Vol. 40 (2009) ACTA PHYSICA POLONICA B No 9

COVARIANT DENSITY FUNCTIONAL THEORY: INCLUSIVE CHARGED CURRENT NEUTRINO–NUCLEUS REACTIONS*

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(Received July 10, 2009)

Covariant Density Functional Theory (CDFT) is used to investigate inclusive neutrino–nucleus cross-sections. The ground state of the even–even nucleus (N, Z) is obtained as the static solutions of the Relativistic Hartree–Bogoliubov (RHB) equations and the final states of the odd–odd nucleus $(N-1, Z+1)$ as well as the relevant transition probabilities are calculated in the Relativistic Quasiparticle Random Phase Approximation (RQRPA). The weak lepton–hadron interaction is expressed in the standard current–current form.

PACS numbers: 21.30.Fe, 21.60.Jz, 23.40.Bw, 25.30.–c

1. Introduction

At low energies neutrino–nucleus reactions depend sensitively on the details of structure of the nuclear ground states and the excited states [1]. This means that one needs a solution of the nuclear many-body problem that includes the strong and electromagnetic interactions. The exact solution of the nuclear many-body problem is only possible for very light nuclei. In all other cases one has to rely on approximations. Weak interaction rates at low energies have been analyzed employing a variety of microscopic approaches, principally in the frameworks of the shell model [2,3], the random phase approximation (RPA) [4], continuum RPA (CRPA) [5], hybrid models of CRPA

* Presented by P. Ring at the 45th Winter School in Theoretical Physics "Neutrino Interactions: From Theory to Monte Carlo Simulations", Lądek-Zdrój, Poland, February 2–11, 2009.

and shell model [6, 7]. Reliable prediction of weak interaction rates in nuclei necessitates a fully consistent description of the structure of ground states and multipole excitations. Among the relevant charge-exchange excitations, the isobaric analog state (IAS) and Gamow–Teller resonance (GTR) have been the subject of extensive experimental and theoretical studies. Much more limited are the data and theoretical predictions for properties of excitations of higher multipolarities at finite momentum transfer. In this work we analyze charged-current neutrino–nucleus reactions by employing a fully consistent microscopic approach based on relativistic energy density functionals. An essential advantage over most current approaches is the use of a single universal effective interaction in calculations of both ground-state properties and multipole excitations of nuclei in various mass regions of the chart of nuclides. Of particular interest for the present study are rates for neutrino–nucleus reactions in the low-energy range below 100 MeV, which play an important role in many astrophysical processes, including stellar nucleosynthesis. A quantitative description of nucleosynthesis of heavy elements during the r-process necessitates accurate predictions of neutrino–nucleus cross-sections not only in stable nuclei, but also in nuclei away from the valley of β-stability. Because nuclei are used as detectors for solar and supernovae neutrinos, as well as in neutrino oscillation experiments, it is important to describe the neutrino detector response in a consistent and fully microscopic theory. Finally, a quantitative estimate of neutrino–nucleus reaction rates will provide information relevant for feasibility studies and simulations of a low-energy beta beam facility, which could be used to produce neutrino beams of interest for particle physics, nuclear physics and astrophysics [8].

2. Neutrino–nucleus cross-sections

We consider the charged-current neutrino–nucleus reactions:

$$\nu_l +_Z X_N \rightarrow_{Z+1} X_{N-1} + l^- \,, \tag{1}$$

where l denotes the charged lepton (electron, muon). Detailed expressions for the reaction rates and the transition matrix elements can be found in Refs [9, 10]. The charged-current neutrino–nucleus cross-section reads

$$\left(\frac{d\sigma_\nu}{d\Omega}\right) = \frac{1}{(2\pi)^2} V^2 p_l E_l \sum_{\mathrm{lepton's\ spin}} \frac{1}{2J_i+1} \sum_{M_i M_f} |\langle f|H_{\mathrm{W}}|i\rangle|^2 \,, \tag{2}$$

where p_l and E_l are the momentum and the energy of the outgoing lepton, respectively. The Hamiltonian H_{W} of the weak interaction is expressed in the standard current–current form, and the transition matrix elements read

$$\langle f|H_{\mathrm{W}}|i\rangle = \frac{G}{\sqrt{2}}\, l^{\lambda} \int d^3r\; e^{-i\boldsymbol{q}\boldsymbol{r}} \langle f|\mathcal{J}_{\lambda}(\boldsymbol{r})|i\rangle\,. \tag{3}$$

The multipole expansion of the leptonic matrix element $l^{\lambda}e^{-i\boldsymbol{q}\boldsymbol{r}}$ determines the operator structure for the nuclear transition matrix elements [9], and the expression for the neutrino–nucleus cross-section. In the extreme relativistic limit (ERL), in which the energy of the outgoing lepton is considered much larger than its rest mass, the differential neutrino–nucleus cross-section takes the form

$$\begin{aligned}
\left(\frac{d\sigma_{\nu}}{d\Omega}\right)_{\mathrm{ERL}} &= \frac{2G_{\mathrm{F}}\cos^2\theta_c}{\pi}\frac{E_l^2}{2J_i+1} \\
&\times \left\{ \left(\frac{q^2}{2\boldsymbol{q}^2}\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2}\right) \sum_{J\geq 1} \left[|\langle J_f||\mathcal{T}_J^{\mathrm{MAG}}||J_i\rangle|^2 + |\langle J_f||\mathcal{T}_J^{\mathrm{EL}}||J_i\rangle|^2 \right] \right. \\
&\times \sin\frac{\theta}{2}\left(\frac{q^2}{\boldsymbol{q}^2}\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2}\right)^{\frac{1}{2}} \sum_{J\geq 1} 2\mathrm{Re}\langle J_f||\mathcal{T}_J^{\mathrm{MAG}}||J_i\rangle\langle J_f||\mathcal{T}_J^{\mathrm{EL}}||J_i\rangle^{*} \\
&\left.\times \cos^2\frac{\theta}{2}\sum_{J\geq 0} |\langle J_f||\mathcal{M}_J - \frac{q_0}{|\boldsymbol{q}|}\mathcal{L}_J||J_i\rangle|^2 \right\}\,,
\end{aligned} \tag{4}$$

where G_{F} is the Fermi constant for the weak interaction, θ_{c} is the Cabibbo's angle, θ denotes the angle between the incoming and outgoing leptons, the energy of the lepton in the final state is E_l, and the 4-momentum transfer is $q = (q_0, \boldsymbol{q})$. The nuclear transition matrix elements between the initial state and final state, correspond to the charge $\mathcal{M}_J$, the longitudinal $\mathcal{L}_J$, the transverse electric $\mathcal{T}_J^{\mathrm{EL}}$, and transverse magnetic $\mathcal{T}_J^{\mathrm{MAG}}$ multipole operators. These are expressed in terms of spherical Bessel functions, spherical harmonics, and vector spherical harmonics. Details are given in Refs [10,11]. They contain the standard set of form factors derived from the assumption of conserved vector current (CVC) [10,12].

The calculations of the neutrino–nucleus cross-section in Eq. (4) requires therefore the evaluation of the transition matrix elements of the various operators between the initial and final states. The initial state is the ground state of the even–even nucleus ${}_ZX_N$ and the final states are the ground state and the various excited states of the final nucleus ${}_{Z+1}X_{N-1}$. Here we use a consistent microscopic theoretical framework for the evaluation of these matrix elements, covariant density functional theory [13]. This will be discussed in the next section.

3. Covariant density functional theory

3.1. The relativistic energy density functional

Covariant density functional theory uses the Walecka model [14] as a vehicle to implement a Lorentz invariant framework for the formulation of the density functional. In this model the nucleus is described as a system of Dirac nucleons coupled to the exchange mesons and the electromagnetic field through an effective Lagrangian. The isoscalar scalar σ-meson, the isoscalar vector ω-meson, and the isovector vector ρ-meson build the minimal set of meson fields that together with the electromagnetic field is necessary for a quantitative description of bulk and single-particle nuclear properties [14–16]. The model is defined by the Lagrangian density

$$\mathcal{L} = \mathcal{L}_N + \mathcal{L}_m + \mathcal{L}_{\text{int}} \tag{5}$$

containing the Lagrangian of free nucleons, of free mesons and the minimal set of interaction terms

$$\mathcal{L}_{\text{int}} = -\bar{\psi}\Gamma_\sigma \sigma\psi - \bar{\psi}\Gamma_\omega^\mu \omega_\mu\psi - \bar{\psi}\vec{\Gamma}_\rho^\mu \vec{\rho}_\mu\psi - \bar{\psi}\Gamma_e^\mu A_\mu\psi \tag{6}$$

containing the vertices

$$\Gamma_\sigma = g_\sigma\,, \qquad \Gamma_\omega^\mu = g_\omega\gamma^\mu\,, \qquad \vec{\Gamma}_\rho^\mu = g_\rho\vec{\tau}\gamma^\mu\,, \qquad \Gamma_e^\mu = q\gamma^\mu\,, \tag{7}$$

with the coupling constants g_σ, g_ω, g_ρ and q (e or 0 for protons or neutrons). Modern versions of covariant density functional theory use density dependent coupling constants $g_\sigma(\rho)$, $g_\omega(\rho)$, and $g_\rho(\rho)$ where the density dependence is carefully adjusted to properties of nuclear matter and finite nuclei [17].

Neglecting retardation effects for the meson fields, which is well justified because of the large meson masses, and using the no-sea approximation, we can express the energy derived from the Lagrangian (5) as a functional of the relativistic single particle density matrix

$$\hat{\rho}\left(\boldsymbol{r},\boldsymbol{r}',t\right) = \sum_{i=1}^{A} \left|\psi_i(\boldsymbol{r},t)\right\rangle \left\langle\psi_i\left(\boldsymbol{r}',t\right)\right|\,, \tag{8}$$

and of various meson fields $\phi_m(\boldsymbol{r},t) \equiv \{\sigma, \omega^\mu, \vec{\rho}^\mu, A^\mu\}$. We thus obtain a covariant density functional

$$E_{\text{RMF}}[\hat{\rho},\phi] = \int d^3r\mathcal{H}(\boldsymbol{r},t) \tag{9}$$

with the energy density

$$\mathcal{H}(\boldsymbol{r},t) = \mathcal{H}_{\text{kin}}(\boldsymbol{r},t) \mp \sum_m \tfrac{1}{2}\left(-\phi_m\Delta\phi_m + m_m^2\phi_m^2\right) + \text{Tr}\left[(\Gamma_m\phi_m)\hat{\rho}(\boldsymbol{r},t)\right]\,, \tag{10}$$

where trace operations includes a summation over the Dirac indices and the $(-/+)$ sign holds for scalar/vector fields. The kinetic energy density is given by

$$\mathcal{H}_{\rm kin}(\boldsymbol{r},t) = {\rm Tr}\left[\left(-i\boldsymbol{\alpha}\boldsymbol{\nabla}+\beta M\right)\hat{\rho}\left(\boldsymbol{r},\boldsymbol{r}',t\right)\right]|_{\boldsymbol{r}=\boldsymbol{r}'} \tag{11}$$

and $\hat{\rho}(\boldsymbol{r},t)=\hat{\rho}(\boldsymbol{r},\boldsymbol{r},t)$ is the local part of $\hat{\rho}$. The energy density functional in Eq. (9) depends on the meson fields, which obey the equations of motion

$$\left(-\Delta+m_m^2\right)\phi_m(\boldsymbol{r},t) = \mp{\rm Tr}\left[\mathit{\Gamma}_m\hat{\rho}(\boldsymbol{r},t)\right], \tag{12}$$

and, therefore, depend implicitly also on the density $\hat{\rho}$. In this way we end up with a relativistic energy density functional $E[\hat{\rho}]$ depending only on the single particle density matrix $\hat{\rho}(t)$ in Eq. (8).

The equations of motion are obtained from the classical variational principle

$$\delta\int_{t_1}^{t_2} dt\left\{\langle\Phi|i\partial_t|\Phi\rangle - E\left[\hat{\rho}\right]\right\} = 0 \tag{13}$$

and have the form of a time-dependent RMF equation

$$i\partial_t\hat{\rho} = \left[\hat{h}(\hat{\rho}),\hat{\rho}\right], \tag{14}$$

where the single-particle Hamiltonian $\hat{h}(\hat{\rho})$ is of Dirac form and is obtained as the functional derivative of the energy with respect to the single-particle density matrix $\hat{\rho}$

$$\hat{h} = \frac{\delta E}{\delta\hat{\rho}}\,. \tag{15}$$

3.2. The response of the system to the weak field of an incoming neutrino

Starting with the energy density functional (9) the ground state $|0\rangle$ of the even–even system is derived as the static solution $\hat{\rho}_0$ of the DFT-equations (14):

$$\left[\hat{h}_0,\hat{\rho}_0\right] = 0\,. \tag{16}$$

The incoming neutrino induces transitions to the states $|\mu\rangle$ in the neighboring $(Z+1,N-1)$ nucleus and we calculate the transition matrix elements $|\langle\mu|\mathcal{O}_J|0\rangle|^2$ of the electric and magnetic multipole operators $\mathcal{O}_J$ in Eq. (4) in linear response approximation. This means, we consider the nuclear many-body system in a time-dependent external field characterized by the operator $\mathcal{O}_J$ oscillating with the frequency ω and solve the time-dependent relativistic DFT-equations (14) in the limit of small amplitudes.

The cross-section is determined by the strength functions:

$$S_{\mathcal{O}}(\omega) = \sum_{\mu} |\langle\mu|\mathcal{O}|0\rangle|^2 \delta(\omega - E_\mu) = -\frac{1}{\pi}\mathrm{Im} \sum_{\alpha\beta\alpha'\beta'} \mathcal{O}_{\alpha\beta^*} R_{\alpha\beta\alpha'\beta'}(\omega) \mathcal{O}_{\alpha'\beta'} \,, \quad (17)$$

for the various operators $\mathcal{O}$ and $R_{\alpha\beta\gamma\delta}(\omega)$ is the response function defined as:

$$R_{\alpha\beta\alpha'\beta'}(\omega) = \sum_{\mu} \left\{ \frac{\langle 0|a^+_\beta a_\alpha|\mu\rangle\langle\mu|a^+_{\alpha'} a_{\beta'}|0\rangle}{\omega - E_\mu + E_0 + i\eta} - \frac{\langle\mu|a^+_\beta a_\alpha|0\rangle\langle 0|a^+_{\alpha'} a_{\beta'}|\mu\rangle}{\omega + E_\mu - E_0 + i\eta} \right\} . \quad (18)$$

It contains the transition densities:

$$\delta\hat{\rho}^\mu = \langle 0|a^+ a|\mu\rangle \,, \quad (19)$$

which can be deduced by Fourier transformation from the time-dependent density matrix $\hat{\rho}(t) = \hat{\rho}_0 + \delta\hat{\rho}(t)$ in Eq. (8). In the small amplitude limit of Eq. (14) we obtain, therefore, the linearized Bethe Salpeter equation for the response function

$$R(\omega) = R^0(\omega) + R^0(\omega) V R(\omega) \,. \quad (20)$$

The residual interaction is found as the second derivative of the energy density functional (9) with respect to the density matrix:

$$V = \frac{\delta^2 E[\hat{\rho}]}{\delta\hat{\rho}\delta\hat{\rho}} \,. \quad (21)$$

and $R^0(\omega)$ is the response of the free system, considering only uncorrelated (ph)-excitations in Eq. (18) and neglecting the residual interaction (21). More details are given in Refs [18, 19].

4. Applications: Cross-sections for neutrino detector response

The theoretical framework described in the previous two sections has been applied in studies of charged-current neutrino reaction rates with target nuclei of arbitrary mass. The inclusive cross-sections, summing up the contributions from transitions to all possible final states, are given as functions of neutrino energy. One of the most extensively studied neutrino–nucleus reactions is $^{12}\mathrm{C}(\nu_e, e^-)^{12}\mathrm{N}$. This reaction is particularly important because $^{12}\mathrm{C}$ is used in liquid scintillator detectors. In order to illustrate the contributions of different multipole excitations, we plot in Fig. 1 the inclusive cross-section for the $^{12}\mathrm{C}(\nu_e, e^-)^{12}\mathrm{N}$ the reaction as function of the neutrino energy, obtained by successively increasing the maximal allowed angular momentum in the sum over J in Eq. (4): from $J_{\max} = 0^\pm$ to $J_{\max} = 7^\pm$.

One notices that the largest contributions arise from $J = 1^{\pm}$ and $J = 2^{\pm}$, and that the contribution of higher multipolarities gradually decreases. In fact, in this figure one cannot distinguish the cross-sections calculated with $J_{\max} = 6^{\pm}$ and $J_{\max} = 7^{\pm}$, for the whole interval of neutrino energies.

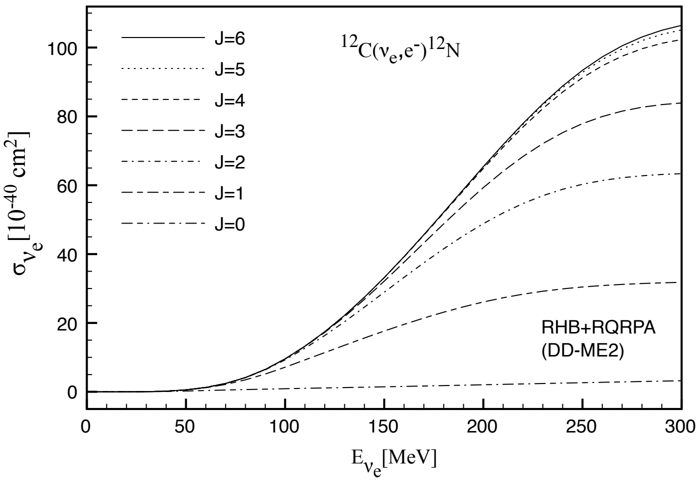

Fig. 1. The RHB plus PN-RQRPA inclusive neutrino–nucleus cross-sections for the $^{12}\mathrm{C}(\nu_e, e^-)^{12}\mathrm{N}$ reaction. The different curves correspond to cross-sections evaluated by successively increasing the maximal allowed angular momentum in the sum over J in Eq. (4): from $J_{\max} = 0^{\pm}$ to $J_{\max} = 7^{\pm}$.

5. Concluding remarks

Detailed microscopic calculations of charged-current and neutral-current neutrino–nucleus reaction rates are of crucial importance for models of neutrino oscillations, detection of supernova neutrinos, and studies of the r-process nucleosynthesis. Covariant density functional theory with a relativistic Hartree–Bogoliubov (RHB) description of nuclear ground states and a quasiparticle RPA (QRPA) treatment of the excited states is applied for a consistent microscopic description of neutrino–nucleus cross-sections. Since it is based on the self-consistent mean-field approach to nuclear structure, the model can be applied to neutrino reactions with target nuclei of arbitrary mass throughout the chart of nuclides. By employing universal effective interactions, with parameters adjusted to global nuclear properties, the calculation of neutrino–nucleus cross-sections is essentially parameter free.

Except at relatively low neutrino energies $E \leq 30$ MeV, for which the reactions are dominated by transitions to IAS and GTR states, at higher energies the inclusion of spin-dipole transitions, and also excitations of higher multipolarities, is essential for a quantitative description of neutrino–nucleus cross-sections. The results for the test cases are in good agreement with the few available data, and with the cross-sections calculated in the shell model for reactions on light nuclei. The advantage of the RHB plus PN-RQRPA model over the shell model approach is, of course, the possibility of performing calculations for higher neutrino energy, for reactions on heavier nuclei, and for reactions on nuclei in regions far from stability.

This work has been supported in part by MZOS — project 1191005-1010, and by the DFG Cluster of Excellence Origin and Structure of the Universe (`www.universe-cluster.de`).

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