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LIMITING FRAGMENTATION IN e^+e^- ANNIHILATION AND ep DEEP INELASTIC SCATTERING*

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The subject of this paper is hadron production in the limiting fragmentation region. The density of particles per unit of rapidity for e^+e^- annihilation is investigated at different energies of the colliding beams using data collected at the PETRA and LEP accelerators. This is compared with $p\bar{p}$ interactions and predictions for ep deep inelastic scattering. The investigation was inspired by a model of hadron production proposed by Bialas and Jezabek. This model describes hadron production via radiation from the colour charges.

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1. Introduction

This paper reports on the search for an observable, a measurable quantity, which exposes differences in hadron production between e^+e^- annihilation, neutral current ep deep inelastic scattering (NC DIS) and $p\bar{p}$ interaction. We also look for some measurable quantity for ep scattering which exposes the features of the exchanged boson by varying its virtuality Q^2. At high four-momentum transfer Q^2 the exchanged boson behaves as a point-like object with increasing contributions from Z^0, whereas at low Q^2 a contribution from the resolved component of the virtual photon provides a good description of the data. One of the possible observables is based on a hypothesis of limiting fragmentation [2] originally introduced for hadron–hadron

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collisions. The proposed observable [3] is a parameter from the Bialas–Jezabek model [1] describing the limiting fragmentation region for hadronic processes.

According to a general picture of hadron–hadron collision there are two distinct sources emitting particles along the collision line. Particles near beam and target rapidity are governed by limiting fragmentation. The momentum distribution of these particles in the rest frame of one of the colliding hadrons becomes energy independent at high enough collision energy. In the concept of limiting fragmentation a Lorentz-contracted hadron passing through the other hadron at rest leaves behind an excited state whose properties do not depend in detail on the energy or identity of the passing hadrons. This exited state fragments into a final state distribution of particles leading to particle production in a restricted window of rapidity with no particles at mid-rapidity. The second source produces particles near mid-rapidity in form of a rapidity plateau with a constant density per a unit of rapidity dn/dy, independent of energy and the nature of the hadrons in the initial collision. The extent of this boost-invariant region is expected to grow with energy [4, 5].

This general picture failed completely for the elementary collisions such as pp but the ansatz of limiting fragmentation has been successfully observed in $pp, p\bar{p}, \pi$-emulsion, p-emulsion and nucleus–nucleus (AA) collisions, for particle production away from mid-rapidity, *e.g.* [6–9]. The limiting fragmentation picture is successful at a very large range of energies extending from $\sqrt{s} = 50\,\mathrm{GeV}$ to $900\,\mathrm{GeV}$ [7, 8]. Thus the data collected at various energies shifted in the rapidity axis by their beam rapidity Y_{beam} tend to some universal curve in the fragmentation region. This universal behaviour is interpreted as a reflection of gluon saturation [10]. The subtraction of Y_{beam} is equivalent to usage of the projectile rest frame as the reference frame.

Bialas and Jezabek proposed a model describing the soft particle production in hadronic collision in terms of multiple gluon exchanges between colliding hadrons. Interacting partons move in opposite directions, each one radiating hadronic clusters from coloured partons distributed uniformly in rapidity. The emission is described by the bremsstahlung process. The model explains two distinct features of the data illustrated in Fig. 1: an increase of rapidity spectra $d\sigma/dy_h \sim \mathcal{A}y_h$ in the regions of limiting fragmentation and the proportionality between the increasing width of the limiting fragmentation region and the height of the central plateau. According to this model a proportionality parameter $\mathcal{A}$ describing the slope is constant, *i.e.* independent from the centre-of-mass energy W and is a product of three quantities λ, a and b which have well-defined physical meanings: λ is the fraction of *active* partons which participated in the collision, b is the parton density per unit of rapidity, and a the density of emitted hadrons per unit of rapidity. The parameters a and b are universal quantities, the same for all QCD processes whereas λ describes the content of the colliding objects. This is a statistical model describing the dominant properties of data.

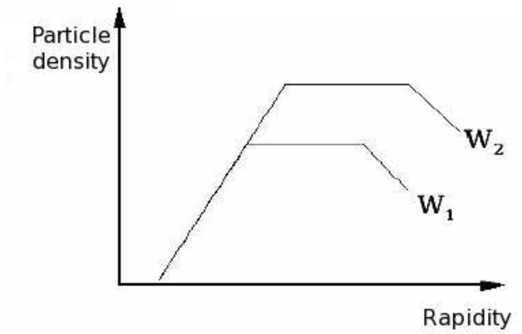

Fig. 1. Illustration of the main features for hadron production according to the Bialas–Jezabek model for two energies in the hadronic centre-of-mass frame $W_1 < W_2$. The slope is described by a linear function of hadron rapidity $\mathcal{A} \cdot (y_h - Y^{\max}) + \mathcal{B}$ with parameters $\mathcal{A}$ and $\mathcal{B}$ being independent of W and $Y^{\max}$ being the maximum value of rapidity.

The model is developed for symmetric projectiles inspired by proton–antiproton ($p\bar{p}$) data collected at ISR and $Sp\bar{p}S$ [7] as well as nucleus–nucleus data collected at RHIC [9]. The linear dependence deduced by Bialas and Jezabek from their assumptions describes well the data.

The concept of universality of parton–parton interactions implies that characteristics of produced hadrons are the same for a variety of processes and in a wide range of W. The elementary process of e^+e^- annihilation into partons and ep scattering with asymmetric projectiles are good laboratories for verification of the limiting fragmentation hypothesis and the basic model features with data from a few to hundreds of GeV.

Measurements of hadron production at similar ranges of hadronic energies W have been carried out in e^+e^- annihilation at PETRA [11] and ep collision at HERA [12–17]. In the first approach hadrons from e^+e^- annihilation should have similar energetic and angular distributions as hadrons from the ep current jet hemisphere. However, only a few processes taking place in ep scattering have their counterparts in e^+e^-. The others like boson–gluon fusion or gluon radiation in the initial state have no matching processes in e^+e^-. Also hadron production in e^+e^- is dominated by hard processes whereas in ep soft processes can have a significant contribution, *e.g.* due to interaction with the proton remnant. In addition, at low Q^2 the exchanged boson can interact via its resolved component; therefore, the hadronic distributions are expected to be different.

The slope $\mathcal{A}$ proposed in Bialas–Jezabek model is a candidate for a measurable quantity which may demonstrate the different aspects of ep interaction and can be used for the comparison with the elementary process of e^+e^- annihilation and with $p\bar{p}$ interactions described well by the multigluon exchange.

In Sec. 2 values of the slope $\mathcal{A}$ in e^+e^- annihilation for different beam energies are estimated. According to the limiting fragmentation hypothesis they should be roughly the same. In Sec. 3 the values of the slope $\mathcal{A}$ are studied for ep interactions for fixed values of Q^2 and for different values of W using the density of hadrons per unit of (pseudo)rapidity in the hadronic centre-of-mass frame. The values are obtained from QCD based models using the Monte Carlo (MC) generator programs ARIADNE [18, 19] and LEPTO [20] because the existing data [12–17] are not precise enough. In Sec. 4 the obtained values of the slope for charged hadron density in rapidity from e^+e^- annihilation are compared with those obtained for ep interactions and for $p\bar{p}$. In these studies the primary charged particle definition includes those produced in the decays of particles with the average lifetime smaller than 3×10^{-11} s. The scattered electrons in ep deep inelastic scattering are excluded.

2. e^+e^- annihilation

Data from the TASSO experiment at PETRA are analysed for the energies in the centre-of-mass of the colliding beams: $2E_{\rm beam} = \sqrt{s_{ee}} = 14$, 22, 35 and 44 GeV [11]. The densities of charged particles as a function of rapidity y_h relative to the event thrust axis are taken assuming the pion mass for all of them. They are folded around hadron rapidity $y_h = 0$; therefore, the fitted slope should be divided by 2. As shown in Fig. 2 the slope $\mathcal{A}$ is roughly energy independent. However, the experimental points do not lay ideally on a straight line and the $\mathcal{A}$ value depends weakly on the number of points taken to fit the line; this is visible for the lowest considered energy. This gives some additional uncertainties included in the final values in Table I. In addition, in e^+e^- experiments particles with lifetime $\tau > 10^{-8}$ s are assumed to be stable contrary to hadron–hadron collisions or DIS where $\tau > 10^{-11}$ s is used. The slope values $\mathcal{A}^{\rm corr}$ with corrections based on PYTHIA [21] are also given in Table I.

The data from the TASSO experiment at PETRA and from the ALEPH experiment at LEP [22] are overlaid in Fig. 3. Both data have been shifted by the respective maximum rapidity of emitted hadrons calculated as $|Y^{\rm max}| = \ln{(E_{\rm beam}/m_\pi)}$ with m_π the pion mass. The aim is to show trends in changes with energy. The essential features of the Bialas–Jezabek model are confirmed although with the large statistics some deviations from linearity are observed characteristic of S-shaped curves with a bend point in the middle. This is illustrated in Fig. 4.

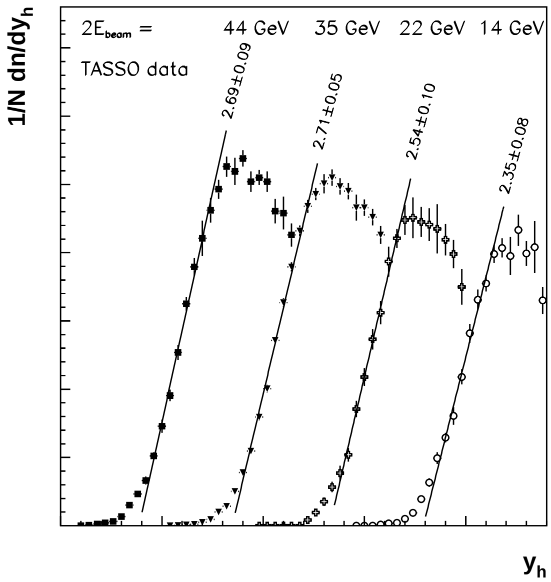

Fig. 2. Density of charged hadrons in rapidity y_h normalized to the number of events N for e^+e^- annihilation from the TASSO experiment for the four energies, E_{beam} of the colliding e^+e^- beams; the data folded together from two hemispheres are taken from Genser's thesis, 1989. The slope values $2*\mathcal{A}$ from the fitted line are quoted with their statistical uncertainties. The scale in the rapidity y_h is arbitrary shifted to present data from the different beam energies on the same plot.

TABLE I

Values of the parameter $\mathcal{A}$ from density of charged hadron production for different energies of colliding electron beams E_{beam} in one hemisphere (Figs 2–3). The uncertainties contain contributions due to the sensitivity of the obtained values to the size of the η^{HCM} interval used in the fit. In addition a correction is introduced in $\mathcal{A}^{\text{corr}}$ for a different definition of stable particles used in the e^+e^- experiments.

$2E_{\text{beam}}$ [GeV]	$\mathcal{A}$ [GeV]	$\mathcal{A}^{\text{corr}}$ [GeV]	Experiment
14	1.18 ± 0.07	1.07 ± 0.06	TASSO [11]
22	1.27 ± 0.07	1.15 ± 0.06	TASSO [11]
35	1.35 ± 0.05	1.22 ± 0.05	TASSO [11]
44	1.34 ± 0.07	1.22 ± 0.06	TASSO [11]
50	1.35 ± 0.05	1.22 ± 0.05	AMY [24]
91.6	1.55 ± 0.01	1.40 ± 0.009	ALEPH [22]
92	1.55 ± 0.02	1.40 ± 0.02	DELPHI [23]

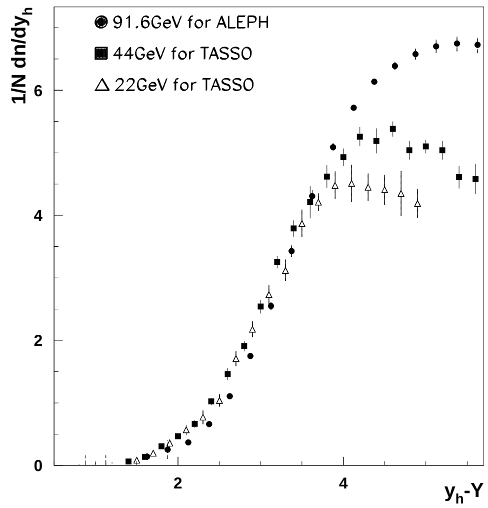

Fig. 3. Charged hadron density in rapidity for e^+e^- annihilation from TASSO and ALEPH plotted *versus* $y_h - Y$ where Y is related to the maximum value of rapidity $Y^{\max}$, *i.e.* $Y = Y^{\max}$ for TASSO and $Y = Y^{\max} + 0.2$ for ALEPH.

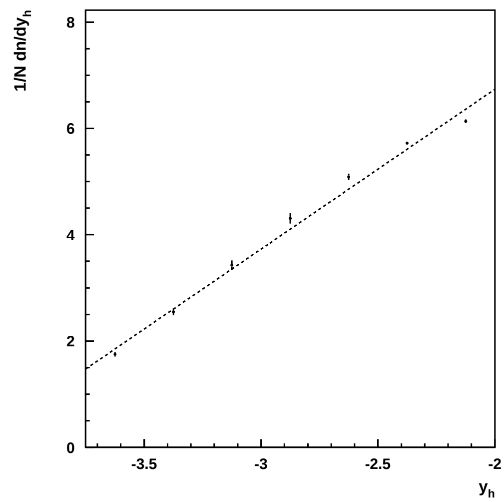

Fig. 4. Charged hadron density in rapidity y_h for e^+e^- annihilation from ALEPH.

This study shows that the slope depends weakly on the energy in the centre of mass system $W = 2E_{\rm beam}$ and changes from $\mathcal{A} = 1.27 \pm 0.07$ GeV for $2E_{\rm beam} = 22$ GeV to $\mathcal{A} = 1.55 \pm 0.01$ GeV for $2E_{\rm beam} = 91.6$ GeV or more as shown in Fig. 4. Saturation is seen with the maximum value of multiplicity per unit of rapidity increasing linearly with energy. The plateau is not well defined with minimum at hadron rapidity $y_h = 0$. Scaling with the maximum value of rapidity $|Y^{\max}|$ suggested by the limiting fragmentation hypothesis and confirmed by nuclear and hadron collisions at high energies is not exactly fulfilled for the ALEPH data where the Z^0 exchange is dominant; a shift of about 0.2 rapidity unit is needed to overlap with the TASSO data as shown in Fig. 3. A similar observation is found also elsewhere [8] based partially on indirect measurements.

We conclude that the Bialas–Jezabek model reproduces only the main features of charged hadron production in e^+e^- annihilation dominated by the elementary process $e^+e^- \to q\bar{q}$.

3. e^+p interactions

Kinematics of the hadronic final state in neutral current deep inelastic scattering are determined by the negative square of the four-momentum of the virtual exchanged boson, $Q^2 = -q^2$ and the invariant mass, W, of the hadronic system. A natural frame to study the dynamics of the hadronic final state in DIS is any frame along the virtual boson–proton collision line such as the hadronic centre-of-mass frame (HCM) or the Breit frame.

In the study the Z-axis is chosen along the incoming proton direction. The current region ($y_h^{\rm Breit} < 0$) in the *ep* Breit frame is analogous to a single hemisphere of e^+e^- annihilation. In $e^+e^- \to q\bar{q}$ the two quarks are produced with equal and opposite momenta, $\pm\sqrt{s_{ee}}/2$. The fragmentation of these quarks can be compared to that of the quark struck from the proton; this quark has an outgoing momentum $-Q/2$ in the Breit frame. For the fixed Q^2 hadron production in the HCM frame at (pseudo)rapidity $\eta^{\rm HCM} \leq \ln{(Q/W)}$ is equivalent to hadron production in the current region defined by the Breit frame. The concept of limiting fragmentation is applied to rapidity distributions but for particle emitted far away from mid-rapidity, this scaling is also expected to apply to pseudorapidity distributions.

The semi-inclusive sample of hadrons in NC DIS is taken to be equivalent to the analysed e^+e^- samples. Thus the distribution of pseudorapidity in the hadronic centre of mass is investigated for three samples of events with the exchanged boson virtuality, Q^2 equal to $92^2\,{\rm GeV}^2$, $14^2\,{\rm GeV}^2$ and $4^2\,{\rm GeV}^2$. The last value is chosen as the lower limit because for $Q^2 < 20\,{\rm GeV}^2$ the resolved photon contribution is expected to be large enough to be measurable [25]. Two subsamples are taken with the hadronic centre-of-mass energies $W = 90\,{\rm GeV}$ and $240\,{\rm GeV}$. The Bialas–Jezabek model suggests that the slope value should be the same at all intervals of W; if the nature of a projectile, *i.e.* the exchanged boson, is the same then the slope value should also be the same for the different values of Q^2.

A study is performed using the generated *ep* events to evaluate the precision of a potential measurement. The accuracy of the available measurements of energy flow [12–16] and charged particle flow [17] is not sufficient to verify the slope dependence on Q^2 and W. In addition the energy flow method does not permit a direct comparison with e^+e^- annihilation and with $p\bar{p}$ interactions. For these data the mean transverse energy as a function of rapidity is needed to convert the energy flow into the charged particle flow. However, the earlier studies [12–17] done for the wide Q^2 range show that out of many investigated descriptions of ep scattering the models included in ARIADNE and LEPTO-MEPS give a satisfactory description of hadron production at $Q^2 > 10\,\mathrm{GeV}^2$.

ARIADNE 4.12 [18,19] and LEPTO-MEPS 6.1.2 [20] are used. LEPTO matches the exact first order QCD matrix elements to DGLAP [26–28] based on leading log parton showers. No option which rearranges the event colour topology is used. ARIADNE simulates parton emission by the Colour Dipol Model (CDM) in which partons are radiated from colour dipoles produced in the hard interaction. This model was tuned to reproduce the high Q^2 data [19]. In all cases, the events were generated using the CTEQ4D next-to-leading order parton density parametrization [29] of the proton. Fragmentation into hadrons is performed according to the JETSET 7.410 code [30,31] based on the Lund string model. These codes do not generate interaction via resolved photons, however some features of this process are expected to be described by the CDM model.

Densities of the charged particles as a function of pseudorapidity generated for positron–proton scattering using both event generators are shown in Figs 5 and 6 scaled with the maximum value of rapidity $|Y^{\max}| = \ln{(W/m_\pi)}$. In Fig. 5 for fixed values of Q^2 the slopes are nearly the same confirming the Bialas–Jezabek model except for the LEPTO predictions at $Q^2 = 92^2$ GeV^2. This is the region with a significant contribution from the Z^0 exchange. The slopes from e^+e^- annihilation agree neither with the ones obtained from the ARIADNE predictions with $Q = E_{\mathrm{beam}}$ nor with the ones deduced from the LEPTO predictions except at low W for ARIADNE at $Q^2 = 14\,\mathrm{GeV}^2$. This is the region with the largest contribution from boson–gluon fusions. This process populates the rapidity region near the origin of the Breit frame marked in Fig. 5. The slope values are sensitive to the lifetime limit used in the definition of charged particles taken as primaries.

In Fig. 6 the region of limiting fragmentation are shown for $Q^2 = 92^2$, 14^2 and 4^2 GeV^2 for fixed values of W. No drastic changes are predicted in the ep slope with Q^2 but the Bialas–Jezabek expectations illustrated in Fig. 1 are not fulfilled. Scaling with the maximum value of rapidity $|Y^{\max}|$ is not fulfilled as in e^+e^- annihilation. If it is confirmed by precise measurements then either the nature of the projectile is changing with Q^2 or leakage from the target region in asymmetric collisions does not follow the simple assumptions of the model [32].

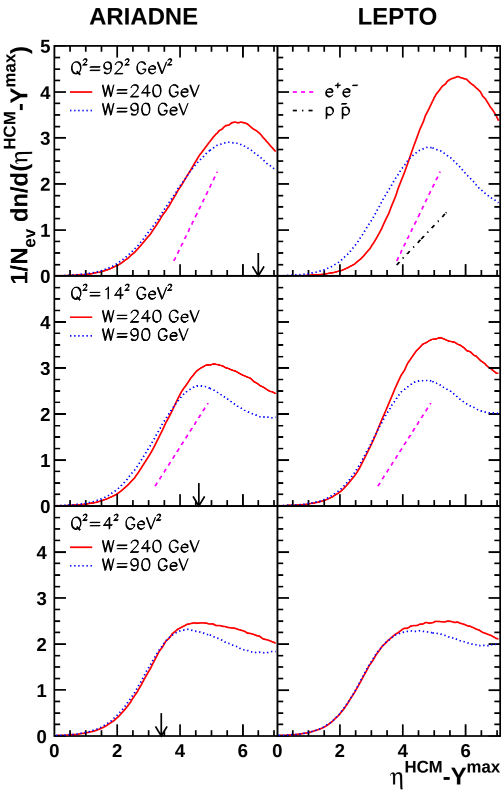

Fig. 5. Density of charged hadrons in generated *ep* NC DIS events normalized to the number of events $N_{\rm ev}$ as a function of relative pseudorapidity $\eta^{\rm HCM} - Y^{\rm max}$ for fixed values of $Q^2 \approx 4^2, 14^2, 92^2\,{\rm GeV}^2$ and for two values of the hadronic energy $W = 90\,{\rm GeV}$ (solid line) and $W = 240\,{\rm GeV}$ (dotted line) as predicted by ARIADNE (on the left side) and LEPTO (on the right side). The measured slopes for e^+e^- annihilation at $2E_{\rm beam} = 92$ and $14\,{\rm GeV}$ from Table I are illustrated by dashed lines put at random $\eta^{\rm HCM} - Y^{\rm max}$. Also the slope for $p\bar{p}$ data is represented as dash-dotted line (see text). The arrows mark the position of the origin of the Breit frame ($\eta_{\rm Breit} = 0$).

A MC study permits interpretation of the data in terms of underlying parton level processes. The region of limiting fragmentation is dominated by gluon radiation with an increase of contribution from boson–gluon fusion. This contribution is predicted to be the largest at Q^2 of hundreds ${\rm GeV}^2$ and decreases with increasing or decreasing Q^2. Thus the steepness of the slope with Q^2 is weakly related to gluon content in proton but together with the plateau height they are sensitive to assumptions on the fragmentation function suggested to be softer by the SMC measurements [33] than the one deduced from the LEP data and implemented in JETSET.

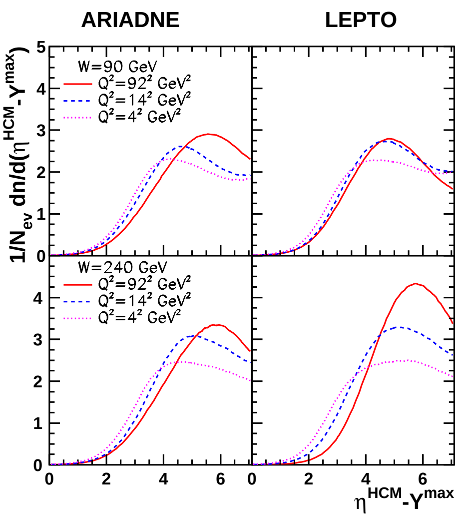

Fig. 6. Density of charged hadrons in the generated ep NC DIS events as a function of relative pseudorapidity $\eta^{\mathrm{HCM}} - Y^{\max}$ for the fixed values of the hadronic energy W and three values of $Q^2 \approx 4^2$, 14^2, $92^2\,\mathrm{GeV}^2$ as predicted by ARIADNE and LEPTO.

Only above $Q^2 \approx 300$ GeV^2 the slope region is totally contained in the current region of the Breit frame, the region in which the hadron production is similar to hadron production in e^+e^- annihilation. Thus the limiting fragmentation region in ep scattering extends beyond the limiting fragmentation extracted from the e^+e^- annihilation.

4. Comparison of e^+e^-, e^+p and $p\bar{p}$

Universality of parton–parton interactions relates hadron production in e^+e^- and ep with $p\bar{p}$. The $p\bar{p}$ data from the UA5 experiment [7] are fitted by a straight line covering the centre-of-mass energies from 53 to 900 GeV. The slope is estimated to be $\mathcal{A} = 0.72 \pm 0.05\,\mathrm{GeV}$ with the uncertainty coming from the spread of data at individual energies. The slopes from e^+e^- annihilation and $p\bar{p}$ interaction are compared with the ep slope in Fig. 5. They are found to be different. The universality of the a and b parameters of the Bialas–Jezabek model suggests that the fraction of "active" partons are $\lambda_{e^+e^-} : \lambda_{p\bar{p}} = \mathcal{A}(e^+e^-) : \mathcal{A}(p\bar{p}) \approx 1 : 0.5$. The main uncertainties in the interpretation of the data come from the different definitions of the stable particles used in measurements, and from choice of axes in e^+e^- events.

The e^+e^- and $p\bar{p}$ data and ep predictions separately confirm the essential features of the Bialas–Jezabek model. It would be of great interest to check the Q^2 dependence for ep in the large amount of data accumulated by the HERA experiments.

In the limiting fragmentation region of hadronic collisions described successfully by multigluon exchange density of charged particles for various energies appears to approach a fixed curve indicating that this universal curve is an important feature of the overall interaction and not simple a break up. The leading order MC predictions do not include some processes with soft hadron production which might lead to the same type of scaling behaviour.

5. Conclusions

Observables describing some general features of the hadronic final state in high energy particle collisions are the density of charged particles in rapidity units. These quantities permit the investigation of emission of gluons from scattered partons as well as interpretation of the data in terms of underlying parton level processes. The hypothesis of limiting fragmentation connects hadron production in the wide range of energies in the region approaching rapidities in the extremes. The model proposed by Bialas and Jezabek relates the e^+e^-, ep and $p\bar{p}$ processes.

Using the published data on rapidity distribution of e^+e^- annihilation at energies from 14 GeV to 92 GeV, hadron production in the limiting fragmentation region has been investigated. Changes of particle density in rapidity, the so-called slope, are found to be weakly dependent on the energy available in the centre-of-mass system. The width of the limiting fragmentation region is increasing with the available energy. The Bialas–Jezabek model reproduces the dominant features of e^+e^- data although this statistical approach is not the obvious description of hadron production for a single $q\bar{q}$ pair.

A comparison of hadron production in e^+e^- annihilation and in ep scattering is done in a similar range of the hadronic centre-of-mass energy. The ep scattering events come from the event generators LEPTO-MEPS and ARIADNE because the existing experimental data are not precise enough. The predictions show a lack of W dependence for ARIADNE and for LEPTO at Q^2 below Z^0 exchange ($Q^2 \ll 92^2\,\mathrm{GeV}^2$). Scaling with the maximum value of rapidity $Y^{\max}$ is not fulfilled. The significant difference in predictions between LEPTO and ARIADNE at $Q^2 \approx M_Z^2$ and the fact that the predicted ep slopes do not agree with the e^+e^- slope causes that it would be of great interest to check the experimental values on the new accumulated data at HERA.

For a multiparticle process it is important to use the variables in terms of which the process is most simply described. This is the case of the slope parameter proposed by Bialas and Jezabek.

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