{g}$ production follows from the dominance of the $gg \rightarrow \tilde{g}\tilde{g}$ channel, and hence larger colour factors. It comes from the fact, that the soft-collinear radiative factor Δ_i grows exponentially with the charge of the incoming parton. In addition, the enhancement coming from the soft non-collinear gluon corrections increases with the total colour charge of the final state, which may be the highest in the $gg \rightarrow \tilde{g}\tilde{g}$ case.

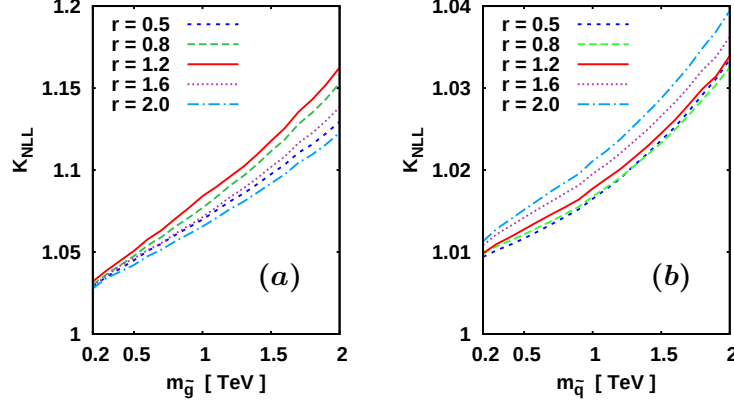


Fig. 1. The NLL K -factor, K_{NLL} , for the $\tilde{g}\tilde{g}$ (a) and the $\tilde{q}\tilde{q}$ (b) total production cross-section at the LHC as a function of gluino and squark mass, respectively; $r = m_{\tilde{g}}/m_{\tilde{q}}$.

We also investigated the dependence of the matched NLL cross-section on the values of factorisation and renormalisation scales, in comparison to the NLO cross-section. To illustrate our results we choose $\mu = \mu_F = \mu_R$ and $r = 1.2$. In Fig. 2(a) and Fig. 2(b) we plot the ratios $\sigma^{\text{NLO}}(\mu = \xi\mu_0)/\sigma^{\text{NLO}}(\mu = \mu_0)$ and $\sigma^{(\text{match})}(\mu = \xi\mu_0)/\sigma^{(\text{match})}(\mu = \mu_0)$, obtained by varying ξ between $\xi = 1/2$ and $\xi = 2$. Due to resummation, the scale sensitivity of the $\tilde{g}\tilde{g}$ production cross-section reduces significantly, by a factor of ~ 3 (~ 2) at $m_{\tilde{g}} = 2$ TeV ($m_{\tilde{g}} = 1$ TeV). At $m_{\tilde{g}} > 1$ TeV the theoretical error of the matched NLL $\tilde{g}\tilde{g}$ cross-section, defined by changing the scale

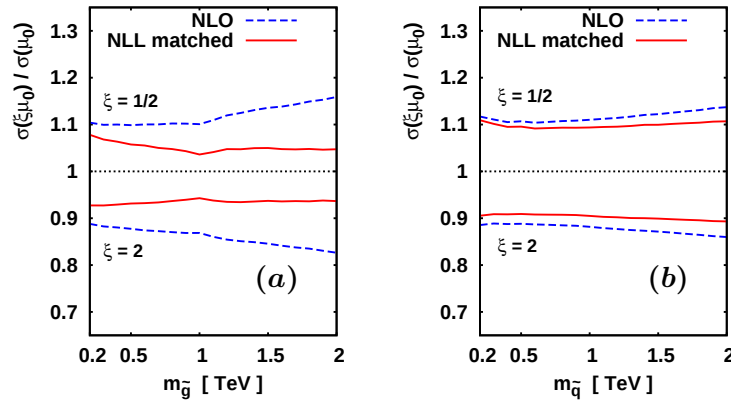


Fig. 2. Scale dependence of the total $\tilde{g}\tilde{g}$ (a) and $\tilde{q}\tilde{q}$ (b) production cross-section at the LHC (see the text for explanation).

$\mu = \mu_F = \mu_R$ around $\mu_0 = m_{\tilde{g}}$ by a factor of 2, is around 5%. In the case of the $\tilde{q}\tilde{q}$ production, the reduction of the scale dependence due to including soft gluon corrections in the theoretical predictions is moderate.

4. Conclusions

Precise predictions for the most dominant production channels of supersymmetric particles at the LHC require that higher-order corrections above NLO are calculated. In this talk, we reported on resummation of soft gluon contributions to the corrections to the $\tilde{q}\tilde{q}$ and $\tilde{g}\tilde{g}$ total hadroproduction rates. We presented our recent results on the soft anomalous dimension matrices governing soft gluon radiation at NLL level in hadronic production process of gluino pairs. It was shown that including soft gluon corrections at NLL accuracy leads to an enhancement of the cross-sections and a reduction of the scale dependence in comparison with the NLO results. The effect is particular prominent for $\tilde{g}\tilde{g}$ production at high $m_{\tilde{g}}$, where the reduction down to the level of $\sim 5\%$ is achieved. Provided that MSSM is realized in nature, the results discussed here may be used for precise determination of the MSSM parameters from the measurements performed at the LHC.

The work of A.K. is supported by the Initiative and Networking Fund of the Helmholtz Association, contract HA-101 (“Physics at the Terascale”). L.M. acknowledges the DFG grant No. SFB 676 and of the Polish Ministry of Education grant No. N202 249235.

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