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FUNDAMENTAL PROBLEMS WITH HADRONIC AND LEPTONIC INTERACTIONS*

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Common beliefs about unitarity are not reliable, and we do not know how to apply DGLAP evolution at small x. Together with the big discrepancy between the measurements of the total cross-section at the Tevatron, a consequence is that the cross-section at the LHC could be anywhere between 90 and 160 mb.

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1. Introduction

The LHC will probe physics under extreme conditions. This means that past physical intuition may be unreliable, and models that depend on intuition may break down. Therefore it is important to ask which of our current beliefs have a sound theoretical basis, and which are just based on folklore.

I will concentrate on two topics where our fundamental understanding is particularly uncertain:

- unitarity,
- DGLAP evolution at small x.

I will show that our lack of understanding of these has serious consequences for what is probably the first thing that will be measured at the LHC, the total cross-section. The best estimate has a huge error:

$$\sigma^{\rm LHC} = 125 \pm 35 \text{ mb}\,. \tag{1.1}$$

My understanding of most of the material that I review here derives from my work over the years with Sandy Donnachie. Further details may be found in our book [1].

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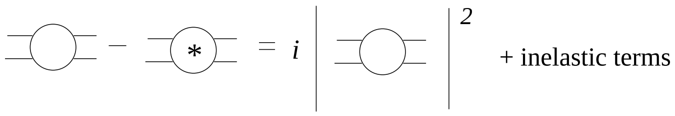
2. Unitarity

For an elastic hadron-scattering amplitude, the unitarity equation reads



or

$$\mathrm{Im}\ a_\ell(s) = |a_\ell(s)|^2 \ + \ \text{inelasticterms} \tag{2.1}$$

so that the partial-wave amplitude obeys

$$|a_\ell(s)| < 1\,. \tag{2.2}$$

A well-known consequence of (2.2) is the Froissart–Lukaszuk–Martin bound [2, 3]:

$$\sigma^{\mathrm{TOT}}(s) < \frac{\pi}{m_\pi^2}\,\log^2\left(\frac{s}{s_0}\right)\,. \tag{2.3}$$

At LHC energies, for reasonable values of the unknown scale s_0, this gives a bound of several barns, and so it is not a useful constraint.

A more useful bound is that of Pumplin [4]:

$$\sigma^{\mathrm{ELASTIC}} < \tfrac{1}{2}\,\sigma^{\mathrm{TOTAL}}\,. \tag{2.4}$$

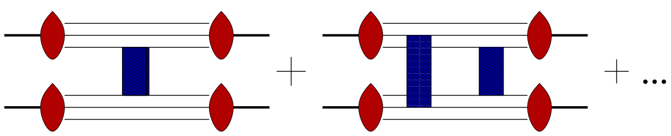
The exchange of a single pomeron exchange $I\!\!P$ gives

$$\sigma^{\mathrm{TOTAL}} \sim s^{\varepsilon}\,, \qquad \varepsilon \approx 0.08\,,$$

and

$$\frac{d\sigma}{dt}^{\mathrm{ELASTIC}}\bigg|_{t=0} \sim s^{2\varepsilon} \tag{2.5}$$

which clearly causes this bound to be violated at large enough s. The remedy is to sum single-$I\!\!P$, double-$I\!\!P$, ... exchanges:



However, even though we started trying to learn how to do this more than 40 years ago, we still do not know how.

The best we can do, although it is certainly wrong [1], is to use an eikonal formalism. Write the amplitude as a 2-dimensional Fourier integral

$$A\left(s, -\boldsymbol{q}^2\right) = 4\int d^2b\, e^{-i\boldsymbol{q}\cdot\boldsymbol{b}}\tilde{A}\left(s, \boldsymbol{b}^2\right)\,. \tag{2.6}$$

Define $\chi(s,b) = -\log(1+2i\tilde{A}/s)$ so that

$$\tilde{A}(s,\boldsymbol{b}^2) = \tfrac{1}{2} is \left(1 - e^{-\chi(s,b)}\right) . \tag{2.7}$$

Then the bound (2.2) on the partial-wave amplitude may be shown to be equivalent to

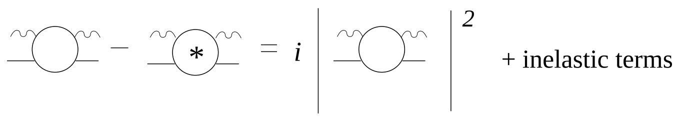
$$\mathrm{Re}\ \chi(s,b) \geq 0 \tag{2.8}$$

which is easy to impose. If we expand the exponential in (2.7) in powers of χ we have

$$\begin{aligned} A(s,-\boldsymbol{q}^2) &= 2is \int d^2b\, e^{-i\boldsymbol{q}.\boldsymbol{b}} \left(1 - e^{-\chi(s,b)}\right) \\ &= 2is \int d^2b\, e^{-i\boldsymbol{q}.\boldsymbol{b}} \left(\chi - \frac{\chi^2}{2!} + \frac{\chi^3}{3!} \ldots\right) . \end{aligned} \tag{2.9}$$

So far, the equations are certainly correct. But we do not know what to take for $\chi(s,b)$. An obvious choice is to approximate it by single-$I\!\!P$ exchange. Although we do not know how to calculate double-$I\!\!P$ exchange, we do know something about its general structure, and the second term in (2.9) has the right structure. Similarly, the third term has the right structure to represent triple-$I\!\!P$ exchange, and so on. Nevertheless, this is wrong, for various reasons [1]. One one is that double-$I\!\!P$ exchange obviously depends on two-quark correlations in the proton wave function, which are absent in this procedure.

There has been a lot of talk about the consequences of unitarity for processes with leptons or photons in the initial state; for example it is believed to lead to what is known as saturation. It is important to understand that basic theory alone does not allow one to conclude anything, without feeding in extra assumptions. That is, there may well be no unitarity bound on $F_2(x,Q^2)$. The reason is that the analogue of (2.1) for $\gamma^* p$ scattering



is not true. This is the case even for γp scattering: for all we know, the γp total cross-section could continue to increase indefinitely with increasing energy.

3. DGLAP evolution at small x

Claims by experimentalists and theorists to be able to extract parton densities with high accuracy from data, for use at the LHC, must be regarded with caution. This is because we do not know how to handle DGLAP evolution at small x.

The singlet DGLAP equation is:

$$\begin{aligned}\frac{\partial}{\partial t}\boldsymbol{u}\left(x,Q^2\right) &= \int\limits_x^1 dz\boldsymbol{P}\left(z,\alpha_{\rm s}\left(Q^2\right)\right)\boldsymbol{u}\left(x/z,Q^2\right)\,,\\ \boldsymbol{u}\left(x,Q^2\right) &= \begin{pmatrix} q\left(x,Q^2\right)\\ g\left(x,Q^2\right)\end{pmatrix}\,.\end{aligned} \tag{3.1}$$

The terms of the perturbation expansion of $\boldsymbol{P}(z,\alpha_{\rm s}(Q^2))$ diverge like $1/z$ at $z=0$. For small x the integration extends to small values of z and therefore it is wrong to expand $\boldsymbol{P}(z,\alpha_{\rm s}(Q^2))$ in powers of $\alpha_{\rm s}$.

A possible exception is when $\boldsymbol{u}(x,Q^2)$ rises steeply with $1/x$. To understand this, approximate $\boldsymbol{u}(x,Q^2)$ in some range of x at some value of Q^2 by

$$\boldsymbol{u}\left(x,Q^2\right)\sim\boldsymbol{f}\left(Q^2\right)x^{-\varepsilon} \tag{3.2}$$

and insert this into the DGLAP equation (3.1). This gives

$$\frac{\partial}{\partial t}\log\boldsymbol{f}\left(Q^2\right)=\tilde{\boldsymbol{P}}(N=\varepsilon\,,\alpha_{\rm s}\left(Q^2\right))-\int\limits_0^x dz z^{\varepsilon}\boldsymbol{P}\left(z,\alpha_{\rm s}\left(Q^2\right)\right) \tag{3.3}$$

with $\tilde{\boldsymbol{P}}$ the Mellin transform of $\boldsymbol{P}$. The last term $\sim x^{\varepsilon}$ and so is negligible at small x if ε is some way above 0. A pole of $\boldsymbol{P}(z,\alpha_{\rm s}(Q^2))$ at $z=0$ reflects itself in a pole of $\tilde{\boldsymbol{P}}(N,\alpha_{\rm s}(Q^2)$ at $N=0$. So if ε is some way above 0 we are at some distance from this pole and then also it should be safe to expand the first term in powers of $\alpha_{\rm s}$. By doing so, and dropping the last term in (3.3), we obtain a simple differential equation for $\boldsymbol{f}(Q^2)$.

The simplest fit to F_2 at small x is a combination of two powers of x, hard-pomeron and soft-pomeron:

$$\begin{aligned}F_2\left(x,Q^2\right) &= f_0\left(Q^2\right)x^{-\varepsilon_0}+f_1\left(Q^2\right)x^{-\varepsilon_1}\,,\\ \varepsilon_0 &\approx 0.4\,,\qquad \varepsilon_1=0.0808\,.\end{aligned} \tag{3.4}$$

The fit has just 5 free parameters, including ε_0. See figure 1.

Donnachie and I made this fit [5] purely phenomenogically, but we then found [7] that its output for the coefficient function $f_0(Q^2)$ in (3.4) obeys the DGLAP evolution differential equation to very high accuracy, both at LO and at NLO. See figure 2. DGLAP evolution is supposed to be valid only for large Q^2 and we can see from the figure that this means at least 5 to 10 GeV^2. It certainly does not make sense to use it down to 1 or 2 GeV^2, as is often done.

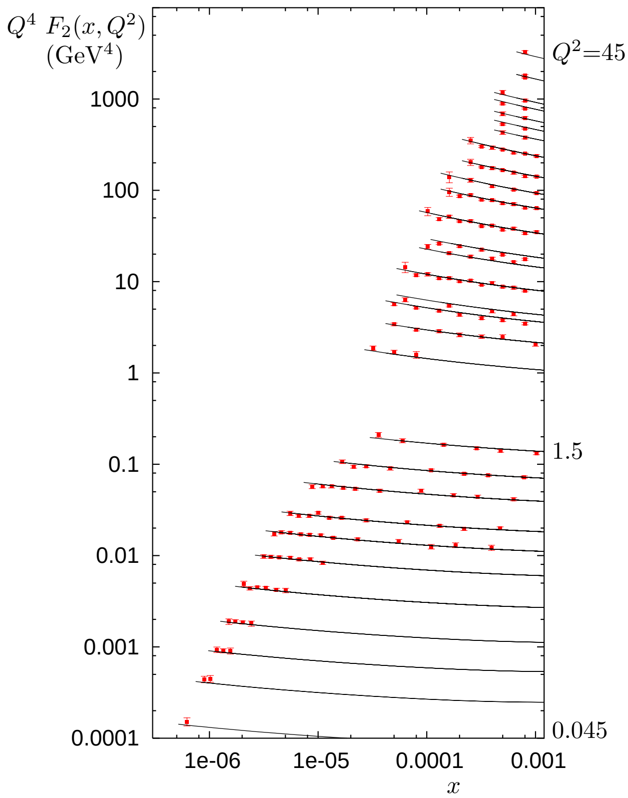


Fig. 1. Simple fit (3.4) to data for $F_2(x,Q^2)$ at small x.

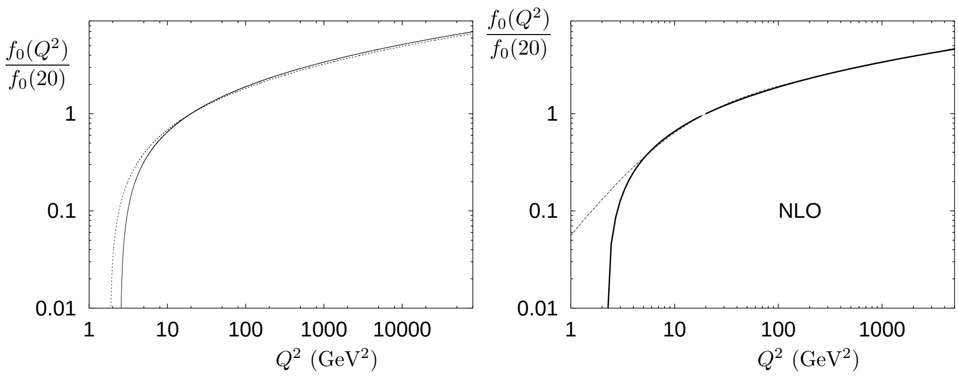


Fig. 2. Comparison of $f_0(Q^2)$ extracted from data with DGLAP evolution at LO and at NLO.

As I have explained, DGLAP evolution breaks down for the soft-pomeron coefficient function $f_1(Q^2)$, as ε_1 is too close to 0. The conventional approach to DGLAP evolution, used by many theorists and experimentalists to extract what are claimed to be highly accurate parton distributions, amounts to ignoring this difficulty.

Our simple fit (3.4) applies only at small x, which is why figure 1 shows data only for $x < 10^{-3}$. If we want to extend it to larger values of x, we should add in a Regge term $x^{-\varepsilon_{\rm R}}$ with $\varepsilon_{\rm R} \approx -\frac{1}{2}$, corresponding to f_2 and a_2 exchange. Also, the simple powers of x in (3.4) must be multiplied by functions that go to 0 as $x \to 1$. We do not know what these should be, so Donnachie and I took [5] just the powers of $(1-x)$ given by the dimensional counting rules. This is certainly too simple to be correct but is better than doing nothing. It is also astonishingly successful: see figure 3, which shows also what the fit gives for the γp total cross-section. Compared with figure 1, only 2 free parameters have been added, both for the Regge term.

4. Total cross-section at the LHC

Given that the data for $F_2(x,Q^2)$ respond so well to a fit that includes a hard pomeron, it is natural [6] to include such a term also in fits to pp and $\bar{p}p$ scattering. That is, for each of $\sigma(pp), \sigma(p\bar{p}), \sigma(\gamma p)$ include hard pomeron, soft pomeron and Regge exchange:

$$\sigma = X_0 s^{\varepsilon_0} + X_1 s^{\varepsilon_1} + X_{\rm R} s^{\varepsilon_{\rm R}}\,. \tag{4.1}$$

The result is shown in figure 4. In the left-hand figure, the lowest curve is the hard-pomeron contribution. The right-hand figure shows the extrapolation to LHC energy of the two fits, this new one and the old fit without a hard pomeron term.

Notice the familiar and long-standing discrepancy between the E710 and CDF measurements at the Tevatron. If the upper CDF measurement should be correct, it surely is a sign that something new is beginning to become important at Tevatron energy, with the consequence that the LHC cross-section will be large. This observation does not depend on any particular theoretical explanation of the data.

However, the large prediction for the LHC total cross-section when the hard pomeron is included will lead some to worry about unitarity. Some even worry about unitarity for the lower curve. Donnachie and I stressed when we made the original DL fit [9] that the power $\varepsilon_1 \approx 0.08$ was an effective power that already, to some extent, includes unitarity corrections; nevertheless Alan Martin and collaborators believe [8] that this is not enough to take account of unitarity and so predict that the LHC cross-section will be about 90 mb.

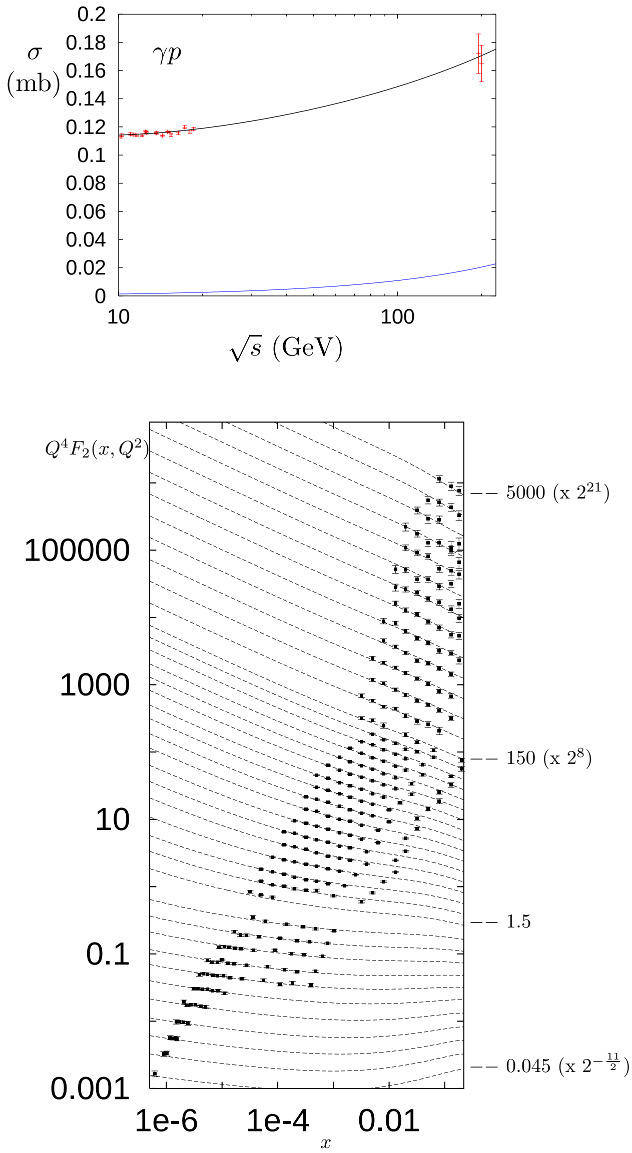


Fig. 3. The fit to F_2 extended to larger values of x, and to real-photon data.

I have explained that nobody knows how to calculate the effects of unitarity. I will now describe an attempt to do so, which should not be taken at all seriously.

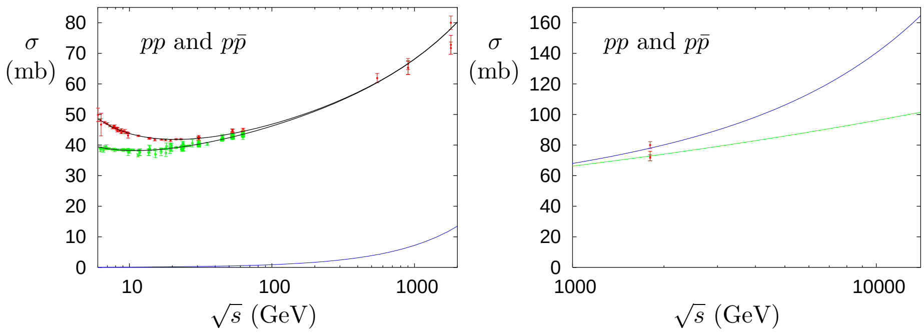


Fig. 4. Fits to pp and $\bar{p}p$ total cross-sections.

Consider pp and $\bar{p}p$ elastic scattering. At existing energies, soft pomeron exchange dominates and its contribution has been known for nearly 40 years:

$$\frac{d\sigma}{dt} = \frac{[3\beta_1 F_1(t)]^4}{4\pi} \left(\alpha_1' s\right)^{2\left(\varepsilon_1+\alpha_1' t\right)} . \tag{4.2}$$

Here $F_1(t)$ is the elastic form factor of the proton and β_1 and ε_1 are known from σ^{TOT}. As long ago as 1973, my then student Jaroskiewicz [10] found that data fix the only free parameter α_1' to be 0.25 GeV^{-2}. This is done by using very-small-t data at some energy: see figure 5(a). Then the formula (4.2) fits the data out to rather larger values of t at that energy, as is seen in figure 5(b). Because $F_1(t)$ is raised to the 4-th power the fit is rather sensitive to it. I do not understand why the proton's electromagnetic form factor should be appropriate, since pomeron exchange has the opposite C parity from photon exchange.

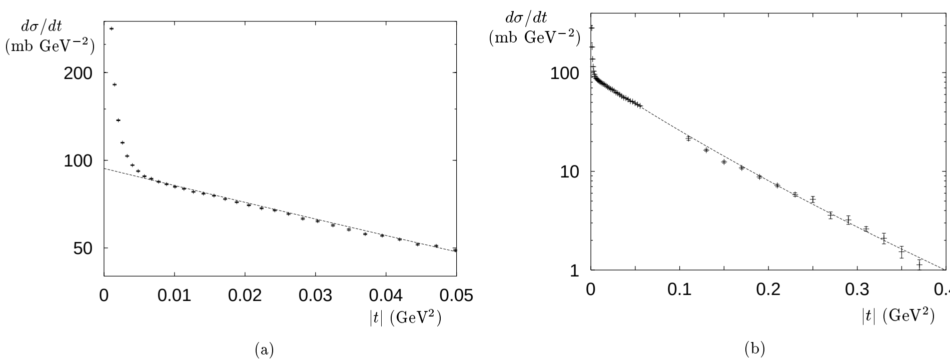


Fig. 5. pp elastic scattering data at $\sqrt{s} = 53$ GeV. The curves do not include photon exchange.

As is well known, at small t pp elastic scattering displays shrinkage: $d\sigma/dt$ becomes steeper as the energy increases. The rate of shrinkage is determined by the value of α_1', and 0.25 GeV^{-2} describes the data very well at all the different energies that have been measured [1].

At large values of $|t|$, greater than about 3 GeV^2, the data take on a different character. They are described very well by

$$\frac{d\sigma}{dt} = 0.09\, t^{-8} \tag{4.3}$$

and are independent of energy. See figure 6. The form (4.3) is what is obtained from the triple-gluon-exchange mechanism of figure 7. This raises an interesting question [11]: what if one replaces each gluon with a hard pomeron? This might provide a mechanism which, while it is too small to be seen at existing energies, grows rapidly with energy and is large at LHC energy. That is, $d\sigma/dt$ at the LHC might be large at large t.

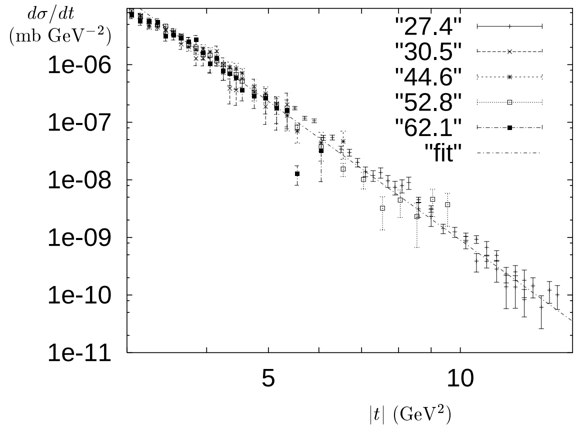


Fig. 6. Large-t data for pp elastic scattering at various energies, with the fit (4.3).

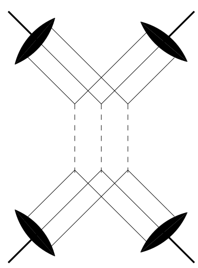


Fig. 7. Triple-gluon exchange.

At energies corresponding to the data plotted in figures 5 and 6, pp elastic scattering data display a striking dip structure at values of t in between those of the two figures. See figure 8. It is not easy to generate a dip: the real and imaginary parts of the amplitude must be very small at the same t, which requires something of an accident. It needs at least three contributions: probably $I\!P$, $I\!P I\!P$ and ggg. However, the third of these three terms has the opposite C parity from the first two, which led us to predict [12] that $\bar{p}p$ scattering should not have a dip. As the figure shows, this was later confirmed. An exchange such as ggg which has negative C parity is known as odderon exchange; it is a mystery why odderon exchange has not been detected at smaller values of t.

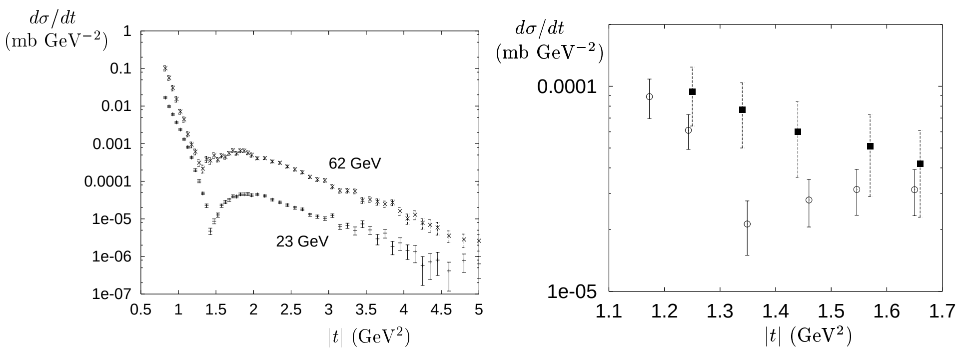


Fig. 8. pp and $\bar{p}p$ elastic scattering data. The left hand figure is for pp and the 62 GeV data are multiplied by 10. The data in the right-hand figure is at 53 GeV and the upper points are $\bar{p}p$, the lower pp.

The fit shown in figure 5 to elastic scattering at relatively low energy remains good at very small t as the energy is increased but becomes less good at larger values of t. Figure 9 shows data for $\bar{p}p$ elastic scattering from the two Tevatron experiments. The curve corresponds to the exchange of

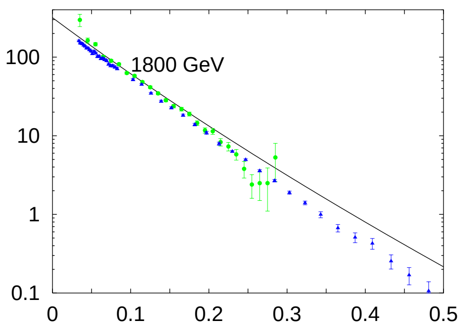


Fig. 9. $\bar{p}p$ elastic scattering data from the Tevatron; the curve corresponds to the exchange of single hard and soft pomerons.

a single soft pomeron plus that of a single hard pomeron. I have explained that we do not know how to calculate the exchange of two pomerons, but we know enough about its general features to know that including this will pull $d\sigma/dt$ down at larger t. As a very crude model that includes the two-pomeron exchanges $I\!P_0 I\!P_0, I\!P_1 I\!P_1$ and $I\!P_0 I\!P_1$ (where again the subscripts 0 and 1 denote the hard and soft pomerons), I calculated the b-space amplitude for the sum of the $I\!P_0$ and $I\!P_1$ exchanges, and squared it to simulate the sum of the $I\!P I\!P$ exchanges:

$$\tilde{A}(s,b) = 2is\left(\chi(s,b) - \lambda[\chi(s,b)]^2\right) . \tag{4.4}$$

I chose the value of λ to cancel the imaginary part of the amplitude so as to get the dips in pp scattering at the right value of t, and added in ggg exchange. The large-t form (6) of the latter needs to be modified at smaller t so that it does not diverge, and one does not know how to do this, but I guessed a form with a single parameter which I chose so as the optimise the fit to the dip structure. My best fit to the data is shown in figure 10.

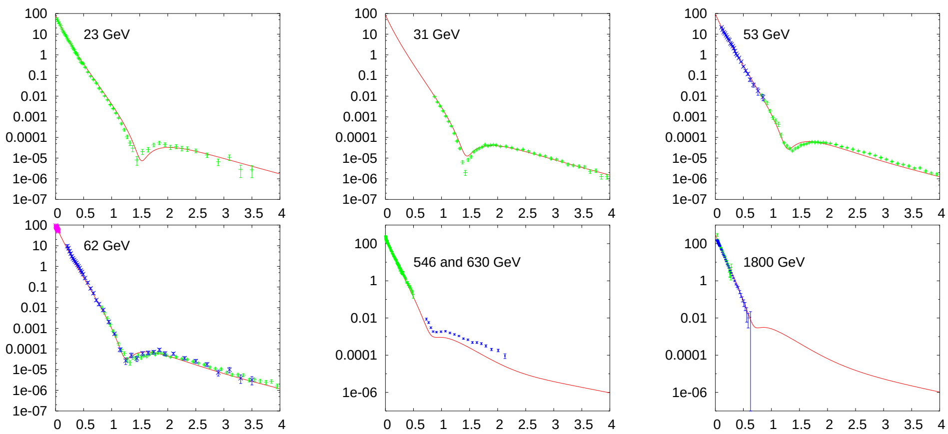


Fig. 10. pp and $\bar{p}p$ elastic scattering data at various with fit $I\!P + I\!P I\!P + ggg$.

The result for the total cross-section at the LHC is that including the two-pomeron exchanges pulls the prediction down from 160 to 125 mb: see figure 11. The lower curve in the figure corresponds to single soft pomeron exchange only. So, since my attempt to include the double exchanges is surely very crude, I have to conclude that the LHC total cross-section could be anywhere between 100 and 160 mb. Remember, though, that it might be even smaller [8].

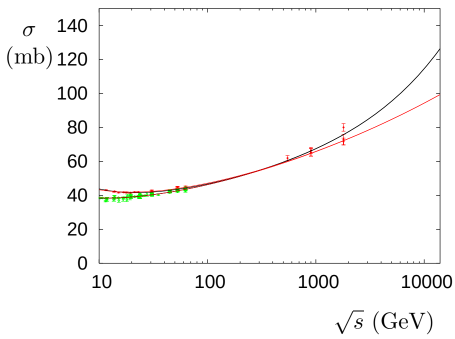


Fig. 11. Extrapolations to LHC energy of fits to the total cross-section. The upper curve corresponds to the fits to the amplitude shown in figure 10, including $I\!P I\!P$ exchanges, while the lower curve omits any hard-pomeron contribution.

5. Summary

- $\sigma^{\rm LHC} = 125 \pm 35$ mb.
- We do not know how usefully to impose unitarity — eikonal-type models are surely too simple.
- Unitarity does not constrain lepton or photon-induced cross-sections.
- We still cannot calculate $I\!P I\!P$ exchange — even after more than 45 years.
- There are severe mathematical problems with DGLAP at small x.
- DGLAP cannot be used below $Q^2 = 5$ GeV^2.
- Regge fits to $F_2(x, Q^2)$ are the simplest — and probably the most correct.
- Elastic scattering at the LHC at large t may be surprisingly large.
- If the CDF Tevatron cross-section is correct, something dramatic must happen — independently of any theory!

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