QCD FACTORIZATION BEYOND LEADING TWIST IN EXCLUSIVE ρ_{T}-MESON PRODUCTION*

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We describe hard exclusive processes involving a transversally polarized ρ meson in the twist 3 approximation, in a framework based on the Taylor expansion of the amplitude around the dominant light-cone direction.

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1. Introduction

The last 15 years have witnessed a tremendous development of the QCD understanding of hard exclusive processes. The basis of this progress has been the derivation of factorization proofs [1] for exclusive amplitudes in various generalized Bjorken kinematical regimes, at the leading twist level. Schematically, the amplitude of a process governed by a large energy scale Q is written as the convolution of a perturbatively calculable subprocess amplitude and a few hadronic matrix elements of light cone operators. For instance, near forward hard electroproduction of a ρ meson depends on the $H(x,\xi,t)$ and $E(x,\xi,t)$ GPDs and on the ρ distribution amplitudes (DA) $\Phi^\rho(z)$.

To extend these factorization procedures beyond the leading twist is mandatory in a number of cases including the phenomenologically important instance of the production of transversely polarized vector mesons. The understanding of the quark–gluon structure of a vector meson is an important task of hadronic physics if one cares about studying confinement dynamics. This quark gluon structure may be described by distribution amplitudes which have been discussed in great detail [2].

Contrarily to the longitudinally polarized case, the leading twist 2 DA of the transversely polarized vector mesons is chirally-odd, a property which opens the way to its use for the study of chirally-odd GPDs [3] (although it is often decoupled because of the vanishing of hard amplitudes [4]). One thus needs to calculate amplitudes at the twist 3 level [5, 6] and this is the source of quite a number of subtle problems, as already discussed in [1]. Two issues are worth emphasizing. Firstly, new non-perturbative objects enter the discussion, as for instance matrix elements of operators containing a transverse derivative. Secondly, the end-point singular behaviour requires careful treatment of kinematical region with soft partons.

Both difficulties should be carefully addressed in turn. Here, we shall not discuss the second one, which may hopefully be treated from a modified collinear factorization point of view [7, 8], including partonic transverse degrees of freedom and subsequent Sudakov resummation techniques. We plan to return to this question later. The first difficulty has been overviewed by most authors, leading sometimes to the use of gauge dependent expressions for the amplitudes. Indeed preserving gauge invariance in a factorization procedure is not guaranteed, as soon as one allows transverse motion of the partons at some intermediate step of the calculation. To preserve the QCD gauge invariance, a subtle interplay of various contributing amplitudes must conspire. For instance, in the case we will detail below, QCD gauge invariance is translated to the statement that the impact factor should vanish when the virtuality of an exchanged t-channel gluon vanishes. We will show that this is indeed the case, provided two and three particle correlators are simultaneously taken into account.

Although some courageous attempts have tried to develop the phenomenology of exclusive vector meson production [5, 8] a careful analysis of the specificities of the transversely polarized case is still lacking. The existing data from HERA (both at high energy with H1 and ZEUS detectors, and at lower energy with HERMES) on the one side, the forthcoming data from JLab and Compass on the other side, deserve a complete study which we aim to perform within the twist 3 approximation. As a first step [9], we show how to use properly the two and three particle distribution amplitudes within a factorized approach.

2. Calculation of the $\gamma^* \to \rho_{\mathrm{T}}$ impact factor

As the simplest example of a hard exclusive reaction involving a transversally polarized vector meson, let us consider the high energy process

$$\gamma^*(q) + \gamma^*(q') \to \rho_{\mathrm{T}}(p_1) + \rho(p_2), \tag{1}$$

where the photons are highly virtual: $q^2, q'^2 = -Q^2, -Q'^2 \gg \Lambda_{\mathrm{QCD}}^2$, and the Mandelstam variables obey the conditions $s \gg Q^2, Q'^2, -t$. Neglecting meson masses, one considers for reaction (1) the vector meson momenta as the light cone vectors ($2\,p_1 \cdot p_2 = s$); the "plus" light cone direction being directed along p_1 and the "minus" light cone direction along p_2 with vector n defined as $p_2/(p_1 \cdot p_2)$. In this Sudakov basis, transverse euclidian momenta are denoted with underlined letters. The virtual photon momentum q reads $q = p_1 - (Q^2/2)\,n$. The impact representation of the scattering amplitude $\mathcal{M}$ for the reaction (1) is

$$\frac{is}{(2\pi)^2} \int \frac{d^2\underline{k}}{\underline{k}^2}\,\Phi_1^{ab}(\underline{k},\,\underline{r}-\underline{k}) \int \frac{d^2\underline{k}'}{\underline{k}'^2}\,\Phi_2^{ab}(-\underline{k}',\,-\underline{r}+\underline{k}')$$

$$\times \int\limits_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i}\left(\frac{s}{s_0}\right)^\omega G_\omega(\underline{k},\underline{k}',\underline{r}),$$

where G_ω is the 4-gluons Green function which obeys the BFKL equation. G_ω reduces to $1/\omega\,\delta(\underline{k}-\underline{k}')\underline{k}^2/(\underline{r}-\underline{k})^2$ within Born approximation. We focus here on the $\gamma^* \to \rho$ impact factor Φ which is the integral of the κ-channel discontinuity of the S-matrix element $\mathcal{S}_\mu^{\gamma_{\mathrm{T}}^* g \to \rho_{\mathrm{T}} g}$ of the subprocess $g(k_1, \varepsilon_1) + \gamma^*(q) \to g(k_2, \varepsilon_2) + \rho_{\mathrm{T}}(p_1)$

$$\Phi^{\gamma^* \to \rho}(\underline{k},\,\underline{r}-\underline{k}) = e^{\gamma^* \mu}\frac{1}{2s}\int \frac{d\kappa}{2\pi}\,\mathrm{Disc}_\kappa\,\mathcal{S}_\mu^{\gamma^* g \to \rho g}(\underline{k},\,\underline{r}-\underline{k}), \tag{2}$$

where $\kappa = (q + k_1)^2$. Considering the forward limit for simplicity, the gluon momenta read

$$k_1 = \frac{\kappa + Q^2 + \underline{k}^2}{s}p_2 + k_{\mathrm{T}},\ \ k_2 = \frac{\kappa + \underline{k}^2}{s}p_2 + k_{\mathrm{T}},\ \ k_1^2 = k_2^2 = k_{\mathrm{T}}^2 = -\underline{k}^2.$$

The impact factor Φ can be calculated within the collinear factorization. It is a convolution of perturbatively calculable hard-scattering amplitudes H and soft correlators S involving relevant ρ-meson distribution amplitudes, symbolically written as

$$\Phi = \int d^4l \cdots \mathrm{Tr}[H(l \cdots)S(l \cdots)], \tag{3}$$

and which involves loop integrations formed by the n partons ($n \geq 2$) entering soft correlators S. Working within the light-cone collinear factorization framework, let us first derive the contribution of the diagrams with the quark–antiquark correlators. The basic tool is to decompose any hard coefficient function $H(\ell)$ around a dominant "plus" direction:

$$H(\ell) = H(yp) + \left.\frac{\partial H(\ell)}{\partial \ell_\alpha}\right|_{\ell=yp} (\ell - y\,p)_\alpha + \dots , \qquad (4)$$

with $(\ell - y\,p)_\alpha \approx \ell_\alpha^\perp$ up to twist 3 accuracy. Here and subsequently α denotes transverse components. This collinear expansion of the hard part should be compared to the expansion of matrix elements in powers of space-time separation $(x^2)^n$ in the coordinate approach.
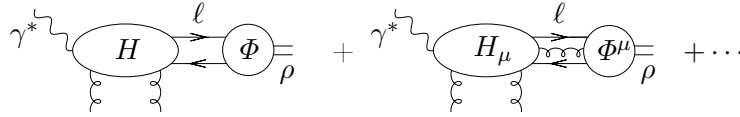

Fig. 1. 2- and 3-body correlators attached to a hard scattering amplitude in the example of the $\gamma^* \to \rho$ impact factor.

The $l^\perp$ dependence of the hard part looks first as an excursion out of the collinear framework. The factorized expression is obtained as the result of an integration by parts which replaces $\ell_\alpha^\perp$ by $\partial_\alpha^\perp$ acting on the soft correlator. This leads to new operators $\mathcal{O}^\perp$ which contain transverse derivatives, such as $\bar{\psi}\partial^\perp\psi$, and thus to the necessity of considering additional correlators $\Phi^\perp(l)$. This procedure leads to the factorization of the amplitude in momentum space. Factorization in the Dirac space can be achieved by a Fierz decomposition. Thus, the amplitude takes the factorized form, see Fig. 1:

$$\mathcal{A} = \int_0^1 dy\, \mathrm{Tr}\left[H(y)\, \Gamma\right] \Phi^\Gamma(y) + \int_0^1 dy_1\, dy_2\, \mathrm{Tr}\left[H^\mu(y_1, y_2)\, \Gamma\right] \Phi_\mu^\Gamma(y_1, y_2). \quad (5)$$

The parameterizations of the vacuum-to-ρ-meson matrix elements needed in the above factorization procedure up to the twist-3 order in the axial (light-like) gauge $n \cdot A = 0$, which we will use in order to get rid of Wilson line, can be written as (here $z = \lambda n$)

$$\langle \rho(p)|\bar{\psi}(z)\gamma_\mu\psi(0)|0\rangle \stackrel{\mathcal{F}_1}{=} m_\rho\, f_\rho \left[\varphi_1(y)\,(e^* \cdot n)p_\mu + \varphi_3(y)\, e_\mu^{*\,T}\right],$$

$$\langle \rho(p)|\bar{\psi}(z)\gamma_5\gamma_\mu\psi(0)|0\rangle \stackrel{\mathcal{F}_1}{=} m_\rho\, f_\rho\, i\varphi_A(y)\, \varepsilon_{\mu\lambda\beta\delta}\, e_\lambda^{*\,T}\, p_\beta\, n_\delta \qquad (6)$$

and

$$\langle \rho(p)|\bar{\psi}(z)\gamma_\mu i \overset{\longleftrightarrow}{\partial^\perp_\alpha} \psi(0)|0\rangle \overset{\mathcal{F}_1}{=} m_\rho\, f_\rho\, \varphi_1^T(y)\, p_\mu e^{*T}_\alpha\,,$$

$$\langle \rho(p)|\bar{\psi}(z)\gamma_5\gamma_\mu i \overset{\longleftrightarrow}{\partial^\perp_\alpha} \psi(0)|0\rangle \overset{\mathcal{F}_1}{=} m_\rho\, f_\rho\, i\, \varphi_A^T(y)\, p_\mu\, \varepsilon_{\alpha\lambda\beta\delta}\, e^{*T}_\lambda\, p_\beta n_\delta\,, \quad (7)$$

where $\overset{\mathcal{F}_1}{=}$ denotes the Fourier transformation $\int_0^1 dy e^{[iy\,p\cdot z]}$. The matrix elements of quark–gluon nonlocal operators can be parameterized as

$$\langle \rho(p)|\bar{\psi}(z_1)\gamma_\mu g A^T_\alpha(z_2)\psi(0)|0\rangle \overset{\mathcal{F}_2}{=} m_\rho\, f^V_{3\rho}\, B(y_1,y_2)\, p_\mu e^{*T}_\alpha\,, \qquad (8)$$

$$\langle \rho(p)|\bar{\psi}(z_1)\gamma_5\gamma_\mu g A^T_\alpha(z_2)\psi(0)|0\rangle \overset{\mathcal{F}_2}{=} m_\rho\, f^A_{3\rho}\, i\, D(y_1,y_2)\, p_\mu \varepsilon_{\alpha\lambda\beta\delta}\, e^{*T}_\lambda\, p_\beta\, n_\delta\,,$$

where $\int_0^1 dy_1 \int_0^1 dy_2\, e^{[iy_1\,p\cdot z_1 + i(y_2-y_1)\,p\cdot z_2]}$ is denoted by $\overset{\mathcal{F}_2}{=}$. The light-cone fractions of the quark, anti-quark and gluon are y_1, $1-y_2$ and y_2-y_1.

The correlators introduced above are not independent, thanks to the QCD equations of motion for the field operators entering them (see, for example, [6]). In the simplest case of fermionic fields, they follow from the vanishing matrix elements $\langle i\hat{D}(0)\psi(0)\,\bar{\psi}(z)\rangle = 0$ and $\langle \psi(0)\,i\hat{D}(z)\bar{\psi}(z)\rangle = 0$ due to the Dirac equation. Denoting $\zeta^{V,A}_{3,\rho} = f^{V,A}_{3\rho}/f_\rho$, we obtain

$$\bar{y}_1\, \varphi_3(y_1) + \bar{y}_1\, \varphi_A(y_1) + \varphi_1^T(y_1) + \varphi_A^T(y_1)$$

$$= -\int_0^1 dy_2 \left[\zeta^V_3\, B(y_1, y_2) + \zeta^A_3\, D(y_1, y_2)\right] \qquad (9)$$

and

$$y_1\, \varphi_3(y_1) - y_1\, \varphi_A(y_1) - \varphi_1^T(y_1) + \varphi_A^T(y_1)$$

$$= -\int_0^1 dy_2 \left[-\zeta^V_3\, B(y_2, y_1) + \zeta^A_3\, D(y_2, y_1)\right]\,. \qquad (10)$$

A crucial point of this approach is that scattering amplitudes do not depend on the specific choice of the vector n which fixes the light-cone direction. As observed for inclusive structure functions [10] and for DVCS [11], this n independence condition leads to non-trivial relations at the twist three level. In the case of exclusive processes [6], this constrains the non-perturbative correlators entering the factorized amplitude. The polarization vector for transverse ρ which enters in the parametrization of twist 3 correlators depends on n according to

$$e^{*T}_\mu = e^*_\mu - p_\mu\, e^* \cdot n\,, \qquad (11)$$

i.e. it is defined with respect to the light-cone vector n. The $n-$independence condition can thus be written as

$$\frac{d\mathcal{A}}{dn^\mu} = 0, \qquad \text{where} \qquad \frac{d}{dn^\mu} = \frac{\partial}{\partial n^\mu} + e^*_\mu \frac{\partial}{\partial(e^* \cdot n)}. \qquad (12)$$

The appearance of the total derivative may be interpreted as a (vector) analog to the renormalization group (RG) invariance when the dependences on the renormalization parameter coming from various sources cancel. One can view this as a RG-like flow in the space of light-cone directions of contributions to the amplitude where the polarization vector plays the role of a beta function. This condition expressed at the level of the *full amplitude* can be reduced to a set of conditions involving only the soft correlators. The general strategy relies on the use of the collinear Ward identities which relate firstly amplitudes with different number of legs and secondly higher order coefficients in the Taylor expansion (4) to lower order ones. The most involved use of these identities occurs in the case of the 3-body correlator. In the case of vector correlator (8), due to (11) the dependency on n enters linearly and only through the scalar product $e^* \cdot n$. Thus, the action on the amplitude of the derivative d/dn defined in (12) can be extracted by the replacement $e^*_\alpha \to p_\alpha$, and after using the Ward identity, it reads

$$(y_1 - y_2)\text{Tr}\,[H_\rho(y_1, y_2)\,p^\rho\,\slashed{p}] = \text{Tr}\,[H(y_1)\,\slashed{p}] - \text{Tr}\,[H(y_2)\,\slashed{p}]\,,$$

as illustrated by Fig. 2. One can apply similar tricks to the 2-body correlators with transverse derivative whose contribution can be viewed as 3-body processes with vanishing gluon momentum. This finally leads to the equations

$$\frac{d}{dy_1}\varphi_1^T(y_1) = -\varphi_1(y_1) + \varphi_3(y_1) - \zeta_3^V \int\limits_0^1 \frac{dy_2}{y_2 - y_1}\,(B(y_1, y_2) + B(y_2, y_1))\,,$$

$$\frac{d}{dy_1}\varphi_A^T(y_1) = \varphi_A(y_1) - \zeta_3^A \int\limits_0^1 \frac{dy_2}{y_2 - y_1}\,(D(y_1, y_2) + D(y_2, y_1))\,. \qquad (13)$$
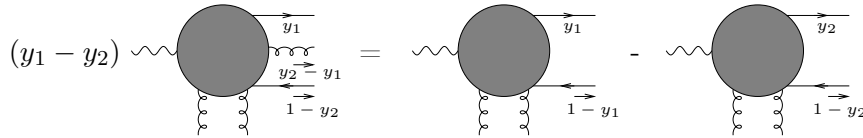

Fig. 2.   Reduction of 3-body correlators to 2-body correlators through Ward identity.

We emphasize that the equations obtained above for the special case of impact factor are universal *i.e.* do not refer to any specific hard process. The equations of motion together with those expressing the n-independence are four equations which constrain seven DAs $(\varphi_1, \varphi_3, \varphi_1^T, \varphi_A, \varphi_A^T, B, D)$. Thus, any hard process involving the exclusive production of a ρ can be expressed, at twist 3, in terms of the three independent DAs which we choose as φ_1, B and D. The solution of Eqs. $(9), (10), (13)$ can be generically written as the sum of the solution in the Wandzura–Wilczek approximation, *i.e.* with vanishing B and D DAs, and the solution describing the genuine twist 3 part. The Wandzura–Wilczek parts take the forms

$$\varphi_{3/A}^{WW}(y) = \frac{1}{2}\left[\int\limits_0^y \frac{dv}{\bar{v}}\varphi_1(v) \pm \int\limits_y^1 \frac{dv}{v}\varphi_1(v)\right],\tag{14}$$

$$\varphi_{1/A}^{T\,WW}(y) = \frac{1}{2}\left[-\bar{y}\int\limits_0^y \frac{dv}{\bar{v}}\varphi_1(v) \pm y\int\limits_y^1 \frac{dv}{v}\varphi_1(v)\right].\tag{15}$$

The remaining genuine twist 3 DAs read:

$$\varphi_3^{\mathrm{gen}}(y) = -\frac{1}{2}\int\limits_y^1 \frac{du}{u}\left[\int\limits_0^u dy_2 \frac{d}{du}\left(\zeta_3^V B - \zeta_3^A D\right)(y_2,\,u)\right.$$

$$-\int\limits_u^1 \frac{dy_2}{y_2-u}\left(\zeta_3^V B - \zeta_3^A D\right)(u,y_2) - \int\limits_0^u \frac{dy_2}{y_2-u}\left(\zeta_3^V B - \zeta_3^A D\right)(y_2,u)\right]$$

$$-\frac{1}{2}\int\limits_0^{y_1} \frac{du}{\bar{u}}\left[\int\limits_u^1 dy_2 \frac{d}{du}\left(\zeta_3^V B + \zeta_3^A D\right)(u,y_2)\right.$$

$$-\int\limits_u^1 \frac{dy_2}{y_2-u}\left(\zeta_3^V B + \zeta_3^A D\right)(u,y_2) - \int\limits_0^u \frac{dy_2}{y_2-u}\left(\zeta_3^V B + v\zeta_3^A D\right)(y_2,u)\right],\tag{16}$$

$$\varphi_1^{T\,\mathrm{gen}}(y) = \int\limits_0^y du\,\varphi_3^{\mathrm{gen}}(u) - \zeta_3^V\int\limits_0^y dy_1\int\limits_y^1 dy_2\frac{B(y_1,y_2)}{y_2-y_1},\tag{17}$$

while the corresponding expressions for $\varphi_A^{\mathrm{gen}}(y)$ and $\varphi_A^{T\,\mathrm{gen}}(y)$ are obtained by the substitutions:

$$\varphi_A^{\mathrm{gen}}(y) \overset{\zeta_3^V B \leftrightarrow \zeta_3^A D}{\longleftrightarrow} \varphi_3^{\mathrm{gen}}(y),\tag{18}$$

$$\varphi_A^{T\,\mathrm{gen}}(y) \overset{\zeta_3^V B \leftrightarrow \zeta_3^A D}{\longleftrightarrow} \varphi_1^{T\,\mathrm{gen}}(y).\tag{19}$$

In summary, we describe consistently exclusive processes at higher twist, in a way which explicitly preserves gauge invariance. The n-independency condition generalized up to the dynamical twist 3 plays a crucial role for the consistency of this approach with the studies of DAs performed in [2]. The present framework opens the way to a systematic and consistent treatment of hard exclusive processes. This does not preclude the solution of the end-point singularity problem [5, 8] which requires a dedicated treatment.

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