QUASI-ELASTIC CHARGED-CURRENT AND NEUTRAL-CURRENT NEUTRINO–NUCLEUS SCATTERING IN A RELATIVISTIC APPROACH*

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Relativistic models developed for the exclusive and inclusive Quasi-Elastic (QE) electron scattering have been extended to Charged-Current (CC) and Neutral-Current (NC) ν-nucleus scattering. The results of different descriptions of Final-State Interactions (FSI) are compared.

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1. Introduction

Several decades of experimental and theoretical work on electron scattering have provided a wealth of information on nuclear structure and dynamics [1]. In these experiments the electron is the probe whose properties are clearly specified, and the nucleus is the target whose properties are under investigation. Additional information on nuclear properties is available from ν-nucleus scattering. Neutrinos can excite nuclear modes unaccessible in electron scattering, can give information on the hadronic weak current and on the strange form factors of the nucleon. Although of great interest, such studies are not the only aim of many neutrino experiments, which are better devised for a precise determination of neutrino properties. In neutrino oscillation experiments nuclei are used to detect neutrinos and a proper analysis of data requires that the nuclear response to neutrino interactions is well under control and that the unavoidable theoretical uncertainties on nuclear effects are reduced as much as possible.

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In recent years different models developed and successfully tested in comparison with electron scattering data have been extended to ν-nucleus scattering. Although the two situations are different, electron scattering is the best available guide to determine the prediction power of a nuclear model. Nonrelativistic and relativistic models have been developed to describe nuclear effects with different approximations. They can be considered as alternative models, but only a relativistic approach is able to account for all the effects of relativity in a complete and consistent way. Relativity is important at all energies, in particular at high energies, and in the energy regime of many neutrino experiments a relativistic approach is required.

Relativistic models for the exclusive and inclusive electron and neutrino scattering in the QE region [2–5] are presented in this contribution. In the QE region the nuclear response is dominated by one-nucleon knockout processes, where the probe interacts with a quasi-free nucleon that is emitted from the nucleus with a direct one-step mechanism and the remaining nucleons are spectators. In electron scattering experiments the outgoing nucleon can be detected in coincidence with the scattered electron. In the exclusive $(e, e'p)$ reaction the residual nucleus is left in a specific discrete eigenstate and the final state is completely specified. In the inclusive (e, e') scattering the outgoing nucleon is not detected and the cross-section includes all the available final nuclear states.

For an incident (anti)neutrino NC and CC scattering can be considered

$$\begin{aligned} &\nu(\bar{\nu}) + A \;\rightarrow\; \nu'(\bar{\nu}') + N + (A-1)\,, && \text{NC} \\ &\nu(\bar{\nu}) + A \;\rightarrow\; l^-(l^+) + p(n) + (A-1)\,. && \text{CC} \end{aligned}$$

In NC scattering only the emitted nucleon can be detected and the cross-section is integrated over the energy and angle of the final lepton. Also the state of the residual $(A-1)$-nucleus is not determined and the cross-section is summed over all the available final states. The same situation occurs for the CC reaction if only the outgoing nucleon is detected. The cross-sections are therefore semi-inclusive in the hadronic sector and inclusive in the leptonic one and can be treated as an $(e, e'p)$ reaction where only the outgoing proton is detected. The exclusive CC process where the charged final lepton is detected in coincidence with the emitted nucleon can be considered as well. The inclusive CC scattering where only the charged lepton is detected can be treated with the same models used for the inclusive (e, e') reaction.

For all these processes the cross-section is obtained in the one-boson exchange approximation from the contraction between the lepton tensor, that depends only on the lepton kinematics, and the hadron tensor $W^{\mu\nu}$, that contains the nuclear response and whose components are given by products of the matrix elements of the nuclear current J^μ between the initial and final

nuclear states, *i.e.*,

$$W^{\mu\nu} = \sum_f \langle \Psi_f \mid J^\mu(\boldsymbol{q}) \mid \Psi_i \rangle \langle \Psi_i \mid J^{\nu\dagger}(\boldsymbol{q}) \mid \Psi_f \rangle \delta(E_i + \omega - E_f) \,, \qquad (1.1)$$

where ω and $\boldsymbol{q}$ are the energy and momentum transfer, respectively. Different but consistent models to calculate $W^{\mu\nu}$ in QE electron and ν-nucleus scattering are outlined in the next sections.

2. Exclusive one-nucleon knockout

Models based on the Relativistic Distorted-Wave Impulse Approximation (RDWIA) have been developed [2, 6, 7] to describe the exclusive reaction where the outgoing nucleon is detected in coincidence with the scattered lepton and the residual nucleus is left in a discrete eigenstate n. In RDWIA the amplitudes of Eq. (1.1) are obtained in a one-body representation as

$$\left\langle \chi^{(-)} \mid j^\mu(\boldsymbol{q}) \mid \varphi_n \right\rangle \,, \qquad (2.1)$$

where $\chi^{(-)}$ is the single-particle (SP) scattering state of the emitted nucleon, φ_n the overlap between the ground state of the target and the final state n, *i.e.*, a SP bound state, and j^μ the one-body nuclear current. In the model the SP bound and scattering states are consistently derived as eigenfunctions of a Feshbach-type optical potential [1,2]. Phenomenological ingredients are adopted in the calculations. The bound states are Dirac–Hartree solutions of a Lagrangian, containing scalar and vector potentials, obtained in the framework of the relativistic mean-field theory [8]. The scattering state is calculated solving the Dirac equation with relativistic energy-dependent complex optical potentials [9]. RDWIA models have been quite successful in describing a large amount of data for the exclusive $(e, e'p)$ reaction [1,2,6,7].

3. Semi-inclusive neutrino–nucleus scattering

The transition amplitudes of the NC and CC processes where only the outgoing nucleon is detected are described as the sum of the RDWIA amplitudes in Eq. (2.1) over the states n. In the calculations [5] a pure Shell-Model (SM) description is assumed, *i.e.*, n is a one-hole state and the sum is over all the occupied SM states. FSI are described by a complex optical potential whose imaginary part reduces the cross-section by $\sim 50\%$. A similar reduction is obtained in the RDWIA calculations for the exclusive one-nucleon knockout. The imaginary part accounts for the flux lost in a specific channel towards other channels. This approach is conceptually correct for an exclusive reaction, where only one channel contributes, but it would be wrong

for the inclusive scattering, where all the channels contribute and the total flux must be conserved. For the semi-inclusive process where an emitted nucleon is detected, some of the reaction channels which are responsible for the imaginary part of the potential are not included in the experimental cross-section and, from this point of view, it is correct to include the absorptive imaginary part. Numerical examples in different kinematics are given in [5].

4. Inclusive lepton–nucleus scattering

In the inclusive scattering where only the outgoing lepton is detected FSI are treated in the Green's Function Approach (GFA) [3,4,10]. In this model the components of the hadron tensor are written in terms of the SP optical model Green's function. This is the result of suitable approximations, such as the assumption of a one-body current and subtler approximations related to the IA. The explicit calculation of the SP Green's function is avoided by its spectral representation, which is based on a biorthogonal expansion in terms of a non Hermitian optical potential $\mathcal{H}$ and of its Hermitian conjugate $\mathcal{H}^\dagger$. Calculations require matrix elements of the same type as the RDWIA ones in Eq. (2.1), but involve eigenfunctions of both $\mathcal{H}$ and $\mathcal{H}^\dagger$, where the different sign of the imaginary part gives in one case an absorption and in the other case a gain of flux. Thus, in the sum over n the total flux is redistributed and conserved. The GFA guarantees a consistent treatment of FSI in the exclusive and in the inclusive scattering and gives a good description of (e, e') data [3].

An example is displayed in Fig. 1, where the $^{16}\mathrm{O}(\nu_\mu, \mu^-)$ cross-sections calculated in GFA are compared with the results of the Relativistic Plane Wave IA (RPWIA), where FSI are neglected. The cross-sections obtained when only the real part of the Relativistic Optical Potential (rROP) is retained and the imaginary part is neglected are also shown in the figure. This approximation conserves the flux, but it is conceptually wrong because the optical potential has to be complex owing to the presence of inelastic channels. The partial contribution given by the sum of all the integrated exclusive one-nucleon knockout reactions, also shown in the figure, is much smaller than the complete result. The difference is due to the spurious loss of flux produced by the absorptive imaginary part of the optical potential.
The analysis of data requires a precise knowledge of ν-nucleus cross-sections, where theoretical uncertainties on nuclear effects are reduced as much as possible. To this aim, it is important to check the consistency of different models and the validity of the approximations. The results of the relativistic models developed by our group and the Madrid–Sevilla group for the inclusive electron scattering are compared in [11]. An example is shown in Fig. 2 for the $^{12}\mathrm{C}(e, e')$ cross-sections calculated with different descriptions for FSI:

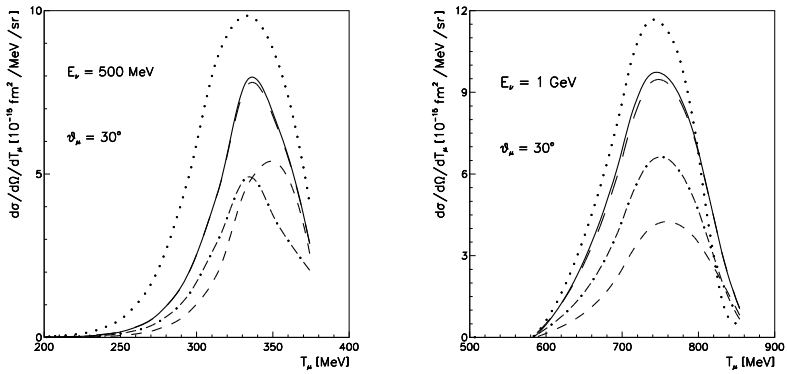


Fig. 1. The cross-sections of the $^{16}\mathrm{O}(\nu_\mu, \mu^-)$ reaction for $E_\nu = 500$ and 1000 MeV at $\theta_\mu = 30^\circ$ as a function of the muon kinetic energy T_μ. Results for GFA (solid) RPWIA (dotted), rROP (long-dashed) are compared. The dot-dashed lines give the contribution of the integrated exclusive reactions with one-nucleon emission. Short-dashed lines give the GFA results for the $^{16}\mathrm{O}(\bar{\nu}_\mu, \mu^+)$ reaction.

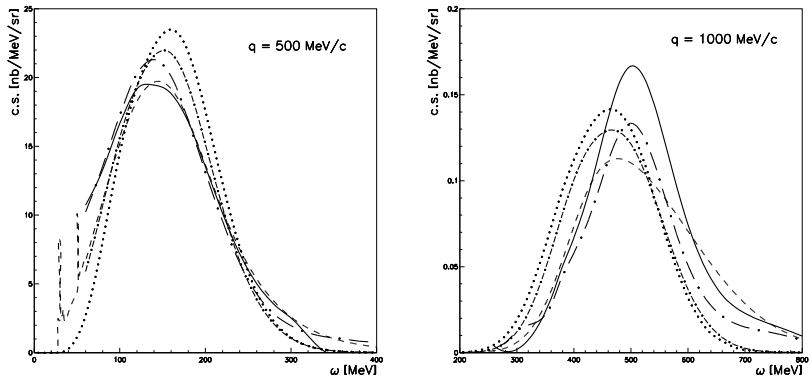


Fig. 2. The cross-sections of the $^{12}\mathrm{C}(e, e')$ reaction for an incident electron energy of 1 GeV, $q = 500$ (left panel) and 1000 MeV/c (right panel), with RPWIA (dotted), rROP (dot-dashed), RMF (dashed), and GFA with two optical potentials, EDAD1 (GF1 solid) and EDA2 (GF2 dot-long-dashed) [9].

RPWIA, rROP, GFA (with two parametrizations of the optical potential), and the Relativistic Mean Field (RMF) [12], where the scattering wave functions are calculated with the same real potential used for the initial bound states. The differences between RMF and GFA increase with q: they are small at $q = 500$ MeV/c and significant at $q = 1000$ MeV/c. The RMF cross-section shows an asymmetry, with a long tail extending towards higher values of ω. A less significant asymmetry is obtained for both GFA cross-sections,

that at $q = 1000\,\mathrm{MeV}/c$ are higher than the RMF one in the maximum region. The enhancement is different for the two optical potentials. The behavior of the RMF and GFA results as a function of q and ω can be understood if we consider that RMF is based on the use of strong energy-independent scalar and vector real potentials, while GFA on a complex energy-dependent optical potential. Different values of q and ω are sensitive to the behavior of the optical potential at different energies, and higher values correspond to higher energies. The GFA results are consistent with the general behavior of the optical potentials and are basically due to their imaginary part. Different parameterizations give similar real terms and the rROP cross-sections are practically insensitive to the choice of the optical potential. The real part decreases increasing the energy and the rROP result approaches the RPWIA one for large values of ω. In contrast, the imaginary part has its maximum strength around 500 MeV and is sensitive to the parameterization of the ROP. The imaginary part gives large differences between GFA and rROP in Fig. 2, while only negligible differences are obtained in the different situation and kinematics of Fig. 1.

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