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EFFECT OF COHERENT-PAIR APPROXIMATION ON NUCLEON PROPERTIES IN THE EXTENDED LINEAR SIGMA MODEL

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Extended chiral sigma model with quark fields and elementary pion and sigma fields is used to describe static properties of the nucleon. The field equations have been solved in the coherent-pair approximation. Better results are obtained for the nucleon properties in comparison with previous work and reasonable agreement with data.

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1. Introduction

Nonperturbative regime of QCD is a mechanism responsible for the formation of baryons which belong to the low-energy. The difficulty in numerical calculations of lattice QCD leading to alternative models are considered some features of QCD. Lagrangians for these models should be derivable from QCD; in practice one constructs them taking into account relevant features of the underlying theory such as confinement, asymptotic freedom and spontaneous broken chiral symmetry [1].

One of the effective models in describing baryons properties is the linear sigma model which has been suggested earlier by Gell-Mann and Levy [2] to describe nucleons interacting via sigma (σ) and pion (π) exchanges. Linear sigma model sets up to understand the structure of nucleon that should respect the constraints imposed by chiral symmetry. Spontaneous and explicit chiral symmetry breaking require the existence of the pion which its mass vanishes in the limit of zero current mass. A few solutions for the Lagrangian of chiral linear sigma model when applied to the nucleon and delta have already been suggested. Birse and Banerjee [3] solved the linear chiral sigma model in mean-field approximation using the hedgehog ansatz for the pion field. After the variation, they performed an approximate projection on angular momentum and isospin ignoring in this procedure the contribution of the pions. Birse [4] and Golli and Rosina [5] evaluated this model further performing proper projections even before the variation in the hedgehog approximation. Fiolhais *et al.* [6] generalized the hedgehog and performed spin and isospin projections as well. In contrast with the mean-field approximation, the coherent-pair approximation is being studied to provide a systematic expansion method for the description of a boson field. In addition, it avoids assumptions like the hedgehog structure of the quark and pion fields [7, 8], Goeke *et al.* [7] obtained the static solitonic solution of the linear sigma model using a coherent pair trial Fock state with proper spin and isospin quantum numbers. The work of Goek *et al.* [7] has been re-examined by Aly *et al.* [8] and they corrected some misprints in it.

In recent years, there has been growing interest in studying nucleon properties, therefore some modifications have been suggested in the linear sigma model in the framework of some aspects of QCD. Broniowski and Golli [9] analyzed a particular extension of the linear sigma model coupled to valence quarks containing an additional term with two ingredients of the chiral fields and investigated the dynamic consequences of this term and its relevance to the phenomenology of soliton models of the nucleons. Dmitrasinovic and Myhrer [10] used an extended linear sigma model [11] in which a pair of extra terms has been added to the original linear sigma model in order to improve pion–nucleon scattering and the nucleon sigma term. Furthermore, Korchin [12] calculated the properties of the nucleon in a non-local sigma model where conserved electromagnetic and vector currents and partially conserved axial vector current are obtained. In the same direction, Rashdan *et al.* [13–15] considered higher-order mesonic interactions in the linear sigma model using mean-field approximation to get a better description of the nucleon properties. Also, Logarithmic mesonic potential is suggested in which some aspects of QCD are taken to get a better description of nucleon properties [16, 17].

The aim of this paper is to estimate the effect of coherent-pair approximation on nucleon properties in the extended chiral quark sigma model is suggested by Broniowski and Golli [9].

This paper is organized as follows: In Sec. 2, extended chiral sigma model is explained briefly. The Fock state in the coherent-pair approximation and the variational principle are presented in Secs 3 and 4, respectively. Derived nucleon properties are explained in Sec. 5. Numerical calculations and results discussion are presented in Sec. 6.

2. Extended chiral quark sigma model

We begin with extended chiral quark sigma model [9] which the Lagrangian density of extended sigma model that describes the interactions between quarks via the σ-and $\boldsymbol{\pi}$-mesons is written [9]

$$L(x) = i\overline{\Psi}\partial_\mu\gamma^\mu\hat{\Psi} + \tfrac{1}{2}\left(\partial_\mu\hat{\sigma}\partial^\mu\hat{\sigma} + \partial_\mu\hat{\boldsymbol{\pi}}\cdot\partial^\mu\hat{\boldsymbol{\pi}}\right) + \tfrac{1}{2}A_0\left(\sigma\partial^\mu\sigma + \hat{\boldsymbol{\pi}}.\partial^\mu\hat{\boldsymbol{\pi}}\right)^2$$
$$+g\hat{\overline{\Psi}}\left(\hat{\sigma} + i\gamma_5\hat{\boldsymbol{\tau}}.\hat{\boldsymbol{\pi}}\right)\hat{\Psi} - U\left(\hat{\sigma},\hat{\boldsymbol{\pi}}\right) \tag{1}$$

with

$$U\left(\hat{\sigma},\hat{\boldsymbol{\pi}}\right) = \frac{\lambda^2}{4}\left(\hat{\sigma}^2 + \hat{\boldsymbol{\pi}}^2 - \nu^2\right)^2 - f_\pi m_\pi^2\hat{\sigma}\,, \tag{2}$$

$$\lambda^2 = \frac{m_\sigma^2 - m_\pi^2}{2f_\pi^2}\,, \tag{3}$$

$$\nu^2 = f_\pi^2 - \frac{m_\pi^2}{\lambda^2}\,, \tag{4}$$

$$\bar{m}_\sigma^2 = \left(1 + f_\pi^2 A_0\right)m_\sigma^2\,, \tag{5}$$

where f_π is the pion decay constant, m_π is the pion mass, and m_σ, g and A_0 are constants to be determined wherever A_0 is constrained by Eq. (5) ($\bar{m}_\sigma^2 \succ 0$). The quark, sigma and $\boldsymbol{\pi}$-mesons are quantum fields denoted by ($\hat{}$). Spontaneous symmetry breaking generates mass for the quark which breaks the chiral symmetry and generates the small pion mass which would be zero, otherwise as the Goldstone boson of the theory (for details, see Ref. [8]).

Now, we can rewrite the Hamiltonian density as in Ref. [8].

$$\hat{H}(r) = \tfrac{1}{2}\left\{\hat{P}_\sigma(r)^2 + (\nabla\hat{\sigma}(r))^2 + \hat{P}_\pi(r)^2 + (\boldsymbol{\nabla}\pi(r))^2 + A_0\left(\sigma\partial^\mu\sigma + \hat{\boldsymbol{\pi}}.\partial^\mu\hat{\boldsymbol{\pi}}\right)^2\right\}$$
$$+U(\hat{\sigma},\hat{\boldsymbol{\pi}}) + \hat{\Psi}^\dagger(r)(-i\alpha\boldsymbol{\nabla})\hat{\Psi}(r) - g\,(r)\,\hat{\Psi}^\dagger(r)(\beta\hat{\sigma}(r) + i\beta\gamma_5\hat{\boldsymbol{\tau}}.\hat{\boldsymbol{\pi}})\hat{\Psi}(r)\,, \tag{6}$$

where α and β are the usual Dirac matrices. In the above expression $\hat{\overline{\Psi}}$, $\hat{\sigma}$, and $\hat{\boldsymbol{\pi}}$ are quantized field operators with the appropriate static angular momentum expansion [8],

$$\hat{\sigma}(r) = \int_0^\infty \frac{d^3k}{[(2\pi)^3 2W_\sigma(k)]^{\frac{1}{2}}}\left[\hat{c}^\dagger(k)e^{-ik.r} + \hat{c}(k)e^{+ik.r}\right]\,, \tag{7}$$

$$\hat{\boldsymbol{\pi}}(r) = \left[\frac{2}{\pi}\right]^{1/2}\int_0^\infty dkk^2\left[\frac{1}{2W_\pi(k)}\right]^{1/2}\sum_{lmw} j_l(kr)Y_{lm}^*(\Omega_r)[\hat{a}_{lm}^{1w\dagger}(k)$$
$$+(-)^{m+w}\hat{a}_{l-m}^{1-w}(k)]\,, \tag{8}$$

$$\hat{\Psi}(r) = \sum_{njmw}\left(\langle r|\,njmw\rangle\,\hat{d}_{njm}^{\frac{1}{2}w} + \langle r|\,\overline{n}jmw\rangle\,\hat{d}_{njm}^{\frac{1}{2}w\dagger}\right)\,, \tag{9}$$

where the $|njmw\rangle$ and $|\overline{n}jmw\rangle$ form a complete set of quark and antiquark spinors with angular momentum quantum numbers and spin–isospin quantum numbers j, m, and w, respectively. The corresponding conjugate momentum fields have the expansion [8],

$$\begin{aligned}\hat{P}_\sigma(r) &= i\int_0^\infty d^3k\left[\frac{W_\sigma(k)}{2(2\pi)^3}\right]^{\frac{1}{2}}\left[\hat{c}^\dagger(k)e^{-\boldsymbol{k}.\boldsymbol{r}} - \hat{c}(k)e^{+\boldsymbol{k}.\boldsymbol{r}}\right],\\ \hat{P}_\pi(r) &= i\left[\frac{2}{\pi}\right]^{\frac{1}{2}}\int_0^\infty dkk^2\left[\frac{W_\pi(k)}{2}\right]^{\frac{1}{2}}\sum_{lmw} j_l(kr)Y^*_{lm}(\Omega_r)\Big[\hat{a}^{1w\dagger}_{lm}(k)\\ &\quad -(-)^{m+w}\hat{a}^{1-w}_{l-m}(k)\Big].\end{aligned} \tag{10}$$

Here $\hat{c}(k)$ destroys a σ-quantum with momentum $\boldsymbol{k}$ and frequency $W_\sigma(k) = \left(k^2+m_\sigma^2\right)^{\frac{1}{2}}$ and $\hat{a}^{1w}_{lm}(k)$ destroys a pion with momentum $\boldsymbol{k}$ and corresponding $W_\pi(k) = (k^2+m_\pi^2)^{\frac{1}{2}}$ in isospin-angular momentum state $\{lm; tw\}$.

3. The Fock state

For convenience one constructs the configuration space pion field functions needed for the subsequent variational treatment by defining the alternative basis operators,

$$\hat{b}^{1w}_{lm} = \int dkk^2\zeta_l(k)\hat{a}^{1w}_{lm}(k)\,, \tag{11}$$

where $\hat{a}^{1w}_{lm}(k)$ are basis operators which create a free massive pion with isospin component w and orbital angular momentum (l,m), and $\zeta_l(k)$ is the variational function. Taking this over to configuration space defines the pion field function [8]

$$\Phi_l = \frac{1}{2\pi}\int_0^\infty dkk^2\frac{\zeta_l(k)}{W_\pi(k)^{\frac{1}{2}}}j_l(r)\,. \tag{12}$$

In the following only the $l=1$ value is used and the angular momentum label will be dropped. The Fock state for the nucleon is taken to be [8]

$$\begin{aligned}|NT_3J_z\rangle &= \Big[\alpha\left(|n\rangle\otimes\left|P^{00}\right\rangle\right)_{T_3J_z} + \beta\left(|n\rangle\otimes\left|P^{11}\right\rangle\right)_{T_3J_z}\\ &\quad +\gamma\left(|\delta\rangle\otimes\left|P^{11}\right\rangle_{T_3J_z}\right)|0\rangle\Big]\left|\sum\right\rangle,\end{aligned} \tag{13}$$

where $|\sum\rangle$ is the coherent sigma field state with the property: $\langle\sum|\hat{\sigma}(r)|\sum\rangle = \hat{\sigma}(r)$, and $|P^{00}\rangle(|P^{1w}_{1m}\rangle)$ are pion coherent-pair states to be determined. The normalization of the nucleon state requires $\alpha^2+\beta^2+\gamma^2=1$. The permutation symmetric form of that SU(2)× SU(2)× SU(2) quark wave functions imply that the source terms in the pion field equations will induce in angular momentum isospin correlation for the pion field (for details, see Ref. [8]).

4. The variational principle

The objective of this section is to seek the minimum of the total energy of baryon is given by

$$E_B = \langle BT_3J_z| \int_0^\infty d^3r : H(r) : |BT_3J_z\rangle \ , \tag{14}$$

where $B = N$ or Δ. The field equations are obtained by minimizing the total energy of the baryon with respect to the variation of the fields, $\{u(r), w(r), \sigma(r), \Phi(r)\}$, as well as the Fock-space parameters, $\{\alpha, \beta, \gamma\}$ subjected to the normalization conditions. The total energy of the system is written as

$$E_B = 4\pi \int_0^\infty dr r^2 \varepsilon_B(r) \, . \tag{15}$$

Writing the quark Dirac spinor as

$$\Psi^{\frac{1}{2}w}_{\frac{1}{2}m}(\boldsymbol{r}) = \begin{pmatrix} u(r) \\ \upsilon(r)\boldsymbol{\sigma}.\hat{\boldsymbol{r}} \end{pmatrix} \chi_{\frac{1}{2}m}\zeta^{\frac{1}{2}w} \, , \tag{16}$$

the energy density is given by

$$\begin{aligned}
\varepsilon_B(r) &= \frac{1}{2}\left(\frac{d\sigma}{dr}\right)^2 + \frac{1}{2}A_0\sigma^2\left(\frac{d\sigma}{dr}\right)^2 + A_0\Phi\frac{d\Phi}{dr}\sigma\frac{d\sigma}{dr} - \frac{\lambda^2}{4}\left(\sigma^2(r)-\nu^2\right)^2 \\
&-m_\pi^2 f_\pi \sigma(r) + 3\left[u(r)\left(\frac{d\upsilon}{dr}+\frac{2}{r}\upsilon(r)\right) - \upsilon(r)\frac{du}{dr} + g\sigma(r)\left(u^2(r)-\upsilon^2(r)\right)\right] \\
&+(N_\pi+x)\left(\left(\frac{d\Phi}{dr}\right)^2 + \frac{2}{r^2}\Phi^2(r)\right) + (N_\pi-x)\Phi_p^2(r) - \alpha\delta g(a+b)u(r)\upsilon(r)\Phi(r) \\
&+\lambda^2\left[x^2+2xN_\pi+81\left(\alpha^2a^2c^2+\left(\beta^2+\gamma^2\right)d^2\right)\right]\Phi^4(r) \\
&+\lambda^2(N_\pi+x)\left(\sigma^2(r)-\upsilon^2\right)\Phi^2(r) + A_0\Phi(r)^2(N_\pi+x)\left(\left(\frac{d\Phi}{dr}\right)^2+\frac{2}{r^2}\Phi^2\right) \, ,
\end{aligned} \tag{17}$$

where N_π is the average pion number

$$N_\pi = 9\left(\alpha^2 a^2 + \left(\beta^2 + \gamma^2\right) c^2\right) , \tag{18}$$

and where δ takes the following values for nucleon or delta quantum numbers:

$$\delta_N = \left(5\beta + 4\sqrt{2}\gamma\right)/\sqrt{3}\,, \qquad \delta_\Delta = \left(2\sqrt{2}\beta + 5\gamma\right)/\sqrt{3}\,. \tag{19}$$

The function $\Phi_p(r)$ is obtained from $\Phi(r)$ by double folding,

$$\Phi_p(r) \;=\; \int\limits_0^\infty w(r,\acute{r})\Phi(\boldsymbol{r})r^2 d\acute{r}\,, \tag{20}$$

$$w(r,\acute{r}) \;=\; \frac{2}{\pi}\int\limits_0^\infty dkk^2 w(k) j_1(kr) j_1(kr^{'})\,. \tag{21}$$

For fixed α, β and γ, the stationary functional variations are expressed by

$$\delta\left[\int\limits_0^\infty drr^2(\varepsilon_B(r) - 3\epsilon\left(u^2(r) + v^2(r)\right) - 2k\Phi\Phi_p(r))\right] = 0\,, \tag{22}$$

where the parameter k enforces the pion normalization condition,

$$8\pi\int\limits_0^\infty \Phi(r)\Phi_p(r) r^2 dr = 1\,, \tag{23}$$

and ϵ fixes the quark normalization,

$$4\pi\int\limits_0^\infty \left(u^2(r) + v^2(r)\right) r^2 dr = 1\,. \tag{24}$$

Minimizing the Hamiltonian yields the four nonlinear coupled differential equations,

$$\frac{du}{dr} \;=\; -2(g\sigma + \epsilon)v(r) - \frac{1}{3}\alpha\delta(a+b)g\Phi(r)u(r)\,, \tag{25}$$

$$\frac{dv}{dr} \;=\; -\frac{2}{r}v(r) - 2(g\sigma(r) - \epsilon)u(r) + \frac{1}{3}\alpha\delta(a+b)g\Phi(r)u(r)\,, \tag{26}$$

$$\begin{aligned}\frac{d^2\sigma}{dr^2} \;=\; & \frac{1}{(1 + A_0\sigma^2)}\Big\{ -\frac{2}{r}\left(1 + A_0\sigma^2\right)\frac{d\sigma}{dr} - m_\pi^2 f_\pi + 3g\left(u^2(r) - v^2(r)\right) \\ & + 2\lambda^2\left(N_\pi + x\right)\Phi^2(r)\sigma(r) + \lambda^2\left(\sigma^2(r) - v^2\right)\sigma(r)\Big\}\,,\end{aligned} \tag{27}$$

$$\begin{aligned}
\frac{d^2\Phi}{dr^2} &= -\frac{2}{r}\frac{d\Phi}{dr}+\frac{2}{r^2}\left(\frac{2N_\pi+A_0\Phi^2\left(N_\pi+x\right)}{4N_\pi+2A_0\Phi^2\left(N_\pi+x\right)}\right)\Phi(r) \\
&\quad+\left(\frac{2N_\pi+2x}{4N_\pi+2A_0\Phi^2\left(N_\pi+x\right)}\right)m_\pi^2\Phi(r)+\frac{1}{4N_\pi+2A_0\Phi^2\left(N_\pi+x\right)} \\
&\quad\times\Big\{4\lambda^2\left[x^2+2N_\pi x+81\left(\alpha^2a^2c^2+\left(\beta^2+\gamma^2\right)d^2\right)\right]\Phi^3(r) \\
&\quad-\alpha(a+b)g\delta u(r)v(r)+2\lambda^2(N_\pi+x)\left(\sigma^2-\nu^2\right)\Phi \\
&\quad-2K\Phi(r)\Big\}+\frac{2A_0\Phi\left(N_\pi+x\right)}{4N_\pi+2A_0\Phi^2\left(N_\pi+x\right)}\left(\left(\frac{d\Phi}{dr}\right)^2+\frac{2}{r^2}\Phi^2\right), \qquad (28)
\end{aligned}$$

with eigenvalue ϵ and k. These consist of two quark equations for u and v where $\sigma(r)$ and $\Phi(r)$ appear as potentials, and two Klein–Gordon equations with $u(r)v(r)$ and $(u^2(r)-v^2(r))$ as source terms. The boundary conditions are for $r \longrightarrow 0$,

$$v=\frac{d\sigma}{dr}=\Phi=\frac{du}{dr}=0\,, \qquad (29)$$

and for $r \longrightarrow \infty$,

$$\begin{aligned}
\left[r\left(gf_\pi-\epsilon\right)^{\frac{1}{2}}+\left(gf_\pi+\varepsilon\right)^{-\frac{1}{2}}\right]u(r)-r\left(gf_\pi+\epsilon\right)^{\frac{1}{2}}v(r) &= 0\,, \qquad (30)\\
\left(2+2m_\pi r+m_\pi^2r^2\right)\Phi(r)+r(1+m_\pi r)\Phi(r) &= 0\,, \qquad (31)\\
r\sigma(r)+\left(\sigma(r)-f_\pi\right)\left(1+m_\sigma r\right) &= 0\,. \qquad (32)
\end{aligned}$$

The field equations are solved for fixed coherence parameter (x) and fixed Fock-space parameter(α,β,γ) as in Ref. [8].

5. The nucleon properties

The expectation value of energy is minimized with respect to (α,β,γ) by diagonalizing of the energy matrix

$$\begin{bmatrix} H_{\alpha\alpha} & H_{\alpha\beta} & H_{\alpha\gamma} \\ H_{\alpha\beta} & H_{\beta\beta} & H_{\beta\gamma} \\ H_{\alpha\beta} & H_{\beta\gamma} & H_{\gamma\gamma}\end{bmatrix}\begin{bmatrix}\alpha\\ \beta\\ \gamma\end{bmatrix}=E\begin{bmatrix}\alpha\\ \beta\\ \gamma\end{bmatrix}, \qquad (33)$$

each H entry of the matrix is related to a corresponding density as follows:

$$H_{\alpha\beta}=4\pi\int r^2E_{\alpha\beta}(r)dr\,, \qquad (34)$$

and analogously for the other entries. The functions for a nucleon are

$$\begin{aligned}
E_{\alpha\alpha} &= E_0(r)+18a^2\Phi_p^2(r)+9a^2\lambda^2\left(2x+9c^2\right)\Phi^4(r) \\
&\quad+9a^2\lambda^2\left(\sigma^2(r)-v^2\right)\Phi^2(r)+9A_0a^2\Phi^2(r)\left(\left(\frac{d\Phi}{dr}\right)^2+\frac{2}{r^2}\Phi^2\right), \qquad (35)
\end{aligned}$$

$$E_{\beta\beta} = E_0(r) + 18c^2\Phi_p^2(r) + 9\lambda^2\left(2xc^2 + 9d^2\right)\Phi^4(r)$$
$$+9c^2\lambda^2\left(\sigma^2(r)-\upsilon^2\right)\Phi^2(r)+9A_0c^2\Phi^2(r)\left(\left(\frac{d\Phi}{dr}\right)^2+\frac{2}{r^2}\Phi^2\right), \quad (36)$$

$$E_{\alpha\beta} = -2g(a+b)\Phi(r)u(r)v(r)\frac{2\sqrt{2}}{\sqrt{3}}, \quad (37)$$

$$E_{\alpha\gamma} = -2g(a+b)\Phi(r)u(r)v(r)\frac{5}{\sqrt{3}}. \quad (38)$$

If $A_0 = 0$, the usual linear sigma model of the above equations are recovered where:

$$E_0(r) = \frac{1}{2}\left(\frac{d\sigma}{dr}\right)^2 + \frac{1}{2}A_0\sigma^2\left(\frac{d\sigma}{dr}\right)^2 + A_0\Phi\frac{d\Phi}{dr}\sigma\frac{d\sigma}{dr} - \frac{\lambda^2}{4}\left(\sigma^2(r) - f_\pi^2\right)^2$$
$$+3g\sigma(r)\left(u^2(r) - \upsilon^2(r)\right)+(N_\pi + x)\left(\left(\frac{d\Phi}{dr}\right)^2 + \frac{2}{r^2}\Phi^2(r)\right)$$
$$+(N_\pi - x)\Phi_p^2(r) - 2xm_\pi^2\Phi(r)^2 + \lambda^2x^2\Phi^4(r) + \frac{m_\pi^2}{4}\left(\sigma(r) - f_\pi\right)^2$$
$$\times\lambda^2x\left(\sigma^2(r)-f_\pi^2\right)\Phi^2(r)+A_0\Phi(r)^2(N_\pi+x)\left(\left(\frac{d\Phi}{dr}\right)^2+\frac{2}{r^2}\Phi^2\right), \quad (39)$$

$$\frac{\mu_p(r)}{4\pi e} = \frac{ruv}{81}\left(54\alpha^2 + 2\beta^2 + \gamma^2 + 32\sqrt{2}\beta\gamma\right)$$
$$+\frac{x}{729a^2}\left(9a^2 + x\right)\left(4\beta^2 + \gamma^2\right)\Phi^2, \quad (40)$$

$$\frac{\mu_n(r)}{4\pi e} = \frac{ruv}{81}\left(-36\alpha^2 - 8\beta^2 + \gamma^2 - 32\sqrt{2}\beta\gamma\right)$$
$$-\frac{x}{729a^2}\left(9a^2 + x\right)\left(4\beta^2 + \gamma^2\right)\Phi^2, \quad (41)$$

$$\frac{g_A}{g_v} = 4\pi\int_0^\infty drr^2\Big[\left(\frac{5}{3}\alpha^2 + \frac{5}{27}\beta^2 + \frac{25}{27}\gamma^2 + \frac{32\sqrt{2}}{27}\beta\gamma\right)$$
$$\times\left(u^2(r) - \frac{\upsilon^2(r)}{3}\right) + \frac{8}{3\sqrt{3}}\alpha\beta(a+b)\frac{d\sigma}{dr}\Phi\Big], \quad (42)$$

$$\sigma(\pi, N) = 4\pi f_\pi m_\pi^2\int_0^\infty drr^2\left(\sigma(r) - f_\pi\right). \quad (43)$$

The change in the magnetic moments, coupling constant $(\frac{g_A}{g_v})$ and sigma commutator $\sigma(\pi N)$ induced by A-term through the dynamics in Eqs (25)–(28) (for details, see Refs [8,9]).

6. Discussion of results

The set of nonlinear differential equations have been solved in the same manner as Aly *et al.* [8]. The iteration procedure is implemented in the following. As to the fixed values α, β and γ, the set of differential equations with the corresponding boundary conditions are solved by using the modified numerical package (COLSYS) as that used in Ref. [8]. The solutions of the system are mixed and repeated until self-consistency is achieved. We reconsider the nucleon properties for different values of sigma mass which consistents with Refs [19,20].

First, we need to show the effect of the A-term on the meson fields and the nucleon and delta masses in comparison with previous calculations. Fig. 1 shows the mesons fields for the $x = 3.0$, $g = 5$ and $m_\sigma = 441$ MeV, where the presence of the A-term weakens the pion field, modifies the shape of the sigma field as well as increasing slightly the soliton size that leads to a stability in the energy of the soliton, therefore the behavior is in an agreement with the behavior in the mean-field approximation as in Ref. [9]. From

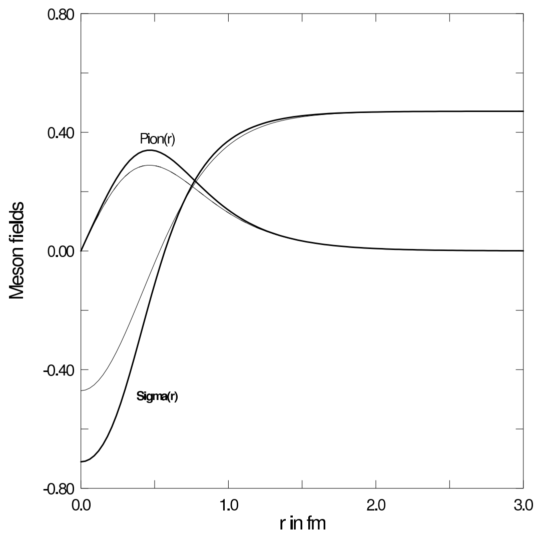

Fig. 1. Sigma and pion fields as functions of the distance r in units f_π for $A_0 = 0.0$ (light curves) and $A_0 = 0.9$ MeV^{-2} (bold curves), $m_\sigma = 441$ MeV and $g = 5.0$.

Fig. 2 we see that the A-term has strong effect on nucleon and delta masses wherever the energy is lowered at high values of coupling constant g. This is a desired effect since many phenomenological approaches have problems in getting the mass in the right ball park if g is too high [9] therefore, the nucleon mass has a good agreement with data and is improved in comparison with the original model [8] (see Table I).

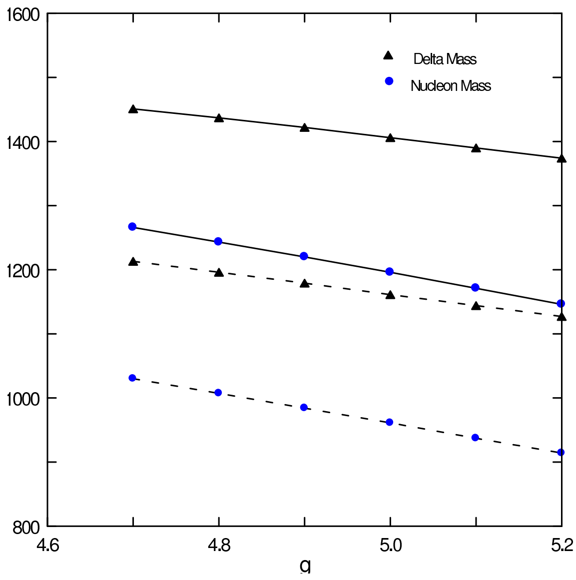

Fig. 2. The soliton energy depends on the coupling constant g for $A_0 = 0$ (solid lines) and $A_0 = 0.9\ \mathrm{MeV}^{-2}$ (dashed lines) for sigma mass = 441 MeV and $x = 1$.

TABLE I

The energy contributions (in MeV) to nucleon and delta when using $g = 5$, $m_\sigma =$ 550 MeV, $A_0 = 0.9\ \mathrm{MeV}^{-2}$ and $x = 1$.

Quantity	Nucleon	Delta	Nucleon [8]	Delta [8]
Quark kinetic energy	1138.49	984.85	1124	975
Sigma kinetic energy	194.03	206.87	304	268
Pion kinetic energy	227.85	180.0	236	185
Quark–meson interaction	−694.19	−319.79	−675	−318
Meson interaction energy	72.58	86.778	84	114
Baryon mass	938.76	1138.71	1073	1224
Nucleon-Delta mass difference		199.9		140

Examining the effect of the degree of coherent state (x) and A_0 -term on the magnetic moments of proton and neutron. From Figs. 3 and 4 we see that the magnetic moments of proton and neutron are decreasing by increasing the value of A_0 but the change is about 7% in comparison with original model [8] ($A_0 = 0.0$) the reason back to the change in the expressions of nucleon magnetic moments are not modified however changes in these quantities are induced by the A_0 through the change of the dynamics of equations of motion as in mean-field approximation in Refs [9,21] but the coherent parameter x appears explicitly in the expressions of nucleon magnetic moments in Eqs (40), (41) so to improve the values of magnetic moments of proton and neutron the coherent parameter is taken ($x = 3.0$) thus we note from the Figs 3 and 4 the values are strongly effect by increasing the coherent parameter (x) also we note from Table II the mesonic contributions have been increased in comparison with original model [8] leading to the change in range 23% thus the improvement in the nucleon magnetic moments are obtained in comparison with previous work [8].

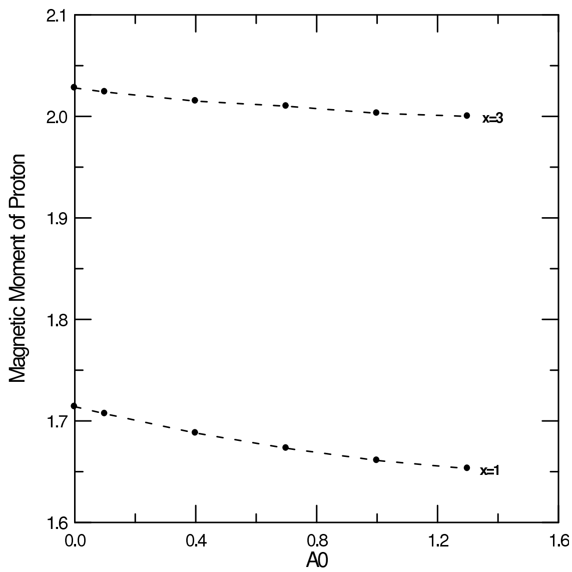

Fig. 3. The dependence of magnetic moment of proton on the constant A_0 MeV^{-2}. Two values of x are used.

TABLE II

Nucleon observables using $x = 3$, $m_\sigma = 441$ MeV, $A_0 = 0.9$ MeV^{-2} and $g = 5$.

Quantity	Quark	Meson	Total	Quark	Meson	Total [8]	Expt.
r_c^2 (proton)(fm^2)	0.684	0.046	0.73	0.533	0.023	0.556	0.7
r_c^2 (neutron)(fm^2)	0.028	−0.108	−0.08	0.019	−0.023	−0.004	−0.12
Magnetic moment (proton)	1.66	0.44	2.1	1.53	0.18	1.71	2.79
Magnetic moment (neutron)	−1.24	−0.46	−1.7	−1.13	−0.18	−1.31	−1.91

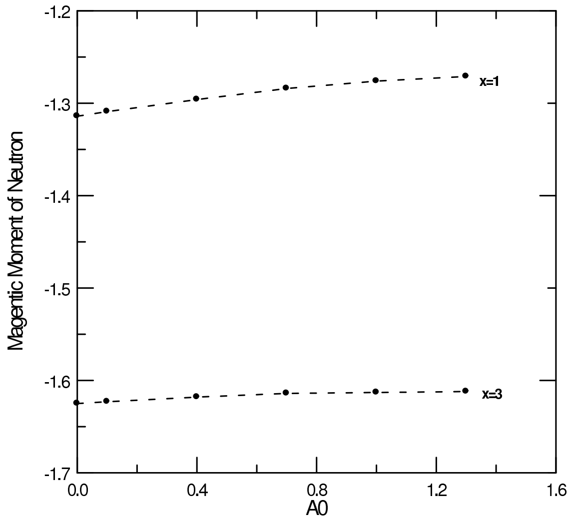

Fig. 4. The dependence of magnetic moment of neutron on the constant A_0 MeV^{-2}. Two values of x are used.

From Table II, the proton and neutron have smallest values in the previous calculation [8] that there is the relative error in the neutron radius is about 96% and 20% in the charge radius of proton in comparison with data by increasing the coherent parameter (x) leading to improve in this quantities that the charge radius of proton is excellent with data and the relative error in the charge radius of neutron is reduce to 33%.

From Table III, the sigma commutator $\sigma(\pi N)$ is improved with the change in about 41% relative to the original model [8] and 24% in comparison with the result in mean-field approximation [21] and a good agreement with data. This quantity $\sigma(\pi N)$ is the fundamental parameter of low-energy hadron physics since it provides a direct measure of the scalar quark conden-

TABLE III

The observables of coupling constant $\frac{g_A(0)}{g_V(0)}$, pion-coupling constant $g_{\pi NN}(0)$ and sigma commutator $\sigma(\pi N)$ using $g = 5$, $m_\sigma = 750$ MeV and $x = 0.65$, $A_0 = 1.6$ MeV^{-2}.

Quantity	Quark	Meson	Total	Quark	Meson	Total [8]	[21]	Expt. [8]
$\sigma(\pi N)$			52			88.9	69	45±5
$\frac{g_A(0)}{g_V(0)}$	1.016	0.364	1.38	1.07	0.39	1.46	1.78	1.25
$g_{\pi NN}(0)$	10.71	3.79	14.5	12.806	2.769	15.57	13.91	12.4

sates in baryons and constitutes a test for the mechanism of chiral symmetry breaking [22]. Note that the meson contribution to the axial vector coupling constant $g_A(0)$ is increased in comparison with the original model [8] by inclusion of the A_0-term that the change in range 7–10% due to the expression for the axial vector current, which does not depend explicitly on the A_0 constant but only on the dynamic of the fields. If one uses the Goldberger Treimen relation and evaluates the $g_{\pi NN}(0)$ as $g_{\pi NN}(0) = \frac{M_n(\frac{g_A}{g_V})}{f_\pi}$ where the ration of the axial-vector coupling constant, to the vector coupling constant g_V is obtained in Table III, therefore we can obtain the value $g_{\pi NN}(0)$ as in Ref. [7] that we note the mesonic contributions in the $g_{\pi NN}(0)$ is increased about 26% in comparison with original model [8] (see Table III).

7. Conclusion

In present work, we examine the effect of coherent-pair approximation in the extended linear sigma model [9]. The coherent-pair approximation has some advantages in comparison with mean-field approximation that provides a systematic expansion method for the description of a boson field. In addition, it avoids assumptions like the hedgehog structure of the quark and pion fields.

From the results, we note that the coherent-pair approximation has been given a good description of the nucleon properties as nucleon mass, the charge radius of proton and sigma commutator are closed with data. The other nucleon properties are improved in comparison with previous calculations [8, 21] that change in range 20–30% and also reasonable agreement with data.

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