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INITIAL CONDITIONS OF HEAVY ION COLLISIONS AND HIGH ENERGY FACTORIZATION*

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The Color Glass Condensate is an effective theory description for the small momentum fraction x degrees of freedom in a high energy hadron or nucleus, which can be understood in terms of strong classical gluon fields. We discuss the resulting picture of the initial conditions in a relativistic heavy ion collision. We describe recent work to show that the leading logarithms of the collision energy can be factorized into the renormalization group evolution of the small x wavefunction. We will then show how this framework can be used to understand the long range rapidity correlations observed by the RHIC experiments.

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1. The little bang of an ultrarelativistic heavy ion collision

Quark gluon plasma is studied in the laboratory in collisions of heavy nuclei at ultrarelativistic energies, presently $\sqrt{s} = 200\ A\,\mathrm{GeV}$ at RHIC in Brookhaven or in the near future 5500 $A\,\mathrm{GeV}$ at the LHC in CERN (where A is the atomic number of the nucleus). The collision process is a complicated one, starting from the formation and equilibration of the matter to its evolution in time and space and ending in the decoupling of the system into the hadrons that are observed in the detectors.

The typical transverse momentum scales of the bulk of particles produced is in the GeV range, much less than the collision energy. Thus the initial conditions depend on the small $x \sim p_{\mathrm{T}}/\sqrt{s} \lesssim 0.01$ part of the nuclear wavefunction. Because of the $\ln 1/x$ enhancement of soft gluon bremsstrahlung this is a dense gluonic system. When the occupation numbers of gluonic

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states in the wavefunction become large enough, of the order of $1/\alpha_s$ (meaning that the gluon field A_μ is of the order of $1/g$), the nonlinear interaction part of the Yang–Mills Lagrangian becomes of the same order of magnitude as the free part. The relevant comparison is between the two terms in the covariant derivative $D_\mu = \partial_\mu + igA_\mu$: the momentum scales $p_\mu = -i\partial_\mu \lesssim gA_\mu$ become nonlinear. In the small x wavefunction the relevant component is the transverse momentum, we are therefore led to the concept of a transverse momentum scale Q_s, the *saturation scale*, below which the system is dominated by nonlinear interactions. When the collision energy is high enough (x small enough), $Q_s \gg \Lambda_{QCD}$ and the coupling is weak: we are faced with a *nonperturbative* strongly interacting system with a *weak coupling constant*. On the other hand, the large occupation numbers mean that the system should behave as a *classical* field. This suggests a way of organizing calculations that differs from traditional perturbation theory. Instead of developing as a series of powers in gA_μ we want to calculate the classical background field $A^\mu_{cl.}$ and loop corrections (which are suppressed by powers of g) to all orders in $gA^\mu_{cl.}$. The classical gluon field will then be radiated by the large x degrees of freedom, which we shall treat as effective classical color charges. This picture of the high energy wavefunction is referred to as the Color Glass condensate (CGC, for reviews see *e.g.* [1]). The collision of two such systems leads, in the early stages $1/\sqrt{s} \ll \tau \lesssim 1/Q_s$, to classical field configurations known as the Glasma [2].

At early times ($\tau \ll R_A$, see Fig. 1 for the coordinate system) the bulk of the system cannot, by causality, be aware of its finite size in the transverse plane. It will therefore be in a longitudinally expanding, to a first approximation boost invariant ($\partial_\eta = 0$) state, a 1-dimensional Hubble expansion. Boost invariance can come in two flavors. As we shall argue in the following, the very early time glasma degrees of freedom are boost invariant at the level of *field configurations*. This means that the longitudinal momenta of particles redshift towards zero $p_z \sim 1/\tau$ while $p_T \sim$ constant and the system becomes very anisotropic in momentum space. This field level invariance is broken by quantum fluctuations suppressed by α_s, which then eventually evolve into a more equilibrated fluid that is isotropic in its local rest frame. What remains is a boost invariant profile of particle flow, as in the Bjorken hydrodynamical picture. What concerns us in this paper is the very earliest glasma stage and the initial quantum fluctuations that serve as the seeds of isotropization.

In the following we shall first discuss the leading order, classical field level, results for the structure of the glasma fields and gluon production. We shall then, in Sec. 3, describe some ingredients of the recent proof [3] that shows how the leading logarithmic divergences of the NLO corrections to these fields can be absorbed into the renormalization group (RG) evolution of the weight functionals describing the hard sources, the *JIMWLK factorization theorem*. In Sec. 4 we shall then describe multigluon correlations in the same framework.

2. Gluon production to leading order and the glasma

The CGC framework is based on a separation of scales between small x and large x degrees of freedom, which are treated as a classical field and an effective color charge density. In practice the classical field is obtained from the equation of motion

$$[D_\mu, F^{\mu\nu}] = J^\nu \,. \tag{1}$$

The current in the case of a nucleus–nucleus consists of two infinitely Lorentz-contracted (this picture will be discussed more below) nuclei on the light cone [4]:

$$J^\mu = \delta^{\mu+}\rho_{(1)}(\boldsymbol{x}_\perp)\delta(x^-) + \delta^{\mu-}\rho_{(2)}(\boldsymbol{x}_\perp)\delta(x^+) \,. \tag{2}$$

The large x degrees of freedom have now been reduced to a classical effective color charge density $\rho(\boldsymbol{x}_\perp)$, which is a static (hence the "glass") stochastic variable. Its values are drawn from a probability distribution $W_y[\rho(\boldsymbol{x}_\perp)]$ which depends on the cutoff rapidity $y = \ln 1/x$ separating large and small x. To a first approximation we can take *e.g.* the Gaussian distribution of color charges that defines the MV [5] model

$$W[\rho(\boldsymbol{x}_\perp)] = \mathcal{N} \exp\left[-\tfrac{1}{2}\int d^2\boldsymbol{x}_\perp \rho^a(\boldsymbol{x}_\perp)\rho^a(\boldsymbol{x}_\perp)/g^2\mu^2\right] . \tag{3}$$

The probability distribution $W_y[\rho(\boldsymbol{x}_\perp)]$ is analogous to a parton distribution function in the DGLAP formalism; it is a nonperturbative input that we are not able to compute from first principles, but one can derive evolution equation for its y-dependence. This equation is known by the acronym JIMWLK.

For a fixed configuration of the color sources ρ the calculation of the Glasma fields proceeds as follows [4]. The solution of the Yang–Mills equations in the regions of spacetime $x^\pm > 0, x^\mp < 0$ that are causally connected to only one of the nuclei (areas (1) and (2) in Fig. 1) is an analytically known pure gauge field. It gives the initial condition for the numerical solution in the forward light cone (3). Working in the temporal gauge $A_\tau = 0$ these initial conditions are

$$A^i|_{\tau=0} = A^i_{(1)} + A^i_{(2)} \,, \tag{4}$$

$$A^\eta|_{\tau=0} = \frac{ig}{2}\left[A^i_{(1)}, A^i_{(2)}\right] \,, \tag{5}$$

where $A^i_{(1,2)}$ are the pure gauge fields that are the solutions of the one-nucleus problem

$$A^i_{(1,2)} = \frac{i}{g} U_{(1,2)}(\boldsymbol{x}_\perp)\partial_i U^\dagger_{(1,2)}(\boldsymbol{x}_\perp)\,. \tag{6}$$

These pure gauge fields are gauge transforms of the vacuum with the *Wilson lines* computed from the color charge density

$$U_{(1)}(\boldsymbol{x}_\perp, x^-) = \mathrm{P}\exp\left\{ -ig \int_{-\infty}^{x^-} dy^- \frac{\rho(\boldsymbol{x}_\perp, y^-)}{\boldsymbol{\nabla}^2_\perp} \right\}, \tag{7}$$

with the Wilson line $U_{(2)}$ given by the analogous formula in terms of the other color charge density.

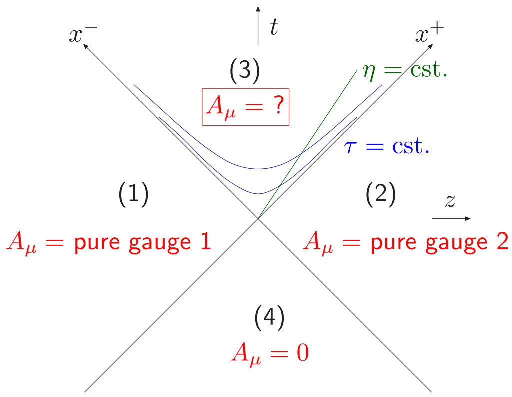

Fig. 1. Spacetime structure of the CGC and glasma fields. It is convenient to use the coordinate system with proper time $\tau = \sqrt{2x^- x^+}$ and the spacetime rapidity $\eta = \frac{1}{2}\ln x^+/x^-$.

These Wilson lines are in fact the most natural variables to describe the soft gluonic field degrees of freedom of the nucleus; they correspond to the eikonal scattering amplitude of a color charge off the strong color fields. For example the dipole cross that determines the structure function measured in deep inelastic scattering is a correlator of these same Wilson lines. The upper limit of the y^--integral in Eq. (7) must be thought of as $x^- \sim e^y$; when the cutoff rapidity y becomes larger (x smaller), smaller momentum p^+ gluons are considered as part of the source, which consequently extends further in the conjugate variable x^-. Thus each infinitesimal step in the renormalization group evolution towards smaller x corresponds to adding a layer in x^- to the color source, or equivalently to multiplying the Wilson line by an SU(3) matrix that is infinitesimally close to identity.

From the point of view of the classical glasma fields in Eq. (6) the Wilson lines are independent of the longitudinal coordinate: the longitudinal structure appears only indirectly in the properties of the probability distribution

$W_y[U]$. The classical fields represent degrees of freedom with a smaller p^+ than the ones integrated out to the Wilson lines and are not able to resolve their structure which is shorter range in x^-. This is the sense in which the δ-functions in the currents of Eq. (2) must be understood.

The numerical method for solving the Yang–Mills equations in the forward light cone was developed in Ref. [6] and the actual computations reported in Ref. [7]. The equations of motion are most conveniently solved in the Hamiltonian formalism. Due to the boost invariance of the initial conditions in the high energy limit the Yang–Mills equations can be dimensionally reduced to a 2+1 dimensional gauge theory with the η-component of the gauge field becoming an adjoint scalar field. With the assumption of boost invariance one is explicitly neglecting the longitudinal momenta of the gluons. In the Hamiltonian formalism one obtains directly the (transverse) energy. By decomposing the fields in Fourier modes one can also define a gluon multiplicity corresponding to the classical gauge fields; the resulting gluon spectrum is shown in Fig. 2. The color fields of the two nuclei are transverse electric and magnetic fields on the light cone. The glasma fields left over in the region between the two nuclei after the collision at times $1 \leq \tau \leq 1/Q_{\mathrm{s}}$ are, however, longitudinal along the beam axis [2] (see Fig. 2).

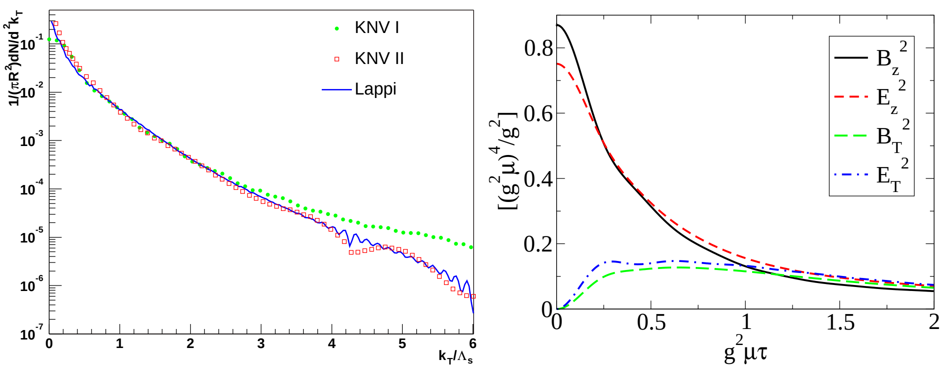

Fig. 2. Left: Gluon spectrum from the leading order classical field computation. Right: the components of the glasma field, the initial condition is a longitudinal electric and magnetic field, the transverse components develop in a time $\sim 1/Q_{\mathrm{s}}$.

3. Factorization

To understand the context of the high energy factorization theorem proven in Ref. [3] it is perhaps useful to look first at the weak field limit of the CGC, where particle production can be computed using k_{T}-factorization ([8], see *e.g.* [9] for an application to heavy ion collisions). The leading order multiplicity is

$$\frac{dN}{d^2\boldsymbol{p}_\perp dy} = \frac{1}{\alpha_s}\frac{1}{\boldsymbol{p}_\perp^2}\int\frac{d^2\boldsymbol{k}_{\perp 1}}{(2\pi)^2}\varphi_y(\boldsymbol{k}_{\perp 1})\varphi_y(\boldsymbol{p}_\perp - \boldsymbol{k}_{\perp 1})\,. \tag{8}$$

To obtain the real part of the leading log correction to this result one must take the corresponding expression for double inclusive gluon production

$$\frac{dN}{d^2\boldsymbol{p}_\perp dy_p d^2\boldsymbol{q}_\perp dy_q} = \frac{1}{\alpha_s}\frac{1}{\boldsymbol{p}_\perp^2\boldsymbol{q}_\perp^2}\int\frac{d^2\boldsymbol{k}_{\perp 1}}{(2\pi)^2}\varphi_y(\boldsymbol{k}_{\perp 1})\varphi_y(\boldsymbol{p}_\perp + \boldsymbol{q}_\perp - \boldsymbol{k}_{\perp 1})\,, \tag{9}$$

and integrate it over the phase space of the second gluon $(\boldsymbol{q}_\perp, y_q)$. Note that at leading log accuracy we have here taken the multi-Regge kinematical limit, assuming that the two produced gluons are far apart in rapidity (see *e.g.* [10]). The integral over y_q diverges linearly (this is the general behavior of the $gg \to gg$ scattering amplitude in the high energy limit t fixed, $s \sim -u \to \infty$). This divergence is compensated (to the appropriate order in α_s) by the real part of the BFKL evolution equation for $\varphi_y(\boldsymbol{k}_{\perp 1})$.

In the fully nonlinear case of AA collisions the k_T-factorization is broken (see *e.g.* [6, 11]), and one must solve the equations of motion to all orders in the strong classical field. The analogue of the unintegrated parton distribution $\varphi_y(\boldsymbol{k}_\perp)$ is the color charge density distribution $W_y[\rho]$. These are similar in the sense that they are not (complex) wavefunctions but (at least loosely speaking) real probability distributions. Factorization can be understood as a statement that one has found a convenient set of degrees of freedom in which one can compute physical observable from only the diagonal elements of the density matrix of the incoming nuclei. The difference is that when in the dilute case these degrees of freedom are numbers of gluons with a given momentum, in the nonlinear case the appropriate variable is the color charge density and the relevant evolution equation is JIMWLK, not BFKL. The kinematical situation, however, remains the same. To produce a gluon at a very large rapidity (or a contribution in the loop integral of the virtual contribution with a large k^+) one must get a large $+$-momentum from the right-moving source. Thus one is probing the source at a large k^+, *i.e.* small distances in x^-, and the result must involve $W_y[\rho]$ at a larger rapidity (see Fig. 3).

The underlying physical reason for factorization is that this fluctuation with a large k^+ requires such a long interval in x^+ to radiated that it must be produced well before and independently of the interaction with the other (left moving and thus localized in x^+) source. The concrete task is then to show that when one computes the NLO corrections to a given observable in the Glasma, all the leading logarithmic divergences can be absorbed into the RG evolution of the sources with the same Hamiltonian that was derived by considering only the DIS process. This is the proof [3, 12, 13] of factorization that we will briefly describe in the following.

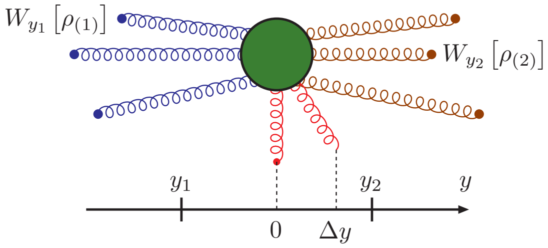

Fig. 3. Factorization of LLog corrections to gluon production: the phase space integral over Δy diverges and is cut off at the separation scales $y_{1,2}$. The dependence of the color charge density distributions $W_{y_{1,2}}$ on the cutoff cancels the leading logarithmic part of the dependence on $y_{1,2}$.

Consider the single inclusive gluon multiplicity which is a sum of probabilities to produce $n+1$ particles, with the phase space of the additional n must be integrated out

$$\frac{dN}{d^3\vec{\boldsymbol{p}}} \sim \sum_{n=0}^{\infty} \frac{1}{n!} \int \left[d^3\vec{\boldsymbol{p}}_1 \ldots d^3\vec{\boldsymbol{p}}_n\right] \; |\langle \vec{\boldsymbol{p}} \; \vec{\boldsymbol{p}}_1 \ldots \vec{\boldsymbol{p}}_n | 0\rangle|^2 \, . \tag{10}$$

Because we have a theory with external color sources of order $\rho \sim 1/g$, all insertions of the sources appear at the same order in g [14]. A calculation using the Schwinger–Keldysh formalism leads to the following results: At LO, the multiplicity is obtained from the *retarded* solution of classical field equations (here $(\ldots)$ includes the appropriate normalization and projection to physical polarizations)

$$\frac{dN_{\mathrm{LO}}}{d^3\vec{\boldsymbol{p}}} = \int d^3\boldsymbol{x} d^3\boldsymbol{y} e^{i\vec{\boldsymbol{p}}\cdot(\vec{\boldsymbol{x}}-\vec{\boldsymbol{y}})} \, (\ldots) \left[\mathcal{A}^\mu(t,\vec{\boldsymbol{x}})\mathcal{A}^\nu(t,\vec{\boldsymbol{y}})\right]\Big|_{t\to\infty} . \tag{11}$$

The NLO contribution includes the one loop correction to the classical field and the $+-$ component of the Schwinger–Keldysh (SK) propagator in the background field

$$\begin{aligned}\frac{dN_{\mathrm{NLO}}}{d^3\vec{\boldsymbol{p}}} &= \int d^3\boldsymbol{x} d^3\boldsymbol{y} e^{i\vec{\boldsymbol{p}}\cdot(\vec{\boldsymbol{x}}-\vec{\boldsymbol{y}})} \, (\ldots) \Big[\mathcal{G}^{\mu\nu}_{+-}(x,y) \\ &\quad + \beta^\mu_+(t,\vec{\boldsymbol{x}}) \, \mathcal{A}^\nu_-(t,\vec{\boldsymbol{y}}) + \mathcal{A}^\mu_+(t,\vec{\boldsymbol{x}}) \, \beta^\nu_-(t,\vec{\boldsymbol{y}})\Big]\Big|_{t\to\infty} .\end{aligned} \tag{12}$$

Now consider a small fluctuation $a^\mu(x)$ of the gluon field around the classical value. The $+-$ (SK index) component of the propagator is bilinear in these small fluctuations satisfying *retarded* boundary conditions. Also the virtual term β satisfies an equation of motion with a retarded boundary condition and a source term involving a loop in the classical background field, see Fig. 4 for a pictorial representation of this structure. One can express the

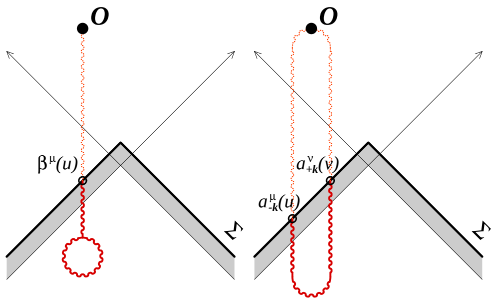

Fig. 4. The one loop one and two point functions in the background field, separated into the parts before the light cone Σ.

propagation of such a small fluctuation $a^\mu(x)$ above the past light cone Σ as a functional derivative $\mathbb{T}_{\boldsymbol{u}}$ of the LO classical field $\mathcal{A}^\mu(x)$ with respect to its initial condition on Σ: $a^\mu(x) = \int_{\vec{\boldsymbol{u}}\in\Sigma} a(\vec{\boldsymbol{u}}) \cdot \mathbb{T}_{\boldsymbol{u}} \mathcal{A}^\mu(x)$. This leads after some rearrangements to the expression for the NLO contribution to the multiplicity as a functional derivative operator acting on the leading order result:

$$\left.\frac{dN}{d^3\vec{\boldsymbol{p}}}\right|_{\mathrm{NLO}} = \left[\frac{1}{2}\int\limits_{\Sigma} d^3\boldsymbol{u}d^3\boldsymbol{v}\mathcal{G}_{\mu\nu}(\boldsymbol{u},\boldsymbol{v})\mathbb{T}^\mu_{\boldsymbol{u}}\mathbb{T}^\nu_{\boldsymbol{v}} + \int\limits_{\Sigma} d^3\boldsymbol{u}\beta_\mu(\boldsymbol{u})\mathbb{T}^\nu_{\boldsymbol{u}}\right] \left.\frac{dN}{d^3\vec{\boldsymbol{p}}}\right|_{\mathrm{LO}} . \quad (13)$$

This expression involves the part of the two point function below the light cone Σ:

$$\mathcal{G}^{\mu\nu}(\vec{\boldsymbol{u}},\vec{\boldsymbol{v}}) \equiv \int \frac{d^3\vec{\boldsymbol{k}}}{(2\pi)^3 2E_{\boldsymbol{k}}}\, a^\mu_{-\boldsymbol{k}}(\boldsymbol{u})\, a^\nu_{+\boldsymbol{k}}(\boldsymbol{v})\,. \quad (14)$$

Here the small fluctuation field $a^\mu(x)$ is the solution of the linearized equation of motion in the classical field background with an initial condition given by a plane wave $\lim_{x^0\to-\infty} a^\mu_{\pm\boldsymbol{k}}(x) = \epsilon^\mu(\boldsymbol{k})e^{\pm ik\cdot x}$.

The leading logarithmic contribution comes from the longitudinal component of the integral over $\boldsymbol{k}$, the momentum of the initial plane wave perturbation (and the corresponding momentum in the one loop source term for the equation of motion satisfied by β). This LLog part of the functional derivative (13) operator turns out to be precisely equivalent to the sum of the JIMWLK Hamiltonians describing the RG evolution of the source distributions $W_y[\rho]$. The JIMWLK Hamiltonian

$$\mathcal{H} \equiv \frac{1}{2}\int d^2\boldsymbol{x}_\perp d^2\boldsymbol{y}_\perp D_a(\boldsymbol{y}_\perp)\eta^{ab}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp)D_b(\boldsymbol{y}_\perp) \quad (15)$$

is most naturally expressed in terms of Lie derivatives $D_a(\boldsymbol{x}_\perp)$ operating on the Wilson lines introduced in Eq. (7). in terms of which the kernel in Eq. (15) is

$$\eta^{ab}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp) = \frac{1}{\pi}\int d^2\boldsymbol{u}_\perp \frac{(\boldsymbol{x}_\perp-\boldsymbol{u}_\perp)\cdot(\boldsymbol{y}_\perp-\boldsymbol{u}_\perp)}{(\boldsymbol{x}_\perp-\boldsymbol{u}_\perp)^2(\boldsymbol{y}_\perp-\boldsymbol{u}_\perp)^2}\Big[U(\boldsymbol{x}_\perp)U^\dagger(\boldsymbol{y}_\perp) - U(\boldsymbol{x}_\perp)U^\dagger(\boldsymbol{u}_\perp) - U(\boldsymbol{u}_\perp)U^\dagger(\boldsymbol{y}_\perp)+1\Big]^{ab}. \tag{16}$$

The fact that no other terms with the same logarithmic divergences appear is the proof of factorization; this is the central result of Ref. [3].

4. Multigluon production

4.1. Short range in rapidity

Let us then consider the probability distribution of the number of gluons produced in a small rapidity interval. It was shown in Ref. [12] that a similar factorization theorem holds for the leading logarithmic corrections to this probability distribution in the sense that we will briefly review here. It is convenient to define a generating functional

$$\mathcal{F}[z(\boldsymbol{p})] = \sum_{n=0}^{\infty}\frac{1}{n!}\int\left[\prod_{i=1}^{n} d^3\boldsymbol{p}_i\,\left(z(\boldsymbol{p}_i)-1\right)\right]\frac{d^nN_n}{d^3\boldsymbol{p}_1\ldots d^3\boldsymbol{p}_n}\,. \tag{17}$$

The Taylor coefficients of $\mathcal{F}$ around $z=1$ correspond to the moments of the probability distribution; integrated over the momenta of the produced gluons they are

$$\langle N\rangle \quad \langle N(N-1)\rangle \quad \ldots \quad \langle N(N-1)\cdots(N-n+1)\rangle\,. \tag{18}$$

The result of Ref. [12] is that when these moments are calculated to NLO accuracy, the leading logarithms can be resummed into the JIMWLK evolution of the sources completely analogously to the single inclusive gluon distribution. The resulting probability distribution can be written as:

$$\frac{d^nP_n}{d^3\boldsymbol{p}_1\ldots d^3\boldsymbol{p}_n} = \int\limits_{\rho_1,\rho_2} W_Y[\rho_1]W_Y[\rho_2]\frac{1}{n!}\frac{dN}{d^3\boldsymbol{p}_1}\cdots\frac{dN}{d^3\boldsymbol{p}_n}e^{-\int d^3\boldsymbol{p}\frac{dN}{d^3\boldsymbol{p}}}\,. \tag{19}$$

Note that the Poissonian-looking form of the result is to some extent an artifact of our choosing to develop and truncate precisely the moments Eq. (17) that are simply $\langle N\rangle^n$ for a Poissonian distribution. Since in our power counting $N\sim 1/\alpha_{\mathrm{s}}$, any contributions that would make the distribution Eq. (19) deviate from the functional form are of higher order in the weak coupling expansion of the moments (17) and are neglected in our calculation unless they are enhanced by large logarithms of x. Nevertheless it should be emphasized that in spite of appearances of Eq. (19) the probability distribution is in fact not Poissonian. To understand the nontrivial nature of this result it must be remembered that the individual factors of $\frac{dN}{d^3\boldsymbol{p}_i}$ in Eq. (19) are all functionals of the *same* color charge densities $\rho_{1,2}$; thus the averaging over the ρ's induces a correlation between them. These correlations are precisely the leading logarithmic modifications to the probability distribution; they have been resummed into the distributions W_y; the functional form of the multigluon correlation function *under* the functional integral in Eq. (19) is the same as at leading order. This is the result of the proof in Ref. [12].

4.2. Long range in rapidity

Let us now relax the restriction that the gluons should be observed only in a small rapidity interval and allow for arbitrary separations in rapidity [15]. There is now phase space available to radiate gluons (and for the corresponding virtual contributions) between the measured gluons, and including this radiation can introduce additional large logarithms of the energy. To develop a physical picture of this situation it is perhaps useful to take a step back and consider a more general picture of JIMWLK evolution in terms of its Langevin formulation derived in Ref. [16]. The original derivation is presented purely as an alternative formulation to generate the single Wilson line probability distribution that solves the JIMWLK equation. The JIMWLK equation as it is usually written, as an equation satisfied by the probability distribution of Wilson lines at a single rapidity y, does not formally give information about correlations between different rapidities. Going back to the derivation one sees, however, that the rapidity correlations are also encoded in the formalism. This is most transparent in the Langevin formulation, where one can identify each trajectory in the Langevin equation with one high energy collision event. In this sense the Langevin formulation contains more physical information than just the JIMWLK equation for the probability distribution at a single rapidity; it also gives the combined probability distribution for Wilson lines at different rapidities

$$W_{y_1 \dots y_n}[U_1(\boldsymbol{x}_\perp), \dots, U_n(\boldsymbol{x}_\perp))] \,. \tag{20}$$

Knowing the general (multiple rapidity) probability distribution will enable us to compute the correlations between Wilson lines, and consequently of physical observables such as the multiplicities, at different rapidities. In the following we will give a more precise formulation of this statement and show that it is consistent with our previous result concerning multigluon production.

In the Langevin formulation the distribution W of the Wilson lines can be obtained by evolving in rapidity the elements of an ensemble of Wilson lines according to

$$U(y+dy,\boldsymbol{x}_\perp) = U(y,\boldsymbol{x}_\perp)e^{-i\alpha(y,\boldsymbol{x}_\perp)dy}\,, \tag{21}$$

where the change in a small step dy in rapidity is given by a deterministic term and a stochastic term,

$$\alpha^a(y,\boldsymbol{x}_\perp) = \sigma^a(\boldsymbol{x}_\perp,y) + \int\limits_{\boldsymbol{z}_\perp} \boldsymbol{e}^{ab}(\boldsymbol{x}_\perp,\boldsymbol{z}_\perp)\zeta^b(\boldsymbol{z}_\perp,y)\,. \tag{22}$$

Here ζ^b is a Gaussian random variable defined by $\langle\zeta_i^a(\boldsymbol{x}_\perp,y)\zeta_j^b(\boldsymbol{y}_\perp,y')\rangle = \delta^{ab}\delta_{ij}\delta^2(\boldsymbol{x}_\perp-\boldsymbol{y}_\perp)\delta(y-y')$ and the square root of the JIMWLK kernel is

$$\boldsymbol{e}^{ac}(\boldsymbol{x}_\perp,\boldsymbol{z}_\perp) \equiv \frac{1}{\sqrt{4\pi^3}}\frac{\boldsymbol{x}_\perp-\boldsymbol{z}_\perp}{(\boldsymbol{x}_\perp-\boldsymbol{z}_\perp)^2}(1-U^\dagger(\boldsymbol{x}_\perp)U(\boldsymbol{z}_\perp))^{ac}\,. \tag{23}$$

This stochastic formulation is the method used in numerical studies of the JIMWLK equation [17].

Knowing that the correlation follows from a Langevin equation imposes an additional structure (of a Markovian process) on the probability distribution:

$$W_{y_p,y_q}\left[U^p,U^q\right] = G_{y_p-y_q}\left[U^p,U^q\right]W_{y_q}\left[U^q\right]\,, \tag{24}$$

where the JIMWLK Green's function G is determined by the initial condition

$$\lim_{y_p\to y_q} G_{y_p-y_q}\left[U^p,U^q\right] = \delta\left(U^p(\boldsymbol{x}_\perp)-U^q(\boldsymbol{x}_\perp)\right) \tag{25}$$

and the requirement that it must satisfy the JIMWLK equation

$$\partial_{y_p}G_{y_p-y_q}\left[U^p,U^q\right] = \mathcal{H}\left(U^p(\boldsymbol{x}_\perp)\right)G_{y_p-y_q}\left[U^p,U^q\right]\,. \tag{26}$$

This JIMWLK Green's function contains all the information, at the leading log level, of long range rapidity correlations in gluon production. This structure follows from the computation of the leading log part of 1-loop corrections to a wide class of observables that can be expressed in terms of correlators of the gluon fields at $\tau=0$ (or equivalently Wilson lines)

$$\langle\mathcal{O}\rangle_{\rm LLog} = \int\left[DU_1(y,\boldsymbol{x}_\perp)\right]\left[DU_2(y,\boldsymbol{x}_\perp)\right]W\left[U_1(y,\boldsymbol{x}_\perp)\right]W\left[U_2(y,\boldsymbol{x}_\perp)\right]\mathcal{O}_{\rm LO}\,. \tag{27}$$

Here we have introduced a continuous rapidity notation $W[U(y,\boldsymbol{x}_\perp)]$ for the probability distribution of Wilson lines (20). This should be understood as a probability distribution for the *trajectories* that the Wilson line $U(\boldsymbol{x}_\perp)$

takes on the group manifold along its evolution forward in y following the Langevin equation. We can formally return from the distribution of trajectories to a distribution of Wilson lines at one individual rapidity as

$$W_y[U(\boldsymbol{x}_\perp)] \equiv \int [DU(y,\boldsymbol{x}_\perp)]\ W[U(y,\boldsymbol{x}_\perp)]\,\delta\left[U(\boldsymbol{x}_\perp) - U(y,\boldsymbol{x}_\perp)\right] . \quad (28)$$

Equation (27) is the central result of Ref. [15], showing that all the leading logarithms of rapidity (either the rapidity intervals between the nuclei and the tagged gluons, or between the various produced gluons) can be absorbed into the probability distributions W for the trajectories of Wilson lines of the two projectiles. However, the crucial point to keep in mind is that it involves an average over y-dependent "trajectories" of Wilson lines, rather than an average over Wilson lines at a given fixed rapidity.

The calculation of multigluon correlation is in fact simplified in the strong field limit, where the leading contribution to particle production corresponds to the classical field and the correlations are encoded in the evolution of the sources [18]. In the pA case where one of the sources is assumed to be dilute, the situation becomes much more complicated, because the disconnected classical contributions are not the only dominant ones any more. This structure is illustrated in Fig. 5.

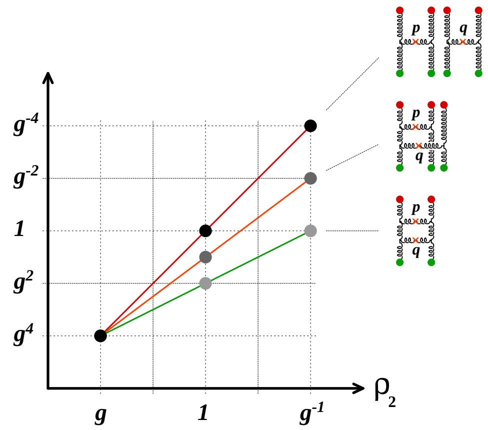

Fig. 5. Relative importance of connected and disconnected diagrams to the two gluon correlation function. One of the color charge densities is considered large, $\rho_1 \sim 1/g$, whereas the other is allowed to vary between the AA case $\rho_2 \sim 1/g$ and the pA one $\rho_2 \sim g$. The order of the disconnected diagram, on top, is $g^4\rho_1^4\rho_2^4$, whereas the interference diagram in the middle is $g^4\rho_1^3\rho_2^3$ and the connected one, lowest, is $g^4\rho_1^2\rho_2^2$. In the AA case the disconnected diagram dominates, for the pA case all three are equally important. In the dilute pp limit only the connected diagram matters and both gluons are produced from the same BFKL ladder.

4.3. Application to multigluon correlations

Let us now specialize Eq. (27) to the case of the single and double inclusive gluon spectra. The single inclusive gluon spectrum $dN_1/d^3\boldsymbol{p}$ at LO depends only on Wilson lines $U_{1,2}(y_p, \boldsymbol{x}_\perp)$ at the rapidity y_p of the produced gluon. One then obtains the known result for the single inclusive gluon spectrum as

$$\left.\frac{dN_1}{d^2\boldsymbol{p}_\perp dy}\right|_{\rm LLog} = \int [DU_1]\,[DU_2]\,W_{y_p}\,[U_1]\;W_{y_p}\,[U_2]\;\left.\frac{dN_1[U_1,U_2]}{d^2\boldsymbol{p}_\perp dy}\right|_{\rm LO}. \qquad (29)$$

For the resummed inclusive two-gluon spectrum, we must recall that at LO it is simply the product of two single gluon spectra, each of which depends on Wilson lines at the rapidity of the corresponding gluon. It is then straightforward to proceed as in the case of the single gluon spectrum in order to obtain:

$$\left.\frac{dN_2}{d^2\boldsymbol{p}_\perp dy_p d^2\boldsymbol{q}_\perp dy_q}\right|_{\rm LLog} = \int [DU_1^p]\,[DU_2^p]\,[DU_1^q]\,[DU_2^q]$$
$$\times W_{y_p,y_q}\,[U_1^p, U_1^q]\,W_{y_p,y_q}\,[U_2^p, U_2^q]\,\left.\frac{dN_1\,[U_1^p, U_2^p]}{d^2\boldsymbol{p}_\perp dy_p}\right|_{\rm LO}\left.\frac{dN_1[U_1^q, U_2^q]}{d^2\boldsymbol{q}_\perp dy_q}\right|_{\rm LO}, \quad (30)$$

where the double probability distribution $W_{y_p,y_q}\,[U_1^p, U_1^q]$ is given by Eqs. (24) and (25).

We have now assembled all the ingredients needed to compute rapidity correlations in the Glasma and address features such as the elongated "ridge" structure in the two particle correlation observed at RHIC [19]. There have already been several boost invariant classical field calculations [20] of this effect and the azimuthal structure, but the inclusion of quantum evolution is needed to understand the rapidity dependence. As a first approximation one should be able to formulate the equivalent of the mean field approximation leading to the BK equation for the JIMWLK propagator. Numerical studies of the JIMWLK equation would then be needed to study the validity of this approximation for rapidity correlations; in the structure of the single rapidity probability distribution the violations from the mean field limit have been observed to be small [21].

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REFERENCES

[1] E. Iancu, R. Venugopalan, arXiv:hep-ph/0303204; H. Weigert, *Prog. Part. Nucl. Phys.* **55**, 461 (2005), arXiv:hep-ph/0501087.

[2] T. Lappi, L. McLerran, *Nucl. Phys.* **A772**, 200 (2006), arXiv:hep-ph/0602189; T. Lappi, *J. Phys. G* **32**, S179 (2006), arXiv:hep-ph/0606090.

[3] F. Gelis, T. Lappi, R. Venugopalan, *Phys. Rev.* **D78**, 054019 (2008), arXiv:0804.2630 [hep-ph].

[4] A. Kovner, L.D. McLerran, H. Weigert, *Phys. Rev.* **D52**, 3809 (1995), arXiv:hep-ph/9505320; Y.V. Kovchegov, D.H. Rischke, *Phys. Rev.* **C56**, 1084 (1997), arXiv:hep-ph/9704201.

[5] L.D. McLerran, R. Venugopalan, *Phys. Rev.* **D49**, 2233 (1994), arXiv:hep-ph/9309289; *Phys. Rev.* **D49**, 3352 (1994), arXiv:hep-ph/9311205; *Phys. Rev.* **D50**, 2225 (1994), arXiv:hep-ph/9402335.

[6] A. Krasnitz, R. Venugopalan, *Nucl. Phys.* **B557**, 237 (1999), arXiv:hep-ph/9809433.

[7] A. Krasnitz, R. Venugopalan, *Phys. Rev. Lett.* **84**, 4309 (2000), arXiv:hep-ph/9909203; *Phys. Rev. Lett.* **86**, 1717 (2001), arXiv:hep-ph/0007108; A. Krasnitz, Y. Nara, R. Venugopalan, *Phys. Rev. Lett.* **87**, 192302 (2001), arXiv:hep-ph/0108092; *Nucl. Phys.* **A717**, 268 (2003), arXiv:hep-ph/0209269; *Nucl. Phys.* **A727**, 427 (2003), arXiv:hep-ph/0305112; T. Lappi, *Phys. Rev.* **C67**, 054903 (2003), arXiv:hep-ph/0303076.

[8] L.V. Gribov, E.M. Levin, M.G. Ryskin, *Phys. Rep.* **100**, 1 (1983).

[9] D. Kharzeev, Y.V. Kovchegov, K. Tuchin, *Phys. Rev.* **D68**, 094013 (2003), arXiv:hep-ph/0307037.

[10] A. Leonidov, D. Ostrovsky, *Phys. Rev.* **D62**, 094009 (2000), arXiv:hep-ph/9905496.

[11] J.-P. Blaizot, Y. Mehtar-Tani, *Nucl. Phys.* **A818**, 97 (2009), arXiv:0806.1422 [hep-ph].

[12] F. Gelis, T. Lappi, R. Venugopalan, *Phys. Rev.* **D78**, 054020 (2008), arXiv:0807.1306 [hep-ph].

[13] F. Gelis, T. Lappi, R. Venugopalan, *Int. J. Mod. Phys.* **E16**, 2595 (2007), arXiv:0708.0047 [hep-ph].

[14] F. Gelis, R. Venugopalan, *Nucl. Phys.* **A776**, 135 (2006), arXiv:hep-ph/0601209.

[15] F. Gelis, T. Lappi, R. Venugopalan, arXiv:0810.4829 [hep-ph].

[16] J.-P. Blaizot, E. Iancu, H. Weigert, *Nucl. Phys.* **A713**, 441 (2003), arXiv:hep-ph/0206279.

[17] K. Rummukainen, H. Weigert, *Nucl. Phys.* **A739**, 183 (2004), arXiv:hep-ph/0309306.

[18] D. Kharzeev, E. Levin, L. McLerran, *Nucl. Phys.* **A748**, 627 (2005), `arXiv:hep-ph/0403271`; N. Armesto, L. McLerran, C. Pajares, *Nucl. Phys.* **A781**, 201 (2007), `arXiv:hep-ph/0607345`.

[19] M. Daugherity [STAR Collaboration], *J. Phys. G* **35**, 104090 (2008), `arXiv:0806.2121 [nucl-ex]`; J. Putschke, *J. Phys. G* **34**, S679 (2007), `arXiv:nucl-ex/0701074`; B. Alver *et al.* [PHOBOS Collaboration], *J. Phys. G* **35**, 104080 (2008), `arXiv:0804.3038 [nucl-ex]`.

[20] A. Dumitru, F. Gelis, L. McLerran, R. Venugopalan, *Nucl. Phys.* **A810**, 91 (2008), `arXiv:0804.3858 [hep-ph]`; S. Gavin, L. McLerran, G. Moschelli, `arXiv:0806.4718 [nucl-th]`.

[21] Y.V. Kovchegov, J. Kuokkanen, K. Rummukainen, H. Weigert, `arXiv:0812.3238 [hep-ph]`.