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REGGE EXCHANGE CONTRIBUTION TO DEEPLY VIRTUAL COMPTON SCATTERING

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Assuming that the energy dependence of quasi on-shell parton–nucleon interactions can be parametrized in terms of J-plane singularities we explore consequences of Regge behavior for Deeply Virtual Compton Scattering (DVCS). In particular we find that resulting generalized parton distributions develop singularities which prevent the use of collinear factorization. The microscopic interpretation is that DVCS is dominated by photon dissociation into a quark–antiquark pair with subsequent re-scattering on target constituents rather than scattering on partons forming the target bound state.

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1. Introduction

1.1. Motivation: Regge behavior and generalized parton distributions

Nowadays it is common to consider deeply virtual exclusive electroproduction of mesons or photons in the context of generalized parton distributions (GPD's) [1–6]. In the case of deeply virtual Compton scattering (DVCS) [7, 8], a single initial quark near its mass shell becomes highly virtual after it interacts with the off-shell photon. This virtual quark is believed to propagate essentially without interaction with the quark and gluon spectators until it radiates a real photon. To produce a final hadron in place of a real photon, the off-shell quark can radiate a hard gluon that enhances its correlation with soft quarks and antiquarks around the target, and hence increases the probability of hadronization into a single meson. In the language of QCD factorization [9, 10], the exchange of a single off-shell quark between the two photons (or photon and meson, in the case of hard exclusive meson production) is said to be sensitive to QCD interactions at momentum scales of the order of Q^2, where Q^2 is the large virtuality of the incoming photon, while all processes involving target constituents occur at scales much smaller than Q^2. If there is scale separation then the physics of hard two-photon–quark interactions can be separated from soft parton–nucleon dynamics.

An alternative approach to exclusive processes involving strong interactions is based on identifying singularities of the scattering amplitude close to the physical region of the relevant kinematical variables. In particular one expects physical states with quantum numbers of the t-channel to dominate processes involving two-to-two particle scattering, $ab \to cd$ at low momentum transfer to the target, $t = (p_d - p_b)^2 < 0$ and large values of s (the square of the center of mass energy) $s = (p_a + p_b)^2 \gg -t$. Inclusion of all allowed t-channel exchanges leads to the Regge-type dependence, of the scattering amplitude $A(s,t)$

$$A(s,t) \sim s^{\alpha(t)} \tag{1}$$

on the center of mass energy s where for low momentum transfer $-t \leq 1\ \mathrm{GeV}^2$. In Eq. (1) the intercept $\alpha(t)$ of the Regge trajectory is a positive number less than one (with exception of diffractive scattering which in this language corresponds to the Pomeron exchange).

If analogous dynamics microscopically occur at the level of parton –nucleon interactions, then, microscopically, exclusive electroproduction differs substantially from that given in terms of generalized parton distributions. Qualitatively, the Regge picture corresponds to the virtual photon dissociating into a quark–antiquark pair followed by rescattering on target constituents. On the other hand dynamics that is commonly associated with the structure functions (pdf's) or generalized distributions, has to do with the parton content of the target nucleon, *i.e.* parton bound states [11, 12]. It was recently shown [13], that, at least in the case of pdf's, both, bound and scattering parton–nucleon states are indeed relevant. In particular it was shown that in light cone gauge structure functions are sensitive to interactions between the anti-quark from the virtual photon and the target spectators. Such interactions occur in the final state and correspond to scattering of a near on-mass shell parton on the proton, as opposed to interactions between target constituents in a bound state. This implies that the pdf's measure not only the target bound state wave function but also probe the parton–nucleon scattering amplitude. Such a microscopic interpretation of low-x_{B} [14] behavior of DIS was proposed long ago in [15, 16] and further explored in [17–19]. In these models the low-x_{B} limit of the structure functions is directly related to the Regge-like behavior of the parton–nucleon scattering amplitude which now as shown in [13] is consistent with expectations from leading-twist QCD.

In this paper we explore consequences of Regge parton–nucleon scattering for DVCS. In particular following [13] we allow for on-shell re-scattering of partons on target constituents in building the DVCS amplitude, *i.e.* for interactions which do not fall with the c.m parton–nucleon energy. A complete amplitude would also involve the bound state component but we do not study it here. The conclusion from [13] is that the QCD analysis is qualitatively similar to that of the pre-QCD models of [15–19]. In the modern language these models describe 0-th order, leading twist two-photon interactions while allowing for an arbitrary form of parton–nucleon interactions *i.e* they allow for both bound and scattering states. Our study builds upon these approaches.

1.2. Phenomenological implications

A distinguishing feature of the GPD mechanism is amplitude scaling in terms of Bjorken variables, *i.e.* at fixed momentum transfer and mass of the produced hadron (or photon) the hadronic part of electroproduction amplitudes [20] is predicted to be a simple function of $-q^2 = Q^2$, the photon virtuality, times a dynamical function of the ratio $x_{\rm B} \equiv Q^2/(2\nu)$ alone, where $\nu = p_a \cdot p_b$ is the energy of the virtual photon in the target rest frame. Furthermore, at fixed Bjorken $x_{\rm B}$ QCD makes specific predictions for the leading order large-Q^2 dependence. In Regge theory, in general the amplitude is expected to be a function of both Q^2 and ν.

Recent results on exclusive vector and pseudoscalar meson production from JLab [21] and from HERMES [22] do not provide conclusive evidence of the Q^2 scaling predicted by QCD factorization. In the case of meson production, the $\gamma^* p \to Mp$ cross-section is predicted to fall as $1/(Q^2)^n$ with $n = 3$, while JLab ω production data and HERMES π^+ data taken in a similar kinematic range give $n \sim 2$. Earlier data [23] on ρ_0 production might be consistent with QCD expectations, but these results appear to be softer than the $n = 3$ predicted by QCD scaling.

DVCS data from Hall A at Jefferson Laboratory [24] and HERMES [22] appear to be consistent with the Q^2-independent amplitude predicted by QCD factorization [25], however, the available Q^2 window is quite small, from 1.5–2.5 GeV^2 and within the published experimental errors one cannot rule out a power-like dependence of the amplitude, $A \propto (Q^2)^\alpha$, with α as large as 0.25. Perhaps even more surprising, "standard" Regge-exchange models have proved successful in describing a variety of differential cross sections [21,26], in the kinematical range where scaling would be expected based on comparisons with deep inelastic scattering (DIS). As we see it, a fundamental question is whether the success of the Regge picture is accidental. If not, this immediately raises the question of how one can disentangle scattering off the meson cloud from effects of nucleon tomography.

It is well known that Regge exchanges also contribute to DIS structure functions [27], but their contribution is restricted to very low $x_{\rm B} \sim 0$. Once it was realized that Regge exchange might play a significant role in exclusive electroproduction, attempts have been made to incorporate Regge effects using analogies with DIS, *i.e.* to restrict Regge contributions in exclusive electroproduction reactions to low-$x_{\rm B}$ so that scaling is not otherwise modified [28–31]. To the best of our knowledge it has not been proven that Regge contributions should only contribute to exclusive amplitudes in this domain, and in fact in Ref. [32] we provide arguments that, in a certain kinematic regime, Regge effects should be substantial even at large $x_{\rm B}$.

In this paper, we investigate further this issue. In Ref. [32] we analyzed hard exclusive reactions by examining the high-energy behavior of t-channel exchange processes. Here we will show that utilization of an s-channel framework, in which one analyzes the "handbag" diagrams that are used in extracting GPDs, leads to the same conclusions reached in Ref. [32], *i.e.* we show that in the region of high energy and small t, Regge effects should make sizeable contributions to hard exclusive amplitudes.

In this s-channel formalism we are able to explore the interplay between Regge behavior in the parton–nucleon amplitude and the hard interaction induced by the virtual photon. We will show that there are crucial differences between DIS and DVCS handbag diagrams which make Regge components of the soft parton–nucleon amplitude much more pronounced for DVCS than for DIS. We find that the difference between these processes arises when one attempts a collinear factorization of the quark propagators occuring in these processes. In the presence of Regge behavior in the parton–nucleon amplitude, the DVCS formalism is ill-defined. We then compute the handbag diagram using the full hard quark propagator. For hard exclusive processes, the divergent terms that are introduced due to the Regge behavior produce a non-analytic, non-scaling dependence on the photon virtuality.

This has the following effect on the hard exclusive amplitudes. First, the breakdown of factorization means that the soft amplitudes are not universal, but are process-dependent. Second, in the region of small t Regge effects will make substantial contributions to DVCS and exclusive meson production. Third, the Q^2 behavior of these hard exclusive processes should be different from that predicted from scaling arguments.

All these effects arise from the one reasonable, although admittedly unproven, hypothesis that the Regge behavior of the quark–nucleon amplitude parallels the Regge behavior of known hadron–hadron amplitudes, that is, that quarks couple to the mesons exchanged in the t-channel as any other hadron would. From this hypothesis we at least know that the standard Regge behavior of forward pdf's can be derived, and will do so in the following. One can then extend the discussion off-forward at no cost (since the formalism is covariant) and describe the entire handbag part of the DVCS amplitude, independently of any factorization (although, where this is applicable at sizeable t, the GPD and factorized DVCS amplitude can be obtained).

Our paper is organized as follows. In the following section we introduce the framework and consider the case of collinear factorization. We review both DIS and DVCS reactions, and we show that the DVCS formalism is ill-defined in the presence of Regge-like behavior in the parton–nucleon amplitude. In Section 3 we compute the handbag diagram with the full hard quark propagator and show how the divergent would-be collinear factorization forces a non-analytical, non-scaling dependence on the photon virtuality. We derive the Q^2 behavior for hard exclusive processes and show how it differs from scaling predictions, and how this Q^2 behavior is related to the leading Regge trajectories. We analyze existing DVCS and exclusive meson data, and show that their Q^2 behavior is, at least qualitatively, consistent with our predictions.

2. Collinear factorization in presence of Regge asymptotics

2.1. Compton amplitude and parton–nucleon amplitude

The hadronic tensor that describes electromagnetic transitions in the doubly virtual, diagonal Compton scattering $\gamma^* p \to \gamma^* p$ or off-diagonal $\gamma^* p \to \gamma p$ DVCS reactions, is given by

$$T^{\mu\nu} = i \int d^4z \, e^{i\frac{q'+q}{2}z} \left\langle p'\lambda' | T J^\mu \left(\frac{z}{2}\right) J^\nu \left(\frac{-z}{2}\right) | p\lambda \right\rangle . \tag{2}$$

In Eq. (2), q is the four momentum of the virtual photon, $q^2 < 0$, $q' = q + p - p' \equiv q - \Delta$, and $q'^2 = 0$ is the momentum of the real photon produced in DVCS. In the case of DIS, $q'^2 = q^2$ and $\Delta = 0$ and the DIS cross-section is proportional to the discontinuity of T across the cut in $(p+q)^2$. Even though we will explicitly consider only the kinematics relevant for either DIS or DVCS the analysis can easily be extended to the more general case of arbitrary time-like q' which is relevant, for example for meson electroproduction. The currents are given by $J^\mu(z) = \sum_q e_q J^\mu_q(z)$, $J^\mu_q(z) = \bar{\psi}(z)\gamma^\mu\psi(z)$ where ψ is the quark field operator and e_q is the quark charge. Throughout this paper we will consider a single quark flavor. For large Q^2 the z-integral peaks at $z^2 \sim 1/Q^2$ and using the leading order operator product expansion of QCD we replace the product of the two currents by a product of two quark field operators and a free propagator between the photon interaction points $(z/2, -z/2)$

$$
\begin{aligned}
T^{\mu\nu} &= -e_q^2 \int \frac{d^4 z d^4 k}{(2\pi)^4} \left[e^{-ikz} - e^{+ikz} (\mu \leftrightarrow \nu) \right] \\
&\times \langle p'\lambda' | T\overline{\psi}(z/2) \frac{\left[\gamma^\mu \left(\not{k} + \frac{\not{q} + \not{q}'}{2} \right) \gamma^\nu \right]}{\left(\frac{q+q'}{2} + k \right)^2 + i\epsilon} \psi(-z/2) | p\lambda \rangle \\
&\equiv -ie_q^2 \int \frac{d^4 k}{(2\pi)^4} \left\{ \frac{\left[\gamma^\mu \left(\not{k} + \frac{\not{q} + \not{q}'}{2} \right) \gamma^\nu \right]_{\alpha\beta}}{\left(\frac{q+q'}{2} + k \right)^2 + i\epsilon} - \frac{\left[\gamma^\nu \left(-\not{k} + \frac{\not{q} + \not{q}'}{2} \right) \gamma^\mu \right]_{\alpha\beta}}{\left(\frac{q+q'}{2} - k \right)^2 + i\epsilon} \right\} \\
&\times A_{\beta\alpha}(k, \Delta, p, \lambda, \lambda') \, . \qquad (3)
\end{aligned}
$$

Here A is the parton–nucleon scattering amplitude un-truncated, with respect to the parton legs [33],

$$
\begin{aligned}
A_{\beta\alpha} &\equiv A_{\beta\alpha}(k, \Delta, p, \lambda', \lambda) \, , \\
A_{\beta\alpha} &= -i \int d^4 z e^{-ikz} \langle p'\lambda' | T \bar{\psi}_\alpha(z/2) \psi_\beta(-z/2) | p\lambda \rangle \, . \qquad (4)
\end{aligned}
$$

As in Refs [15–19], we assume that despite its non-physicality, the analytical properties of the parton–nucleon amplitude display structures in the complex plane similar to conventional hadron scattering amplitudes. In the light cone gauge this assumption captures the physics of final state interactions between the antiquark and nucleon spectators which are responsible for the low-$x_{\rm B}$ behavior of the DIS cross-section [13]. This is necessary if such amplitudes are to be of any use at all, *i.e.* if they are to be connected to asymptotic properties of QCD[1].

The T-ordered product could then be replaced by a normal ordered product corresponding to generalized parton distributions [35]. For the purpose of our study it will be more efficient to deal directly with the T-ordered amplitudes. The parton–nucleon amplitude is a function of four variables and the nucleon helicities. The variables are $k_1^2 = (\Delta/2 - k)^2$, $k_2^2 = (-\Delta/2 - k)^2$ the (virtual) masses of the incoming and outgoing partons ($\Delta = p' - p = q - q'$), $s = (p + k_1)^2 = [(p + p')/2 - k]^2$ is the square of the center of mass energy in the s-channel, $u = (p' - k_1)^2 = [(p' + p)/2 + k]^2$ is the square of the center of

[1] Recently similar amplitudes (*e.g.* gluon and quark propagators in Landau gauge) were explored [34] and they were found to have usual threshold branch points associated with the production of colored quasiparticles (such as additional gluons or ghost pairs).

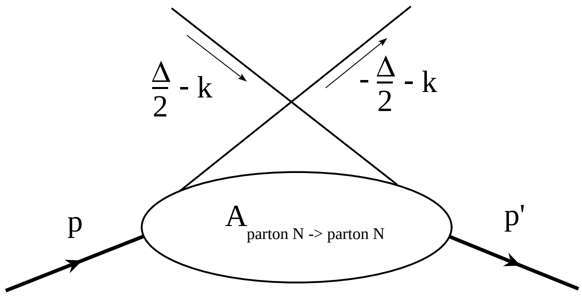

Fig. 1. Parton–nucleon scattering amplitude.

mass energy in the u-channel. Together with the four-momentum transfer, $t = (p' - p)^2 = \Delta^2$ they satisfy $s + t + u = k_1^2 + k_2^2 + 2M^2$ where M is the nucleon mass.

To obtain DIS scaling relations it is necessary to assume that the parton–nucleon amplitude has cuts for positive s and u. We will be interested primarily in the implications of the high-s or u behavior at low t where the amplitude is expected to be helicity conserving. Furthermore to reproduce the scaling limit of DIS and to preserve current conservation, the dependence on the parton spin (Dirac) indices must be of the form $A_{\beta\alpha} \propto \not{k}_{\beta\alpha}$, or $[\gamma_5 \not{k}]_{\beta\alpha}$ The former (latter) contributes respectively to the symmetric (antisymmetric) parts of the hadronic tensor $T^{\mu\nu}$. Finally for fixed-t we arrive at the general representation of the parton–nucleon amplitude in the form,

$$A = A^+ \frac{\not{k}_{\beta\alpha}}{4} \delta_{\lambda'\lambda} + A^- \frac{[\gamma_5 \not{k}]_{\beta\alpha}}{4} \tau^3_{\lambda'\lambda} \,, \tag{5}$$

with the factor of 1/4 introduced for later convenience and with the amplitudes $A^\pm$ having the Mandelstam representation,

$$A^\pm = (2\pi)^4 \int \frac{d\beta^2 dm^2}{(\beta^2 - k_1^2 - i\epsilon)(\beta^2 - k_2^2 - i\epsilon)} \left[\frac{\rho_s^\pm(\beta^2, m^2, t)}{m^2 - s - i\epsilon} \pm \frac{\rho_u^\pm(\beta^2, m^2, t)}{m^2 - u - i\epsilon} \right] + \text{subtractions} \,. \tag{6}$$

At asymptotically high energies, the quark and antiquark structure functions are becoming identical which implies that the s and u-channel spectral functions become identical, and so for large m^2 $\rho_u^\pm \sim \rho_s^\pm$. These amplitudes are in principle different in the valence (finite m^2) region. Even though we are primarily interested in the large-m^2 region we will distinguish between the s and u spectral functions in order to be able to keep track of quark and antiquark contributions. The dependence of the spectral density $\rho^\pm$ on β determines the dependence of the parton–nucleon scattering amplitude on parton virtualities. In perturbation theory [15] one would have

$\rho \propto \delta(\beta^2 - \mu^2)$ where μ is the bare quark mass. The free quark propagators are needed in order to represent final state interactions between one of the partons from the virtual photon and target spectators. We will return to this point in Sec. 2.5 where we discuss current conservation and positivity constraints. Before the real photon is emitted, partons hadronize, and thus the ultraviolet behavior of the loop integrals over the parton momenta is softened. This implies that at least the 0-th moment of ρ vanishes [36, 37]

$$\int d\beta^2 \rho^{\pm}_{u,s}(\beta^2) = 0\,. \tag{7}$$

The spin structure of A could be more complicated than given by the two terms in Eq. (5), for example there could be terms proportional to $\not{p}$, $\not{p}'$, or $\not{p}\gamma_5$ *etc.* [16]. As will be clear from the discussion that follows, however, it is the terms proportional to $\not{k}$ that lead to the Regge behavior of the structure functions and thus will be considered here. Without loss of generality we can take

$$\rho^{\pm}_{u,s}(\beta^2, m^2, t) \to \rho^{\pm}_{u.s}(m^2, t)(\mu^2)^n \frac{d^n}{d(\mu^2)^n} \delta(\beta^2 - \mu^2)\,, \tag{8}$$

with $n \geq 1$, where for simplicity we use a single scale μ for both partons (inclusion of charge symmetry breaking effects is an obvious generalization). The most general spectral density can always be written as a linear combination of functions of this type $\rho = \sum_n c_n \rho_n$. Henceforth, we will omit the subindex on ρ_n. As we have already discussed, for low m^2 this amplitude is expected to be sensitive to poles and cuts associated with low energy resonances and few-particle production thresholds. For large m^2 it is expected to be dominated by the leading Regge trajectory,

$$\rho^{\pm}_{u,s}(m^2, t) = \rho^{\pm}_{u,s,V}(m^2, t) + \rho^{\pm}_{u,s,\mathrm{R}}(m^2, t)\,. \tag{9}$$

For large m^2, the valence part $\rho^{\pm}_{u,s,V}(m^2)$ falls off with m^2 and does not require subtractions, on the contrary for large m^2 the Regge part, $\rho^{\pm}_{u,s,\mathrm{R}}(m^2)$ behaves as

$$\rho^{\pm}_{u,s,\mathrm{R}}(m^2, t) \to \beta^{\pm}_{u,s}(t) \left(\frac{m^2}{\mu^2}\right)^{\alpha^{\pm}_{u,s}(t)}\,, \tag{10}$$

where $0 < \alpha^{\pm}_{u,s}(t) < 1$ for small t and requires one subtraction in Eq. (6).

Here we consider only the quark contribution, as opposed to gluon exchanges which lead to diffractive, Pomeron-type contributions with $\alpha > 1$. These could also be effectively included but would require additional subtractions. As we are interested in the low-t limit, we have approximated the intercepts and residues by their values in the limit $t \to 0$.

In the following we will be interested in the role of the Regge (high energy) component and thus the parton–nucleon amplitude can be written,

$$A^{\pm}(s,u,k_1^2,k_2^2) = (2\pi)^4 \int dm^2 I_n \frac{1}{(\mu^2 - k_1^2 - i\epsilon)(\mu^2 - k_2^2 - i\epsilon)} \times \left\{ \left[\frac{\rho_s^{\pm}(m^2)}{m^2 - s - i\epsilon} - \frac{\rho_{s,\mathrm{R}}^{\pm}(m^2)}{m^2 - i\epsilon} \right] \pm (s \to u) \right\}, \quad (11)$$

where in Eq. (11), $I_n = (\mu^2)^n d^n / d(\mu^2)^n$. It should be noted that as long as s and u channel spectral functions are identical, subtractions are really only necessary for A^+ while they cancel in A^-. We are now in position to evaluate the two diagrams (direct and crossed) that contribute to the hadronic tensor W as shown in Fig. 2. For the symmetric part the leading contribution in the Bjorken limit is given by

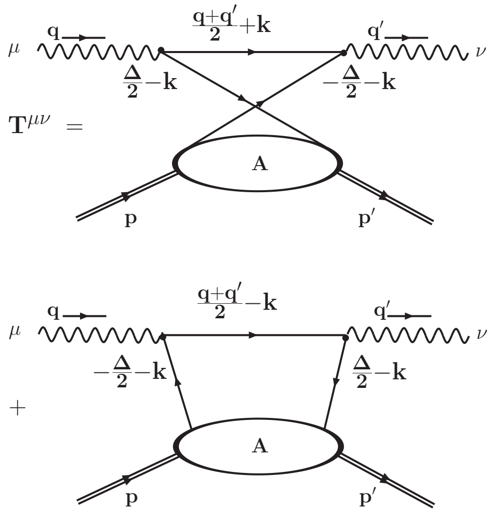

Fig. 2. u and s channel contributions to the DVCS amplitude.

$$
\begin{aligned}
T_{+}^{\mu\nu} &= i\delta_{\lambda'\lambda}\, e_q^2 \int \frac{d^4k dm^2}{im^2} \left[\frac{\rho_{\rm s}^{+}(m^2)\left(\frac{p+p'}{2}-k\right)^2}{\left[\left(\frac{p+p'}{2}-k\right)^2 - m^2 + i\epsilon\right]} + (s\to u, k\to -k)\right] \\
&\times I_n \left[\frac{1}{\left[(\frac{\Delta}{2}-k)^2-\mu^2+i\epsilon\right]\left[(\frac{\Delta}{2}+k)^2-\mu^2+i\epsilon\right]}\right] \\
&\times \left[\frac{\left(k+\frac{q+q'}{2}\right)^{\mu} k^{\nu} + (\mu\leftrightarrow\nu) - g^{\mu\nu}\left(k+\frac{q+q'}{2}\right)\cdot k}{\left(\frac{q+q'}{2}+k\right)^2 + i\epsilon} \right. \\
&\left. - \frac{\left(-k+\frac{q+q'}{2}\right)^{\mu} k^{\nu} + (\mu\leftrightarrow\nu) - g^{\mu\nu}\left(-k+\frac{q+q'}{2}\right)\cdot k}{\left(\frac{q+q'}{2}-k\right)^2 + i\epsilon} \right] . \qquad (12)
\end{aligned}
$$

Similarly the leading contribution to the antisymmetric part can be written,

$$
\begin{aligned}
T_{-}^{\mu\nu} &= i\tau^3_{\lambda'\lambda} e_q^2 \int \frac{d^4k dm^2}{m^2} \left[\frac{\rho_{\rm s}^{-}(m^2)\left(\frac{p+p'}{2}-k\right)^2}{\left(\frac{p+p'}{2}-k\right)^2 - m^2 + i\epsilon} - (s\to u, k\to -k)\right] \\
&\times I_n \left[\frac{-i\epsilon^{\mu\rho\nu\eta}}{\left[(\frac{\Delta}{2}-k)^2-\mu^2+i\epsilon\right]\left[(\frac{\Delta}{2}+k)^2-\mu^2+i\epsilon\right]}\right] \\
&\times \left[\frac{\left(k+\frac{q+q'}{2}\right)_{\rho} k_{\eta}}{\left(\frac{q+q'}{2}+k\right)^2 + i\epsilon} + \frac{\left(-k+\frac{q+q'}{2}\right)_{\rho} k_{\eta}}{\left(\frac{q+q'}{2}-k\right)^2 + i\epsilon} \right] . \qquad (13)
\end{aligned}
$$

To obtain an expression in terms of structure functions or generalized parton amplitudes, one applies a collinear factorization to the quark propagator in the last square bracket in Eqs (12) and (13). We will first consider the diagonal case, $q = q'$. In this case T is the analog of the hadronic amplitude for forward virtual Compton scattering, whose imaginary part is proportional to the DIS cross-section.

2.2. The DIS reaction $\gamma^* p \to \gamma^* p$

In this subsection we will show how our minimal assumption about the parton–nucleon amplitude, namely that it supports Regge behavior as any other hadron–hadron amplitude, automatically yields the standard Regge behavior of conventional pdf's, namely a $x^{-\alpha}$ divergence at low x. This is an encouraging feature noted in [15–19] among others, and we repeat the discussion here for completeness.

It is convenient to express all momenta in terms of light cone components, $a^\mu = (a^+, a^-, a_\perp)$ with $a^\pm = a^0 \pm a^z$, $a_\perp = (a^1, a^2)$ and to choose a frame in which, $p = p' = (P^+, M^2/P^+, 0_\perp)$, $q = q' = (0, Q^2/x_{\rm B}P^+, Q_\perp)$, with $-q^2 = -q'^2 = Q^2 = Q_\perp^2$. Since the nucleon mass M does not play a role in our discussion, for simplicity we will set it to zero. The hard quark propagators (the term in the last square bracket in Eqs (12,13)) become

$$\frac{1}{\left(\frac{q+q'}{2} \pm k\right)^2 + i\epsilon} \to \frac{x_{\rm B}}{Q^2} \frac{1}{(-x_{\rm B} \pm \frac{k^+}{P^+} + i\epsilon)} \,, \tag{14}$$

where following the collinear approximation $k \propto P$ we have ignored terms of order $|k_\perp|/\sqrt{Q^2}$. The leading contribution to the numerator comes from the terms that maximally involve the photon momentum; the term in the last square bracket in Eq. (12) can be written as

$$[\ldots] = t^{\mu\nu} \frac{(k^+/P^+)^2}{(k^+/P^+)^2 - {x_{\rm B}}^2 + i\epsilon} \,, \tag{15}$$

where we have introduced the vectors, $n^\mu = (0^+, 2, 0_\perp)$ $(n \cdot a = a^+)$ and $\widetilde{p}^\mu \equiv p^\mu/P^+$ and introduced $t^{\mu\nu} \equiv n^\mu \widetilde{p}^\nu + n^\nu \widetilde{p}^\mu - g^{\mu\nu}(n \cdot \widetilde{p})$. In the next step we combine all of the propagators using the Feynman parametrization, and we obtain

$$\begin{aligned} T_+^{\mu\nu} &= i\delta_{\lambda'\lambda} e_q^2 t^{\mu\nu} \int d^4k \int dm^2 \int_0^1 dx \left[\frac{(k^+/P^+)^2}{\left(\frac{k^+}{P^+}\right)^2 - {x_{\rm B}}^2 + i\epsilon} \right] \\ &\times \left[\rho_{\rm s}^+(m^2) I_n \left(\frac{2(1-x)}{[(k-xp)^2 - xm^2 - (1-x)\mu^2 + i\epsilon]^3} - \frac{1}{-m^2(k^2-\mu^2+i\epsilon)^2} \right) \right. \\ &\left. + (s \to u, k \to -k) \right] \,. \end{aligned} \tag{16}$$

Finally we perform the k^- and $k_\perp$ integrals using[2]

$$\int \frac{dk^- d^2k_\perp}{2i(k^2 + a^2 + i\epsilon)^\alpha} = \pi^2 \frac{\Gamma(\alpha-2)}{\Gamma(\alpha)} \frac{\delta(k^+)}{(a^2 + i\epsilon)^{\alpha-2}} \tag{17}$$

[2] For positive (negative) k^+, the singularities of the integrand in the complex k^- plane are all in the lower (upper) half-plane and the k^- integral vanishes. If $k^+ = 0$ the integrand is k^- independent and the k^- integration is divergent. Thus the result has to be proportional to $\delta(k^+)$. The coefficient can be determined by integrating over k^+ and comparing with the covariant result.

to obtain

$$T_{+}^{\mu\nu} = \delta_{\lambda'\lambda} e_q^2 t^{\mu\nu} \int_0^1 dx \, \frac{2x}{{x_{\rm B}}^2 - x^2 - i\epsilon} \Big[f_q(x) + \bar{f}_q(x) \Big] \, . \tag{18}$$

Here $f_q(x), \bar{f}_q(x)$ are the quark and antiquark structure functions, respectively, which are given by

$$\begin{aligned} f_q(x) &= f_V(x) + f_{\rm R}(x) \\ &= \frac{\pi^2}{2} \mu^2 \theta(1-x) \int dm^2 \rho_{\rm s}^{+}(m^2) I_{n-1} \frac{x(1-x)^2}{[xm^2 + (1-x)\mu^2]^2} \, , \\ \bar{f}_q(x) &= \bar{f}_{\rm R}(x) = \frac{\pi^2}{2} \mu^2 \theta(1-x) \int dm^2 \rho_{u}^{+}(m^2) I_{n-1} \frac{x(1-x)^2}{[xm^2 + (1-x)\mu^2]^2} \, . \end{aligned} \tag{19}$$

There is no "valence" contribution to the antiquark distribution. Increasing n produces more powers of $(1-x)$ that soften the propagator, form factors, and simultaneously the $x \to 1$, end-point behavior of the parton distribution functions (PDFs), as dictated by the Drell–Yan–West relation [38]. The valence part of the spectral function vanishes in the limit of large-m^2, which implies that the valence structure functions are proportional to x as $x \to 0$. The low-x behavior originating from the Regge part of the spectral function is given by

$$\begin{aligned} f_{\rm R}(x) &= (\mu^2)^{1-\alpha_{\rm s}^{+}} \frac{x\pi^2 \beta_{\rm s}^{+}}{2} I_{n-1} \int_0^\infty \frac{dm^2 (m^2)^{\alpha_{\rm s}^{+}}}{(xm^2 + \mu^2)^2} \\ &\to (\mu^2)^{1-\alpha_{\rm s}^{+}} I_{n-1} \frac{\pi^2 \beta_{\rm s}^{+}}{2(\mu^2)^{1-\alpha_{\rm s}^{+}}} \left[\frac{\pi \alpha_{\rm s}^{+}}{\sin \pi \alpha_{\rm s}^{+}} \frac{1}{x^{\alpha_{\rm s}^{+}}} \right] \equiv \frac{\gamma_{\alpha_{\rm s}^{+}}}{x^{\alpha_{\rm s}^{+}}} \end{aligned} \tag{20}$$

and for the antiquark distribution $\bar{f}_q(x)$ one needs to replace $s \to u$. As expected the small-x behavior of the structure function is determined by the leading high-energy behavior of the parton–nucleon amplitude.

A similar analysis for the antisymmetric part, $T_{-}^{\mu\nu}$, gives

$$T_{-}^{\mu\nu} = i e_q^2 \epsilon^{\mu\nu}{}_{03} \tau^3_{\lambda'\lambda} \int_0^1 \frac{2x_{\rm B}}{{x_{\rm B}}^2 - x^2 - i\epsilon} \left[\Delta f_q(x) + \Delta \bar{f}_q(x) \right] \, , \tag{21}$$

where

$$\begin{aligned}
\Delta f_q(x) &= \Delta f_{\rm V}(x) + \Delta f_{\rm R}(x) \\
&= \frac{\pi^2}{2}\mu^2\theta(1-x)\int dm^2 \rho_{\rm s}^-(m^2) I_{n-1} \frac{x(1-x)^2}{[xm^2+(1-x)\mu^2]^2}\,, \\
\Delta \bar{f}_q(x) &= \Delta \bar{f}_{\rm R}(x) \\
&= \frac{\pi^2}{2}\mu^2\theta(1-x)\int dm^2 \rho_{u}^-(m^2) I_{n-1} \frac{x(1-x)^2}{[xm^2+(1-x)\mu^2]^2}\,.
\end{aligned} \tag{22}$$

Since antiquarks are expected to dominate in the sea region, the valence part $\rho_{u,V}^-$ can be neglected in this region. The low-x behavior of the spin dependent structure functions is determined by the Regge part and is proportional to $1/x^{\alpha_u^-}$ or $1/x^{\alpha_{\rm s}^-}$ for $\Delta f_q(x)$ or $\Delta \bar{f}_q(x)$, respectively. We note that the subtraction terms do not contribute to the hadronic tensor. This is related to the small-x behavior of the structure functions, which are integrable over the low-x region since we assume $\alpha < 1$.

The hard propagators in the collinear approximation do not spoil the convergence of the integrals over low-x. It is important to realize, however, that this need not be the case in general. For example in the scalar model it was shown that the full $T^{\mu\nu}$ amplitude has a constant component (independent of Q^2 and $x_{\rm B}$), the so-called $J = 0$ pole contribution in the language of Regge phenomenology. This component originates from the seagull coupling of both photons to the quark at the same space-time point, as required by QED gauge invariance. This interaction alone leads to a divergent contribution of the form $\int_0 dx f_q(x)/x$ (as opposed to $\int_0 dx f_q(x)$ found above) which gets regulated as $x \to 0$ precisely by the subtraction term [17–19]. Thus in the scalar case the subtraction term is essential for producing a finite Compton amplitude. The $J = 0$ pole is a finite piece in the amplitude originating from subtraction of the leading Regge contribution. It is also expected to contribute in the spin-1/2 case. The reason why in Eq. (18) there is no trace of the Regge subtraction term is because in Eq. (16) after integrating over k^- the Regge subtraction term becomes proportional to $x^2\delta(x)/(x^2 - x_{\rm B}^2)$ and vanishes for $x_{\rm B} \neq 0$. However it is finite for $x_{\rm B} = 0$ and it is this value that provides a Q^2-independent contribution to T_+ at finite $x_{\rm B}$ that can be identified with the $J = 0$ pole [39].

From this discussion it should be clear that the convergence of the low-x integration may be a special rather than a general feature of these amplitudes. In Sec. 2.4 we show that convergence arises for DVCS in a different manner than for DIS.

2.3. Normalization

The structure functions $f_q(x)$ and $\bar{f}_q(x)$ represent probability densities for finding a quark or antiquark of a particular flavor q in the nucleon and as such need to be normalized to the net number of quarks of that flavor in the proton, (*e.g.* $n_q = (0,1,2)$ for s, d and u quarks in the proton, respectively)

$$\int\limits_0^1 dx[f_q(x) - \bar{f}_q(x)] = n_q \,. \tag{23}$$

Below we verify that this is consistent with the normalization of the vector current which is also sensitive to quark densities. The normalization of the diagonal matrix element of the electromagnetic current, $J^+(0) = e_q\bar{\psi}(0)\gamma^+\psi(0)$, is given by

$$\langle p\lambda'|e_q J_q^+(0)|p\lambda\rangle = 2P^+\delta_{\lambda'\lambda}e_q F^q \,. \tag{24}$$

The factor of 2 on the r.h.s of Eq. (24) comes from the relativistic normalization of states and F^q is the contribution to the proton charge from the particular quark flavor. In terms of the parton–nucleon amplitudes defined in Eq. (4), the vector current matrix element is given by

$$\begin{aligned}
\langle p\lambda'|e_q J_q^+(0)|p\lambda\rangle &= -e_q\int\frac{d^4k}{i(2\pi)^4}\,\mathrm{Tr}[\gamma^+ A] = -e_q\int\frac{d^4k}{i(2\pi)^4}\,\mathrm{Tr}\left[\gamma^+ A^+\frac{\not{k}}{4}\right] \\
&= -ie_q\delta_{\lambda'\lambda}\int\frac{d^4k dm^2}{m^2}k^+ I_n\frac{1}{(k^2-\mu^2+i\epsilon)^2} \\
&\quad\times\left[\frac{\rho_{\mathrm{s}}^+(m^2)\,(p-k)^2}{(p-k)^2-m^2+i\epsilon} - (s\to u, k\to -k)\right] \\
&= 2P^+\delta_{\lambda'\lambda}e_q\int\limits_0^1 dx[f_q(x)-\bar{f}_q(x)]\,.
\end{aligned} \tag{25}$$

Thus as expected the quark and antiquark structure functions contribute with opposite signs. We also note that the subtraction terms do not contribute, since for these terms the integrand is antisymmetric in k^+. The normalization of the spin dependent structure functions is related to the axial current matrix element $J_5^+(0) = \bar{\psi}(0)\gamma^+\gamma_5\psi(0)$

$$\langle p\lambda'|J_{5q}^+(0)|p\lambda\rangle = 2P^+ g_A^q \tau^3_{\lambda'\lambda}\,. \tag{26}$$

In Eq. (26) g_A^q denotes the contribution from a single quark flavor to the nucleon axial charge, and in terms of the spin-dependent structure functions should be given by

$$g_A^q = \int\limits_0^1 dx \left[\Delta f_q(x) + \Delta \bar{f}_q(x)\right] . \tag{27}$$

Indeed, expressing the axial current matrix element in terms of the parton–nucleon amplitude we obtain

$$\begin{aligned}
\langle p\lambda'|J^+_{5q}(0)|p\lambda\rangle &= -\int \frac{d^4k}{i(2\pi)^4}\,\mathrm{Tr}\left[\gamma^+\gamma_5 A\right] = -\int \frac{d^4k}{i(2\pi)^4}\,\mathrm{Tr}\left[\gamma^+ A^- \frac{\not{k}}{4}\right] \\
&= -i\tau^3_{\lambda'\lambda}\int \frac{d^4k dm^2}{m^2} k^+ I_n \frac{1}{(k^2-\mu^2+i\epsilon)^2} \\
&\times \left[\frac{\rho_{\mathrm{s}}^-(m^2)\,(p-k)^2}{(p-k)^2 - m^2 + i\epsilon} - (s\to u, k\to -k)\right] \\
&= 2P^+\tau^3_{\lambda'\lambda}\int\limits_0^1 dx \left[\Delta f_q(x) + \Delta \bar{f}_q(x)\right] .
\end{aligned} \tag{28}$$

In the following section we will consider the collinear approximation for the DVCS amplitude.

2.4. The DVCS reaction $\gamma^* p \to \gamma p$

In this subsection we attempt to write a collinearly factorized formula for off-forward DVCS. We will show that in presence of Regge behavior, s^α in the parton–nucleon amplitude that for positive α (small t) grows with parton–nucleon c.m. energy, the collinear approximation introduces a singularity in the longitudinal momentum integral of the handbag diagram for DVCS. This is because the Regge behavior of the parton–nucleon amplitude results in a GPD that is singular at the break points. We note that it is the same parton–nucleon amplitude which in the case when the virtualities of both photons are large does lead to scaling.

When $\Delta \neq 0$ it is convenient to choose a frame with the following momentum coordinates [40] (where again we ignore the nucleon mass) $p = [P^+, 0, 0_\perp], p' = [(1-\zeta)P^+, \Delta_\perp^2/(1-\zeta)P^+, \Delta_\perp]$, $q = [0, (Q_\perp - \Delta_\perp)^2/\zeta P^+ + \Delta_\perp^2/(1-\zeta)P^+, Q_\perp]$, $q' = [\zeta P^+, (Q_\perp - \Delta_\perp)/\zeta P^+, Q_\perp - \Delta_\perp]$. In the Bjorken limit, at small momentum transfer, $\zeta = x_{\mathrm{B}} + O(-t/Q^2)$ and $-t = \Delta_\perp^2/(1-\zeta)$. Since we are interested in the small-t region we also set $\Delta_\perp = 0$ which also implies $\Delta^2 = 0$ ($\Delta \to [-\zeta P^+, 0, 0_\perp]$). To facilitate comparison with standard formulas it is convenient to shift the integration variable in Eqs (12), (13) from k to $\tilde{k} \equiv k + \Delta/2$. In the collinear approximation, the hard propagators become

$$\frac{1}{\left(\frac{q+q'}{2}+\widetilde{k}-\frac{\Delta}{2}\right)^2} \pm \frac{1}{\left(\frac{q+q'}{2}-\widetilde{k}+\frac{\Delta}{2}\right)^2} = \frac{1}{(q'+\widetilde{k})^2+i\varepsilon} \pm \frac{1}{(q-\widetilde{k})^2+i\varepsilon}$$
$$= \frac{x_{\rm B}}{Q^2}\left[\frac{1}{\frac{\widetilde{k}^+}{P^+}+i\epsilon} \pm \frac{1}{-x_{\rm B}-\frac{\widetilde{k}^+}{P^+}+i\epsilon}\right]. \quad (29)$$

Next we combine the two soft propagators,

$$\frac{1}{\left[(\Delta-\widetilde{k})^2-\mu^2\right]\left[\widetilde{k}^2-\mu^2\right]} = \int_0^1 dr \frac{1}{\left[(\widetilde{k}-r\Delta)^2-\mu^2+i\epsilon\right]^2} \quad (30)$$

and for $T_+^{\mu\nu}$ we obtain,

$$\begin{aligned} T_+^{\mu\nu} &= i\delta_{\lambda'\lambda}e_q^2\frac{1}{2}t^{\mu\nu}\int d^4\widetilde{k}dm^2\int_0^1 drdx\left[\frac{\left(\frac{\widetilde{k}^+}{P^+}-\frac{\Delta^+}{2P^+}\right)}{\frac{\widetilde{k}^+}{P^+}+i\epsilon} - \frac{\left(\frac{\widetilde{k}^+}{P^+}-\frac{\Delta^+}{2P^+}\right)}{-x_{\rm B}-\frac{\widetilde{k}^+}{P^+}+i\epsilon}\right] \\ &\times\left[\rho_{\rm s}^+(m^2)I_n\left(\frac{2(1-x)}{[(\widetilde{k}-xp'-(1-x)r\Delta)^2-xm^2-(1-x)\mu^2+i\epsilon]^3}\right.\right. \\ &\left.-\frac{1}{-m^2[(\widetilde{k}-r\Delta)^2-\mu^2+i\epsilon]^2}\right) \\ &+\rho_u^+(m^2)I_n\left(\frac{2(1-x)}{[(\widetilde{k}+xp-(1-x)r\Delta)^2-xm^2-(1-x)\mu^2+i\epsilon]^3}\right. \\ &\left.\left.-\frac{1}{-m^2[(\widetilde{k}-r\Delta)^2-\mu^2+i\epsilon]^2}\right)\right] \end{aligned} \quad (31)$$

and after integrating over $\widetilde{k}^-$ and $\widetilde{k}_\perp$, we obtain a formal relation that is reminiscent of the standard leading-twist DVCS formula in terms of GPD's.

$$T_+^{\mu\nu} = -e_q^2\delta_{\lambda'\lambda}t^{\mu\nu}\int_0^1 dx\, H^+(x,x_{\rm B})\left(\frac{1}{x-i\epsilon}+\frac{1}{x-x_{\rm B}+i\epsilon}\right). \quad (32)$$

The hadronic tensors, spectral functions and generalized parton distributions here all represent the contribution from a single quark flavor; we have not included the quark flavor indices but they are implicit. As will be discussed shortly, this expression for DVCS fails to be convergent in the presence of Regge behavior. In Eq. (32) the generalized parton distribution is given by

$$H^+(x,x_{\rm B}) = \Big(x-\frac{x_{\rm B}}{2}\Big)\int_0^1 dr\int_0^1 dy$$
$$\times\left\{\delta\left(x-y-(1-y)rx_{\rm B}\right)\left[\frac{f_q(y)+\bar f_q(y)}{y}\right]-\delta(x-rx_{\rm B})\left[\frac{f_0(y)+\bar f_0(y)}{y}\right]\right\},\quad(33)$$

with f_0 and $\bar f_0$ given by Eq. (19) without $(1-x)^2$ in the numerator. Just as in the case of DIS analyzed in the previous section the contribution given by the quark (antiquark) distribution $f_q(x)$ $(\bar f_q(x))$ comes from the s-channel (u-channel) spectral function respectively. The δ-functions which arise after $\widetilde k^-$ integration fix $\widetilde k^+/P^+$ in terms of the Feynman parameter-x, and lead to both positive and negative $\widetilde k^+/P^+$. We immediately note that the first term under the double integral is divergent for Regge-behaved pdf's when $y\to 0$, independent of the values of x and $x_{\rm B}$. This singularity is precisely canceled by the last term in Eq. (33), which originates from the subtractions in the parton–nucleon amplitude needed for the parton–nucleon amplitude with Regge asymptotic behavior. The generalized parton distribution H^+ is thus perfectly well defined, however, as we will discuss below it leads to a singular expression for the DVCS amplitude. The GPD appearing in Eq. (32) is the C-*even* generalized parton distribution [6]. The factor $(x-x_{\rm B}/2)$ in Eq. (33) makes the expression vanish at $x=x_{\rm B}/2$, this is, however, not necessarily true for the total GPD, but only for this portion carrying the Regge behavior, since the factor can be traced back to $\not k$ in Eq. (5).

It can easily be checked that H^+ satisfies the correct normalization conditions

$$\int_0^1 dx\frac{H^+(x,x_{\rm B})}{1-x_{\rm B}/2}=\int_0^1 dx[f_q(x)+\bar f_q(x)] \quad (34)$$

and

$$H^+(x,0)=f_q(x)+\bar f_q(x)\,. \quad (35)$$

Finally changing variables in Eq. (33) to (α,β) where $\alpha=2(1-y)r-1+y$, $\beta=y$, that equation can be represented in terms of a double distribution [3,8,41,42]:

$$H^+_{\rm s}(x_{\rm s},\eta) \;=\; \int_0^1 d\beta\int_{-1+\beta}^{1-\beta} d\alpha\delta(\beta+\eta\alpha-x_{\rm s})\left[f(\beta)+\eta g(\beta,\alpha)\right]\,, \quad (36)$$

$$\text{with} \quad f(\beta) = \frac{f_q(\beta) + \bar{f}_q(\beta)}{2(1-\beta)},$$

$$g(\beta,\alpha) = \frac{\alpha}{2(1-\beta)} \left[\frac{f_q(\beta)+\bar{f}_q(\beta)}{\beta} - \delta(\beta) \int_0^1 d\beta' \frac{f_0(\beta')+\bar{f}_0(\beta')}{\beta'} \right]. \quad (37)$$

Here we used the symmetric variables of [2] $x_{\rm s} \equiv (x - x_{\rm B}/2)/(1 - x_{\rm B}/2)$, $\eta \equiv x_{\rm B}/2/(1 - x_{\rm B}/2)$ and $H^+_{\rm s}(x_{\rm s}, \eta) \equiv H^+(x, x_{\rm B})$.

Even though H^+ is well defined, the DVCS amplitude in Eq. (32) is not. This is easily seen by first simplifying the expression for H^+ in Eq. (33); the y integral is done using the δ function and the variable r is changed to z where $z = (x - r x_{\rm B})/(1 - r x_{\rm B})$. This leads to

$$H^+(x, x_{\rm B}) = \frac{x - \frac{x_{\rm B}}{2}}{x_{\rm B}} \left\{ \theta(x - x_{\rm B}) \int_{\frac{x - x_{\rm B}}{1 - x_{\rm B}}}^{x} dz \frac{f_q(z) + \bar{f}_q(z)}{z(1-z)} \right.$$

$$\left. + \; \theta(x_{\rm B} - x) \left[\int_0^x dz \frac{f_q(z)+\bar{f}_q(z)}{z(1-z)} - \int_0^1 dz \frac{f_0(z)+\bar{f}_0(z)}{z} \right] \right\}. \quad (38)$$

Since for $z \to 0$, $f_q(z)(\bar{f}_q(z)) \to f_0(z)(\bar{f}_0(z))$ the singularities at $z \to 0$ cancel between the last two integrals. For finite $x_{\rm B}$ and $x \to 0$ H^+ is determined by the $\theta(x_{\rm B} - x)$ term in Eq. (38). Since the low-z Regge behavior of the quark and antiquark structure functions is $f_{q(0)}(z) \sim 1/z^{\alpha_{\rm s}^+}$ and $\bar{f}_{q(0)}(z) \sim 1/z^{\alpha_u^+}$, in the limit $x \to 0$ the last two integrals in Eq. (38) result in H^+ of the general form

$$H^+(x \sim 0) \sim -\frac{1}{2\alpha} \frac{1}{x^\alpha}. \quad (39)$$

The integral over the first hard propagator in Eq. (32) thus gives a contribution to the DVCS amplitude

$$\int_0 dx \frac{H^+(x, x_{\rm B})}{x - i\epsilon} \sim O\left(\frac{1}{\epsilon^\alpha}\right), \quad (40)$$

which is divergent for $0 < \alpha < 1$. Similarly, as $x \to {x_{\rm B}}^+$ the first term in Eq. (38) for H^+, proportional to $\theta(x - x_{\rm B})$, by virtue of the Regge form for $f_q(z)$ and/or $\bar{f}_q(z) \propto 1/z^\alpha$, is dominated by the lower limit of the integral over z, leading to

$$H^+(x \sim {x_{\rm B}}^+) \sim \frac{1}{2\alpha} \frac{(1 - x_{\rm B})^\alpha}{(x - x_{\rm B})^\alpha}. \quad (41)$$

Using the form of H^+ from Eq. (41) in Eq. (32), and integrating over the second hard propagator, $(1/[x - x_{\rm B} + i\epsilon])$, it gives a contribution to the DVCS amplitude of the form

$$\int\limits_{x_{\rm B}} dx \, \frac{H^+(x, x_{\rm B})}{x - x_{\rm B} + i\epsilon} \sim (1 - x_{\rm B})^\alpha O\left(\frac{1}{\epsilon^\alpha}\right) . \tag{42}$$

Even though these singular terms contribute to $T_+^{\mu\nu}$ with opposite signs, they do not cancel because of the extra factor $(1 - x_{\rm B})^\alpha$. These residual singularities must originate from the collinear approximation since after Regge subtraction there is no reason to expect that the expression for $T_+^{\mu\nu}$ in Eq. (12) will be singular. In other words, to properly regularize those singularities it will be necessary to retain the full momentum dependence of the hard propagators.

We note that the problem arises from the Regge contribution to the soft part of the handbag diagram. The valence spectral functions do not require subtraction, thus their contributions to $T_+^{\mu\nu}$ do not have the singularity associated with the $(f_0(x) + \bar{f}_0(x))/x$ term in Eq. (33). Furthermore valence structure functions vanish at small-x. As a result, the valence contributions vanish in the regions $H^+(x \sim 0)$ and $H^+(x \sim {x_{\rm B}}^+)$, so no singularities appear of the type given in Eqs (40) and (42).

A similar analysis of the antisymmetric contribution yields,

$$\begin{aligned}
T_-^{\mu\nu} &= -e_q^2 \epsilon^{\mu\nu}{}_{03} \tau^3_{\lambda'\lambda} \frac{1}{2} \int d^4\widetilde{k}\, dm^2 \int\limits_0^1 dr dx \left[\frac{\left(\frac{\widetilde{k}^+}{P^+} - \frac{\Delta^+}{2P^+}\right)}{\frac{\widetilde{k}^+}{P^+} + i\epsilon} + \frac{\left(\frac{\widetilde{k}^+}{P^+} - \frac{\Delta^+}{2P^+}\right)}{-x_{\rm B} - \frac{\widetilde{k}^+}{P^+} + i\epsilon} \right] \\
&\times \left[\rho_{\rm s}^+(m^2) I_n \left(\frac{2(1-x)}{[(\widetilde{k} - xp' - (1-x)r\Delta)^2 - xm^2 - (1-x)\mu^2 + i\epsilon]^3} \right. \right. \\
&- \left. \frac{1}{-m^2 \left[(\widetilde{k} - r\Delta)^2 - \mu^2 + i\epsilon\right]^2} \right) \\
&- \rho_u^+(m^2) I_n \left(\frac{2(1-x)}{[(\widetilde{k} + xp - (1-x)r\Delta)^2 - xm^2 - (1-x)\mu^2 + i\epsilon]^3} \right. \\
&- \left. \left. \frac{1}{-m^2 \left[(\widetilde{k} - r\Delta)^2 - \mu^2 + i\epsilon\right]^2} \right) \right] ,
\end{aligned} \tag{43}$$

$$T_-^{\mu\nu} = e_q^2 \epsilon^{\mu\nu}{}_{03} \tau^3_{\lambda'\lambda} \int\limits_0^1 dx \widetilde{H}^+(x, x_{\rm B}) \left(\frac{1}{x - i\epsilon} - \frac{1}{x - x_{\rm B} + i\epsilon} \right) . \tag{44}$$

The $\widetilde{H}^+$ parton distribution appearing in Eq. (44) is given by the same equation as H^+ from Eq. (38) with the replacements $f_q \to \Delta f_q$ and $\bar{f}_q \to \Delta\bar{f}_q$, thus there are the same left-over singularities in the DVCS amplitude as in the case of $T_+^{\mu\nu}$. These originate from the $1/x^{\alpha_{\bar{u},s}}$ behavior of $\widetilde{H}^+$ near $x \sim 0$ and the $1/(x - x_{\rm B})^{\alpha_{\bar{u},s}}$ behavior for x near $x_{\rm B}$.

The origin of the break point singularity has a simple explanation. Consider first the symmetric DIS reaction discussed in Sec. 2.2. The invariant parton–nucleon mass is proportional to the light-cone k^- energy of the parton which, for a quasi-on-shell particle, is in turn proportional to $1/k^+ \propto 1/x$. Thus the parton–nucleon amplitude becomes large when $x \to 0$. In the case of DVCS, however the parton light-cone energies are proportional to either $1/x$ or $1/(x - x_{\rm B})$ thus the amplitude becomes large at finite $x_{\rm B}$ as long as $x \to 0$ or $x \to x_{\rm B}$.

2.5. Current conservation and the parton–nucleon amplitude

Electromagnetic current conservation imposes constraints on the coupling of the photon to the electric charge of the quarks. To ensure current conservation the quark propagating between the two photons has to be treated in the same way as the antiquark emitted by the virtual photon that scatters off nucleon spectators. This justifies the use of the free quark propagators in the parton–nucleon amplitude. More specifically, in place of A given by Eqs (5),(6) we can consider the following representation for Eq. (4) which has the same spin-1/2 propagators as the hard one

$$A'_u = \frac{(2\pi)^4}{4}\delta_{\lambda'\lambda}\int d\beta\, dm^2 \left[\frac{1}{\not{k}_1 - \beta + i\epsilon}\ \Gamma_u \frac{1}{\not{k}_2 - \beta + i\epsilon}\right]_{\beta\alpha}. \tag{45}$$

Here $\Gamma_u = \Gamma_u[\beta, (k + (p+p')/2)^2, m^2, t] = \sum_i \Gamma^i_{\alpha\beta} F$ with Γ_i representing combinations of Dirac matrices and 4-vectors, $k, (p+p'), \Delta$ and F are scalar form factors. For simplicity we consider only the u-channel contribution to the symmetric hadronic tensor (the treatment of the s-channel and spin-flip contributions is analogous)

$$\begin{aligned} T_+^{\mu\nu} &= i\delta_{\lambda'\lambda} e_q^2 \int d^4k\, d\beta\, dm^2 \frac{1}{4}\,\mathrm{Tr}\left\{\Gamma_u\left[\beta, \left(k + \frac{p+p'}{2}\right)^2, m^2, t\right]\right. \\ &\times \left[\frac{1}{-\not{k}+\not{\Delta}/2-\beta+i\epsilon}\gamma^\mu \frac{1}{-\not{k}-(\not{q}+\not{q}')/2-\beta+i\epsilon}\gamma^\nu \frac{1}{-\not{k}-\not{\Delta}/2-\beta+i\epsilon}\right. \\ &\left.\left. + \frac{1}{-\not{k}+\not{\Delta}/2-\beta+i\epsilon}\gamma^\nu \frac{1}{-\not{k}+(\not{q}+\not{q}')/2-\beta+i\epsilon}\gamma^\mu \frac{1}{-\not{k}-\not{\Delta}/2-\beta+i\epsilon}\right]\right\}, \end{aligned} \tag{46}$$

where we included the (soft) quark mass in the hard propagator. Contractions with q_μ vanishes (and similarly contraction with q'_ν) as expected

$$
\begin{aligned}
q_\mu T_+^{\mu\nu} &= i\delta_{\lambda'\lambda} e_q^2 \int \frac{d^4k}{i} d\beta dm^2 \frac{1}{4} \operatorname{Tr} \left\{ \Gamma_u \left[\beta, \left(k + \frac{p+p'}{2} \right)^2, m^2, t \right] \right. \\
&\times \left[\frac{1}{-\not{k} - (\not{q} + \not{q}')/2 - \beta + i\epsilon} \gamma^\nu \frac{1}{-\not{k} - \not{\Delta}/2 - \beta + i\epsilon} \right. \\
&\left.\left. - \frac{1}{-\not{k} + \not{\Delta}/2 - \beta + i\epsilon} \gamma^\nu \frac{1}{-\not{k} + (\not{q} + \not{q}')/2 - \beta + i\epsilon} \right] \right\} \qquad (47) \\
&= e_q \Gamma^\nu(p', p' + q') - e_q \Gamma^\nu(p - q', p) , \qquad (48)
\end{aligned}
$$

where

$$
\begin{aligned}
\Gamma^\nu(p-q,p) &= i\delta_{\lambda'\lambda} e_q \int d^4k d\beta dm^2 \\
&\times \frac{1}{4} \operatorname{Tr} \left[\frac{1}{-\not{k} - \not{q} - \beta + i\epsilon} \gamma^\nu \frac{1}{-\not{k} - \beta + i\epsilon} \Gamma_u(\beta, (k+p)^2, m^2, t) \right] . \qquad (49)
\end{aligned}
$$

Finally the relation $q_\mu T_+^{\mu\nu} = 0$ is obtained after Born diagrams with nucleon exchange are added [19].

The amplitudes derived in Secs 2.2–2.4 are obtained by choosing Γ_u proportional to the identity in the Dirac space. It is this choice that leads to an appropriate behavior of the structure functions at low-x_{B}. For example, should Γ_u be proportional to the single Dirac γ matrix (*e.g.* via $\not{k}_1$ or $\not{k}_2$) an additional power of x would result in in the numerator of Eq. (18) resulting in $f_q(x) \propto 1/x^{\alpha-1}$ $(0 < \alpha < 1)$ at low-x. Such a spin dependence of the parton–nucleon amplitude has also been been used elsewhere [36,43]. Current conservation could not be achieved if the quark that scatters off the nucleon and the quark being exchanged between the photons were not described by the same quark propagator. This is typically the case where the quark which interacts with the nucleon forms part of the nucleon bound state and is described by a soft amplitude, as discussed for example in [44], however, if the quasi-free scattering of the parton on the nucleon contributes to the structure functions as discussed in [13] such final state interactions cannot be accounted for by the nucleon wave function alone. Finally we note that Regge behavior of GPD's derived here violates the constraint on the upper bound of the GPD [45] (and references therein). The GPD being constrained from above by the structure function follows from the assumed finite norm of the states obtained by removing a quark from the target nucleon. After the virtual photon is absorbed, final state rescattering in the target can influence the cross-section and proceeds via on-shell intermediate states that have infinite Hilbert norm. Such contributions are regularized at the level of the full DVCS amplitude and not by the nucleon state alone. Therefore, it is not surprising that the constraints on the upper limit of GPD's are not satisfied in the presence of on-shell scattering in the final state.

3. DVCS amplitude without collinear approximation

In the previous section we noticed that the *C-even* part of the DVCS amplitude is singular when evaluated in collinear approximation and expressed in terms of the H^+ or $\tilde{H}^+$ GPD's, provided that the parton–nucleon amplitude has a high energy behavior typical of hadronic amplitudes, commensurate with the Regge type scaling behavior of the form s^α with $0 < \alpha < 1$ (we also showed that this problem does not arise for the structure functions).

However, the Compton amplitude we started with was perfectly finite. From the discussion above it is also clear that the singularity in the DVCS amplitude appears with the collinear approximation to the denominators of the hard quark propagator exchanged between photon interaction points. Thus in the following we use the collinear approximation only for the numerators and keep the full k-dependence of the denominators while performing the d^4k integral in Eqs (12) and (13). Then the Regge part of the spectral function, that is now finite and dominant at low t in the DVCS amplitude $T_+^{\mu\nu}$ gives:

$$
\begin{aligned}
T_+^{\mu\nu} &= -\delta_{\lambda'\lambda} e_q^2 \frac{1}{2} t^{\mu\nu} \frac{Q^2}{x_{\mathrm{B}}} \int_0^\infty d\xi \int_0^1 \frac{dx}{x} \int_0^1 dr \int_0^1 dz 2\pi^2 (1-x)(1-z)^2 \\
&\times \Bigg\{ \frac{\mu^2 \beta_{\mathrm{s}}^+}{(x\mu^2)^{\alpha_{\mathrm{s}}^+}} I_{n-1} \\
&\times \Bigg[\left(\frac{\xi^{\alpha_{\mathrm{s}}^+} (1-x)^2 [x(1-x_{\mathrm{B}}) + \frac{x_{\mathrm{B}}}{2} - (1-x)(1-z) r x_{\mathrm{B}} - (1-x) z x_{\mathrm{B}}]}{[\xi + (1-x)(1-z)\mu^2 - (2p\cdot q + q^2) x z (1-x) - z(1-z)(1-x)^2 r q^2 - i\epsilon]^3} - (x=0) \right) \\
&- \left(\frac{\xi^{\alpha_{\mathrm{s}}^+} (1-x)^2 (x(1-x_{\mathrm{B}}) + \frac{x_{\mathrm{B}}}{2} - (1-x)(1-z) r x_{\mathrm{B}})}{[\xi + (1-x)(1-z)\mu^2 + (2p\cdot q) x z (1-x) - z(1-z)(1-x)^2 (1-r) q^2 - i\epsilon]^3} - (x=0) \right) \Bigg] \\
&+ \frac{\mu^2 \beta_u^+}{(x\mu^2)^{\alpha_u^+}} I_{n-1} \\
&\times \Bigg[\left(\frac{\xi^{\alpha_u^+} (1-x)^2 (-x + \frac{x_{\mathrm{B}}}{2} - (1-x)(1-z) r x_{\mathrm{B}} - (1-x) z x_{\mathrm{B}})}{[\xi + (1-x)(1-z)\mu^2 + (2p\cdot q) x z (1-x) - z(1-z)(1-x)^2 r q^2 - i\epsilon]^3} - (x=0) \right) \\
&- \Bigg(\frac{\xi^{\alpha_u^+} (1-x)^2 (-x + \frac{x_{\mathrm{B}}}{2} - (1-x)(1-z) r x_{\mathrm{B}})}{[\xi + (1-x)(1-z)\mu^2 - (2p\cdot q + q^2) x z (1-x) - z(1-z)(1-x)^2 (1-r) q^2 - i\epsilon]^3} \\
&- (x=0) \Bigg) \Bigg] \Bigg\} . \qquad (50)
\end{aligned}
$$

Here following Ref. [19] we changed the m^2 variable to ξ, with $m^2 \to \xi/x$. The large m^2 contribution to the integral corresponds to small-x thus we ignore x in all terms of the form $(1-x)$, and terms proportional to x in the numerator, and we extend the x integral to infinity. We change the x variable so as to bring each denominator to the same form as in the subtraction terms (those with $(x = 0)$). In particular for the term written explicitly in the third line of Eq. (50) we replace $x \to x'$ given by,

$$x = x' \frac{\xi + (1-z)\mu^2 - z(1-z)rq^2}{z(2p\cdot q + q^2)}\,, \tag{51}$$

in the fourth line,

$$x = x' \frac{\xi + (1-z)\mu^2 - z(1-z)(1-r)q^2}{2p\cdot qz}\,, \tag{52}$$

in the sixth line

$$x = x' \frac{\xi + (1-z)\mu^2 - z(1-z)rq^2}{2p\cdot qz}\,, \tag{53}$$

and in the seventh line

$$x = x' \frac{\xi + (1-z)\mu^2 - z(1-z)(1-r)q^2}{z(2p\cdot q + q^2)}\,. \tag{54}$$

We note that since $q^2 < 0$ and $2p\cdot q + q^2 > 0$ these transformations are non-singular. After this change of variables we obtain,

$$\begin{aligned}
T_+^{\mu\nu} &= -\delta_{\lambda'\lambda} e_q^2 \frac{1}{2} t^{\mu\nu} Q^2 \int_0^\infty d\xi \int_0^\infty \frac{dx'}{x'} \int_0^1 dr \int_0^1 dz 2\pi^2 (1-z)^2 \\
&\times \left\{ \frac{\mu^2 \beta_{\mathrm{s}}^+}{(x'\mu^2)^{\alpha_{\mathrm{s}}^+}} I_{n-1} \right. \\
&\times \left[\left[(2p\cdot q + q^2)z\right]^{\alpha_{\mathrm{s}}^+} \frac{\xi^{\alpha_{\mathrm{s}}^+} \left[\frac{1}{2} - (1-z)r - z\right]}{[\xi + (1-z)\mu^2 + z(1-z)rQ^2]^{3+\alpha_{\mathrm{s}}^+}} \left(\frac{1}{(1-x'-i\epsilon)^3} - 1 \right) \right. \\
&- \left. \left[(2p\cdot q)z\right]^{\alpha_{\mathrm{s}}^+} \frac{\xi^{\alpha_{\mathrm{s}}^+} \left[\frac{1}{2} - (1-z)r\right]}{[\xi + (1-z)\mu^2 + z(1-z)(1-r)Q^2]^{3+\alpha_{\mathrm{s}}^+}} \left(\frac{1}{(1+x'-i\epsilon)^3} - 1 \right) \right]
\end{aligned}$$

$$+\frac{\mu^2\beta_u^+}{(x'\mu^2)^{\alpha_u^+}}I_{n-1}\left[\left[(2p\cdot q)z\right]^{\alpha_u^+}\frac{\xi^{\alpha_u^+}\left(\frac{1}{2}-(1-z)r-z\right)}{\left[\xi+(1-z)\mu^2+z(1-z)rQ^2\right]^{3+\alpha_u^+}}\right.$$
$$\times\left(\frac{1}{(1+x'-i\epsilon)^3}-1\right)\left[\left(2p\cdot q+q^2\right)z\right]^{\alpha_u^+}$$
$$\left.\times\frac{\xi^{\alpha_u^+}\left(\frac{1}{2}-(1-z)r\right)}{[\xi+(1-z)\mu^2+z(1-z)(1-r)Q^2]^{3+\alpha_u^+}}\left(\frac{1}{(1-x'-i\epsilon)^3}-1\right)\right]\Bigg\}. \tag{55}$$

The ξ integral in Eq. (55) can be performed analytically yielding,

$$T_+^{\mu\nu}=-\delta_{\lambda'\lambda}e_q^2\frac{1}{2}t^{\mu\nu}Q^2\int_0^\infty dx'\int_0^1 dr\int_0^1 dz2\pi^2$$
$$\times\left\{\frac{z^{\alpha_s^+}\beta_s^+(\mu^2)^{1-\alpha_s^+}}{(1+\alpha_s^+)(2+\alpha_s^+)}I_{n-1}\frac{\frac{1}{2}-(1-z)r-z}{[\mu^2+zrQ^2]^2}\frac{1}{x'^{1+\alpha_s^+}}\right.$$
$$\times\left[\left[2p\cdot q+q^2\right]^{\alpha_s^+}\left(\frac{1}{(1-x'-i\epsilon)^3}-1\right)+[2p\cdot q]^{\alpha_s^+}\left(\frac{1}{(1+x'-i\epsilon)^3}-1\right)\right]$$
$$+\frac{z^{\alpha_u^+}\beta_u^+(\mu^2)^{1-\alpha_u^+}}{(1+\alpha_u^+)(2+\alpha_u^+)}I_{n-1}\frac{\frac{1}{2}-(1-z)r-z}{[\mu^2+zrQ^2]^2}\frac{1}{x'^{1+\alpha_u^+}}$$
$$\left.\times\left[[2p\cdot q]^{\alpha_u^+}\left(\frac{1}{(1+x'-i\epsilon)^3}-1\right)+\left[2p\cdot q+q^2\right]^{\alpha_u^+}\left(\frac{1}{(1-x'-i\epsilon)^3}-1\right)\right]\right\}. \tag{56}$$

The crucial ingredient which leads to the difference between the DVCS amplitude given in Eq. (56) and the DIS case studied in [19] is the presence of the r factor in the zrQ^2 terms in the denominators. In the absence of this factor, the integral over z would be dominated by the region $z\sim\mu^2/Q^2$. In that case the factors of $z^\alpha(2p\cdot q+q^2)^\alpha$ and $z^\alpha(2p\cdot q)^\alpha$ would become Q^2-independent; this would produce a Q^2-independent expression for $T_+^{\mu\nu}$ as expected from scaling. This additional r-dependence is of the same type as found in Ref. [32]. In that paper, Regge behavior was introduced by utilizing a t-channel approach, and not through the s or u-channel formalism as employed here. The factor of r in zrQ^2 makes r peak around μ^2/Q^2, and this produces an overall $(Q^2)^\alpha$ dependence for the DVCS amplitude. In particular we can write the symmetric tensor $T_+^{\mu\nu}$ in the form

$$T_+^{\mu\nu}=-\delta_{\lambda'\lambda}e_q^2t^{\mu\nu}\left[\left(\frac{Q^2}{x_{\rm B}\mu^2}\right)^{\alpha_s^+}F_{\rm s}^+(x_{\rm B})+\left(\frac{Q^2}{x_{\rm B}\mu^2}\right)^{\alpha_u^+}F_u^+(x_{\rm B})\right], \tag{57}$$

where in Eq. (57) we have introduced the quantities

$$F^{+}_{s,u}(x_{\rm B}) \equiv \frac{\pi^2}{2}\frac{(1-\alpha^{+}_{\rm s,u})\Gamma(\alpha^{+}_{\rm s,u})}{(1+\alpha^{+}_{\rm s,u})\Gamma(3+\alpha^{+}_{\rm s,u})}\beta^{+}_{s,u}\left[\mu^2 I_{n-1}\frac{1}{\mu^2}\right] \times \left[\xi^{+}_{\alpha^{+}_{\rm s,u}} + (1-x_{\rm B})^{\alpha^{+}_{\rm s,u}}\xi^{-}_{\alpha^{+}_{\rm s,u}}\right] \tag{58}$$

and in Eq. (58) we define

$$\xi^{\pm}_{\alpha} \equiv \int_0^\infty \frac{dx'}{x'^{1+\alpha}}\left[\frac{1}{(1\pm x'-i\epsilon)^3}-1\right]. \tag{59}$$

We call the functions $F(x_{\rm B})$ introduced in Eq. (58) the Regge Exclusive Amplitudes, since they contain the information from the coupling of the relevant Regge trajectories to a particular exclusive process, in this case DVCS. Unfortunately, the loss of factorization in this process makes this and analogous functions non-universal, unlike the generalized parton distributions or GPDs. However the Regge Exclusive Amplitudes do convey information regarding the exponents α of the relevant Regge trajectories that are indeed universal. These amplitudes also allow a comparison between hard exclusive processes and high-energy total cross-sections. Alternatively one can directly employ the t-channel formulation of the hard process in terms of a single (or a few) Regge trajectories.

Finally a similar analysis for the antisymmetric amplitude yields a form

$$T^{\mu\nu}_{-} = ie_q^2\epsilon^{\mu\nu}{}_{03}\tau^3_{\lambda'\lambda}\left[\left(\frac{Q^2}{x_{\rm B}\mu^2}\right)^{\alpha^-_{\rm s}} F^-_{\rm s}(x_{\rm B}) + \left(\frac{Q^2}{x_{\rm B}\mu^2}\right)^{\alpha^-_u} F^-_u(x_{\rm B})\right], \tag{60}$$

where the relevant Regge Exclusive Amplitudes are defined as

$$F^{-}_{s,u}(x_{\rm B}) \equiv \frac{\pi^2}{2}\frac{(1-\alpha^{-}_{s,u})\Gamma(\alpha^{-}_{s,u})}{(1+\alpha^{-}_{s,u})\Gamma(3+\alpha^{-}_{s,u})}\beta^{-}_{s,u}\left[\mu^2 I_{n-1}\frac{1}{\mu^2}\right] \times \left[\xi^{+}_{\alpha^{-}_{s,u}} - (1-x_{\rm B})^{\alpha^{-}_{s,u}}\xi^{-}_{\alpha^{-}_{\rm s,u}}\right]. \tag{61}$$

We note the familiar structure. The finite constants $\xi^{\pm}_{\alpha}$ encode the integration over the hard propagators from the collinear approximation, and contribute with a relative factor of $\pm(1-x_{\rm B})^{\alpha}$ to the symmetric and antisymmetric DVCS amplitudes, respectively. This is the same factor that arises from the singularities of the collinear approximation. The regularization of the collinear approximation leads to an increase of the hard exclusive amplitude by a factor of $(Q^2/x_B\mu^2)^\alpha$ relative to the DIS amplitude. This is the same enhancement factor that was found in Ref. [32][3]. In the more general case, when the nucleon and/or quark masses are kept finite or more than one scale appears in the parton–nucleon amplitude, the single quantity μ would be replaced by some combination of quantities. The functions $F^{+}_{s,(u)}$ describe the quark (antiquark) helicity averaged contribution to the DVCS amplitude. Similarly, the functions $F^{-}_{s,(u)}$ describe the quark (antiquark) helicity-dependent contribution to the DVCS amplitude.

As was discussed in Sec. 1, we have carried out a preliminary study of photon-induced exclusive processes. We have shown that Regge amplitudes should make significant contributions at large values of x_B, and not just at small x_B. A major result of our formalism is the prediction of scaling violation in these hard exclusive processes. At intermediate energies the Bethe–Heitler (BH) amplitude is generally substantially larger than DVCS, so the DVCS amplitudes must be extracted via their interference with the BH term. A group at Hall A in Jefferson Laboratory [24] has recently performed a test of QCD scaling in spin-dependent $\vec{e}p$ scattering. They measured the beam-spin azimuthal asymmetry [5, 6], which is proportional to interference between BH and DVCS amplitudes. After removing the Q^2-dependence associated with the BH term, they extracted twist-2 and -3 Compton form factors which by QCD scaling should be Q^2-independent. In Fig. 3 we plot the twist-2 Compton form factor $C^I(F)$ *versus* Q^2; this term

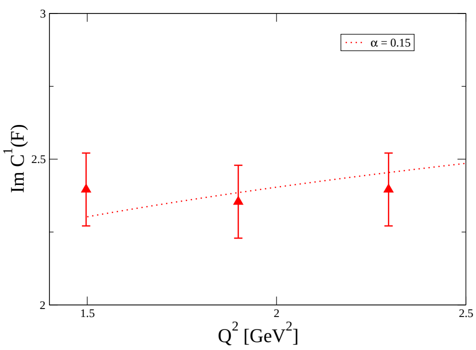

Fig. 3. (Color online) Comparison with DVCS results from Jefferson Lab [24]. The data points represent the twist-2 Compton form factor extracted from beam-spin asymmetry measurements in $\vec{e}p$ scattering, *versus* Q^2. The data have been averaged over t. The dotted curve is a $(Q^2)^\alpha$ fit with $\alpha = 0.15$.

[3] We have recently found an heuristic argument by Bjorken and Kogut [48] that leads them to the same conclusion (see their equation 3.25) that the exponents of Q^2 and ν are different and thus scaling is absent in exclusive processes in the presence of Regge behavior.

has been averaged over t. Although the data show very little Q^2 dependence, they correspond to a limited range of Q^2 and are also in good agreement with our predicted behavior $(Q^2)^\alpha$. In Fig. 3 the dotted line corresponds to $(Q^2)^\alpha$ with $\alpha = 0.15$. Because the data points were averaged over t it is not obvious what value of α to choose, but over this range of Q^2 our predicted behavior is in agreement with the Hall A points.

In Fig. 4 we compare our predictions with the data on exclusive meson electroproduction. Scaling arguments predict that the reduced π^+ cross-section should fall off at fixed $x_{\rm B}$ as $1/Q^2$. We predict a behavior $(Q^2)^{2\alpha-1}$ with $0 < \alpha < 1$. Fitting π^+ data from HERMES [22] in the range $0.26 < x_{\rm B} < 0.8$ gives $\alpha = 0.13 \pm 0.1$. Similarly for ω electroproduction cross-section from the CLAS collaboration at Jefferson Lab [21] we find $\alpha = 0.6 \pm 0.4$ for the range $0.52 < x_{\rm B} < 0.58$.

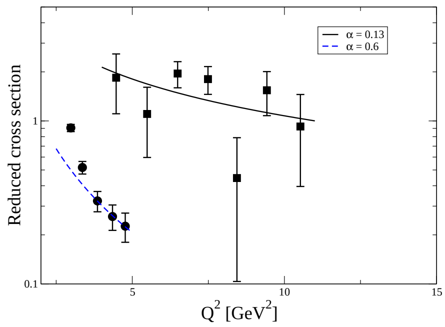

Fig. 4. (Color online) A simple fit to electroproduction data for mesons, π^+ results from HERMES (squares, [22]), and ω results from the CLAS Collaboration at Jefferson Lab (circles, [21]). In the case of π^+ production the cross-section reduced by the photon flux is plotted (in arbitrary units).

We see that for both DVCS and exclusive meson electroproduction, not only are the data consistent with scaling violations, but the additional Q^2 dependence is softer than predicted by scaling and in agreement with our predicted factor of $(Q^2)^\alpha$ with $0 < \alpha < 1$. At this point it is difficult to compare the Regge exponents α obtained from the fit with total cross-section data, since the electroproduction data was taken at different values of t. However, we find this trend encouraging, and we believe that it warrants further phenomenological studies. QCD scaling predicts that agreement with scaling should become progressively better with increasing Q^2. However, we have shown that scaling violations should persist regardless of the size of Q^2.

We should point out, however, that in the absence of collinear factorization the new expressions for the DVCS amplitudes are non-universal and sensitive to non-perturbative physics. It remains to be seen how QCD corrections behave in presence of GPD's that are singular at the break points.

For this reason it is hard to compare our predictions with the $x_{\rm B} \to 0$, HERA data, where it is known that NLO QCD corrections are necessary for obtaining the observed Q^2 dependence [30]. This can be seen from Refs [46, 47], where a fit to HERA data [49–51] within a dipole-model of diffractive scattering has been performed using several different parameterizations.

4. Summary and outlook

In this paper we examined Regge behavior in Compton scattering. We started with the generic hadronic tensor for DVCS which was then expressed in terms of a parton–nucleon amplitude with Regge behavior. We have found that if the parton–nucleon amplitude increases with energy, which is expected for low momentum transfer of hadronic scattering amplitudes, the resulting generalized parton distributions become singular at the break points and thus the standard factorization formula breaks down. This is to be expected, since the proof of factorization in DVCS [9] is based on the assumption that the parton–nucleon amplitude vanished with increasing s or u of the parton nucleon system. The full amplitude, without collinear approximation is, however, well defined and leads to non-scaling with the DVCS amplitude proportional to ν^α, where α is a positive Regge intercept. If the quasi-onshell parton–nucleon states are relevant in DVCS then we would argue that scaling might still take place but at a sizable momentum transfer $t < t_0 \sim -m_\rho^2$. This would occur because Regge intercepts decrease with increasing $|t|$ and once they become negative scaling sets in. A preliminary examination of experimental data on DVCS and hard meson production indicates that non-scaling behavior cannot yet be ruled out and future experiments should soon be able to establish the Q^2 dependence. A similar problem could potentially arise in other channels. For example it is commonly assumed that hadron (*e.g.* pion) distribution amplitudes vanish at the end-points. And such behavior is in general necessary for factorization theorems to be applicable. The vanishing of distribution amplitudes near the endpoints has only been shown within perturbation theory. The end-point region, however, may be dominated by soft scattering which could enhance the end-point contribution [52].

Note added in revision. Following the first version of this paper a criticism of our analysis was posed on the `arXiv.org` [53]. The main point of the criticism is related to the treatment of the DVCS amplitude that follows from our Regge-dominated description of the GPD. The authors of [53] propose that the singularities in DVCS originating from the break-point region of the x-integral in Eq. (32) should be removed. While this is a possible way out, it seems to us more of a mathematical trick rather than a physically motivated argument since we cannot identify a microscopic origin of the would-be counter-terms. In contrast the relevant subtraction terms which make parton–nucleon and the DVCS amplitudes finite have already being identified and included in the construction of the hadronic tensor in Eqs (12), (13). The singularity in the DVCS amplitude discussed in Sec. 2.4 is in fact artificially generated after a particular approximation to the integral over parton momenta (collinear factorization) is made. Otherwise a finite, albeit non-scaling behavior is obtained as shown in Sec. 3.

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