PROPAGATING ν-INTERACTION UNCERTAINTIES VIA EVENT REWEIGHTING*

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We present an event reweighting scheme for propagating neutrino cross-section and intranuclear hadron transport model uncertainties which has been developed for the GENIE-based [C. Andreopoulos *et al.*, `arXiv:0905.2517[hep-ph]`] neutrino physics simulations. We discuss the motivations, implementation and validation of the scheme and show an example application where it is used to evaluate the associated systematic uncertainties for neutral current π^0 production.

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1. Introduction

Neutrino interaction Monte Carlo (MC) generators are an integral part of many modern neutrino experiments. The complex and multiplicative nature of the physics they model, combined with a need to simulate events within detailed realistic detector geometries, places a high demand on computational resources, and the production time for large MC-data sets representing whole lifetimes of an experiment can be significant[1].

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[1] A recent GENIE MC production run for the T2K [2] experiment took 200 CPU's ~ 3 weeks to generate a set of data corresponding to ~ 5 years of experimental running.

Neutrino generators have many input parameters defining a large physics configuration space. Uncertainties on the nominal values[2] of these input parameters propagate into the observable distributions we extract from the MC simulations. Whether using the MC data within the context of an oscillation fit or as a tool to tune, validate or measure cross-sections, it is vital to understand and take into account these sources of systematic uncertainty.

We have developed a reweighting scheme that allows one to obtain the physics MC predictions for another point in the physics configuration space without re-running the time consuming MC. This opens up many possibilities such as generating error envelopes that fully take into account correlations[3] between MC input parameters or the use of these input parameters as nuisance terms in fits to data. The reweighting scheme described here is geared towards ν_μ–CC interactions in the few GeV region. It has been developed in the context of the T2K [2] physics analysis, and the reweighting methods will soon be made available with the GENIE neutrino generator.

2. Reweighting schemes

At present the reweighting scheme can be thought of as two separate schemes. The first is for evaluating cross-section systematics and the second deals with intranuclear hadron transport systematics. We now describe both schemes and their validation and then in section 3 we present an example application of relevance to current neutrino experiments. Full details of both schemes can be found in the GENIE users manual[4].

2.1. Neutrino cross-section systematics

Of the numerous GENIE cross-section model parameters we are considering changes to parameters controlling the rate of Quasi-Elastic (QEL) and Resonant (RES) interactions and of the 1π and 2π non-resonant background in the resonance region. The most important of these parameters are shown in the first part of Table I. Full details of the physics models used in GENIE can be found in [1].

The neutrino event weight, w_σ^{evt}, to account for changes in physics parameters controlling neutrino cross-sections is calculated as:

[2] Typically these nominal values are based on fits to global data or previous experimental measurements and may have large uncertainties.

[3] This requires scanning over many points in the physics configuration space, over the volume defined by the uncertainties on the input parameters. A conservative choice of 5 input parameters scanned over 5 points within their systematic uncertainties would require regeneration of 3125 MC-data sets.

[4] Available at `http://www.genie-mc.org/`

$$w_{\sigma}^{\text{evt}} = \frac{(d^n\,\sigma'_{\nu}/dK^n)}{(d^n\,\sigma_{\nu}/dK^n)}\,, \tag{2.1}$$

where $d^n\sigma/dK^n$ is the nominal differential cross-section for the process at hand and $d^n\sigma'/dK^n$ is the differential cross-section computed using the modified input physics parameters. For each event the differential cross-section needs only to be evaluated at a single point in the kinematical phase space. This avoids MC selection over the entire phase space making it much faster than full event generation.

In order to reweight an event it is essential that the reweighting scheme matches exactly the physics used in the original simulation, otherwise the denominator of Eq. (2.1) will not cancel with the original probability for producing that event. For this reason the reweighting scheme reconfigures and accesses the cross-section models through GENIE.

TABLE I

First part shows cross-section reweighting knobs: Not shown are 14 other scaling factors for non-RES background for all (ν,$\bar{\nu}$)+(CC,NC)+(1π,2π)+(p, n) combinations. Second part shows intranuclear hadron transport reweighting knobs: only 4 of the 5 fates parameters can be varied freely due to the unitary requirement with the 5th one acting as a cushion term. By default x_{el} is the cushion term.

Phys. param.	Short description	Def. value	Error (1σ)
M_A^{QEL}	QEL axial mass	0.99 GeV	$\sim 15\%$
M_V^{QEL}	QEL vector mass	0.84 GeV	$\sim 5\%$
M_A^{RES}	RES axial mass	1.12 GeV	$\sim 20\%$
M_V^{RES}	RES vector mass	0.84 GeV	$\sim 5\%$
$R^{\text{bkg}}_{\nu p;CC1\pi}$	Scales non-RES bkg for νp $CC1\pi$	0.1	$\sim 50\%$
$R^{\text{bkg}}_{\nu p;CC2\pi}$	Scales non-RES bkg for νp $CC2\pi$	1.0	$\sim 50\%$
$\vdots$	Plus 14 more non-RES bkg scales	$\vdots$	$\vdots$
x^N_{mfp}	Tweaks nucleon mean free path	0.0	10%
x^N_{cex}	Tweaks nucleon charge exchange prob.	0.0	10%
x^N_{el}	Tweaks nucleon elastic reaction prob.	0.0	10%
x^N_{inel}	Tweaks nucleon inelastic reaction prob.	0.0	10%
x^N_{abs}	Tweaks nucleon absorption prob.	0.0	10%
x^N_{π}	Tweaks nucleon π-production prob.	0.0	10%
$\vdots$	Plus another 5 similar but for π's	$\vdots$	$\vdots$

2.2. Intranuclear rescattering systematics

Hadrons produced in the nuclear environment may rescatter on their way out of the nucleus. These re-interactions can significantly modify the observable distributions seen by experiments which makes understanding the associated systematics important. Typically neutrino generators handle intranuclear hadron transport using cascade MC. Each simulation step gives rise to a large number of outcomes and the probabilities of these outcomes are conditional upon the hadron transport history up to that point. This makes it difficult to evaluate the probability for a generated multi-particle final state, given a primary hadronic multi-particle system, without resorting to a MC method. As a result it is generally not possible to build comprehensive reweighting schemes for intranuclear hadron-transport simulations. GENIE's INTRANUKE/hA effective model [1] is unique in that regard due to the simplicity of the simulation strategy which makes it amenable to reweighting. Thus a full systematic analysis of the model is possible providing a unique tool in the analysis of neutrino data.

INTRANUKE/hA is a data driven model anchored to a range of hadron–nucleus and hadron–nucleon data. During event generation, for each hadron being propagated within the nuclear environment its rescattering probability, $P^h_{\rm rescat}$ (and survival probability $P^h_{\rm surv}$), is calculated as

$$P^h_{\rm rescat} = 1 - P^h_{\rm surv} = 1 - \int e^{-r/\lambda^h(\vec{r},h,E_h)}\, dr\,, \tag{2.2}$$

where λ^h is the mean free path and the integral is evaluated along the hadron trajectory. The mean free path is a function of the hadron type, h, the hadron energy, E_h, and its position, $\vec{r}$, within the target nucleus[5].

Once it is determined that a particular hadron is to be rescattered, then a host of scattering modes are available to it[6]. We will refer to these modes as the *hadron fates*. They are: elastic, inelastic, charge exchange and absorption/nuclear knockout. For a detailed description of INTRANUKE/hA see [1].

We consider two types of systematic parameters: Those that change the overall rescattering probability (mean free path), and those that modify the relative probability of various rescattering outcomes (fates). They are treated separately for pions and nucleons and are shown in the second part of Table I. In a similar manner to cross-section reweighting the weight for a hadron, reflecting a modified set of input parameters, is evaluated as the ratio of the new modified probability to the old probability (corresponding to the nominal set of input parameters). Thus a hadron h receives two weights:

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[5] This is where the hadron–nucleon data is used.

[6] These are based on hadron–nucleus scattering data.

one corresponding to the rescattering probability and one corresponding to the particular fate chosen for that hadron, $w^h_{\rm mfp}$ and $w^h_{\rm fate}$, respectively.

In the case of rescattering-rate reweighting the scheme allows the mean free path, λ^h, for a hadron type h to be modified in terms of its corresponding error, $\delta\lambda^h$:

$$\lambda^h \to \lambda^{h\prime} = \lambda^h \left(1 + x^h_{\rm mfp}\,\frac{\delta\lambda^h}{\lambda^h}\right) , \tag{2.3}$$

where $\lambda^{h\prime}$ is the modified mean free path and $x^h_{\rm mfp}$ is a tweaking knob. The single hadron rescattering-rate weight is given by

$$w^h_{\rm mfp} = \begin{cases} \dfrac{1 - P^{h\prime}_{\rm surv}}{1 - P^h_{\rm surv}} & \text{if } h \text{ re-interacts}\,, \\[2ex] \dfrac{P^{h\prime}_{\rm surv}}{P^h_{\rm surv}} & \text{if } h \text{ escapes}\,, \end{cases} \tag{2.4}$$

where $P^h_{\rm surv}$ is the hadron survival probability corresponding to mean free path λ^h, and $P^{h\prime}_{\rm surv}$ is the hadron survival probability corresponding to a tweaked mean free path $\lambda^{h\prime}$.

For fate reweighting the probability for a given fate f is $P^h_f = \sigma^{hA}_f/\sigma^{hA}_{\rm total}$, where σ^{hA}_f is the hadron–nucleus cross-section[7] for that particular fate and $\sigma^{hA}_{\rm total}$ is the total hadron–nucleus cross-section. The hadron–nucleus cross-section for a particular fate is also modified in terms of its corresponding error, $\delta\sigma^{hA}_f$:

$$\sigma^{hA}_f \to \sigma'^{hA}_f = \sigma^{hA}_f \left(1 + x^h_f\,\frac{\delta\sigma^{hA}_f}{\sigma^{hA}_f}\right) , \tag{2.5}$$

where x^h_f is the fate tweaking knob. It follows that the single-hadron fate weight is

$$w^h_{\rm fate} = \sum_f \delta_{f;f'}\, x^h_f\, \frac{\delta\sigma^{hA}_f}{\sigma^{hA}_f} , \tag{2.6}$$

where f runs over all possible fates, $f\prime$ is the actual fate for that hadron, as determined during the simulation, and $\delta_{f;f'}$ is a factor which is 1 if $f = f'$ and 0 otherwise. Not all 5 hadron fates may be tweaked simultaneously. Since the sum of all fractions should add up to 1 then, at most, only 4 out of 5 fates may be tweaked directly. The 5th fate (*cushion* term) is adjusted automatically to conserve the sum. The choice of which fate to act as a cushion term is configurable.

[7] These cross-sections are a function of the hadron kinetic energy.

An important difference to the cross-section reweighting is that the choice of how to weight each hadron depends critically on its intranuclear transport history. Consider the case where a neutrino event has 2 primary hadrons, h_1 and h_2, one of which (h_1) re-interacts while the other (h_2) escapes. Had the mean free path been larger than the one used in the simulation (and therefore, had the the interaction probability been lower) then h_1's history would have been more unlikely while, on the other hand, h_2's history would have been more likely. Therefore, in order to account for an increase in mean free path, h_1 has to be weighted down while h_2 has to be weighted up (and vice versa for a mean free path decrease).

So far we have described the calculation of single-hadron weights taking into account the effect that modified hadron–nucleon and hadron–nucleus cross-sections would have had on that hadron. The total single-hadron weight is $w^h = w^h_{\mathrm{mfp}}\, w^h_{\mathrm{fate}}$ and the corresponding hadron transport (HT) related weight for a neutrino interaction event, $w^{\mathrm{evt}}_{\mathrm{HT}}$, is given by the product of single-hadron weights $w^{\mathrm{evt}}_{\mathrm{HT}} = \prod_j w^h_j$.

An important point worth mentioning is that we expect the inclusive leptonic distributions to be unaffected by changes to the hadron transport parameters. To an observer who is blind to the hadronic system emerging from the nucleus, and measures only the primary lepton, one can easily assert that, from the perspective of that observer, the hadron transport reweighting scheme should have no effect on the leptonic system characteristics of samples[8]. This means we expect the shape and normalization (this will be referred to as the unitary constraint and is the same as requiring that the average weight is one) of the leptonic distributions to remain unchanged for intranuclear rescattering reweighting.

2.3. Validation

To validate the schemes a sample generated with the nominal set of physics parameters (*'nominal'* sample) is reweighted to a tweaked set of physics parameters (*'tweak_reweighted'* sample). This is then compared to another sample, for which GENIE itself was reconfigured, generated using the tweaked set of physics parameters (*'tweak_generated'* sample). As the goal of event reweighting is to emulate what the physics simulation would have produced had the physics assumptions been different then the validity of the scheme is determined entirely on the basis of the agreement between *'tweak_reweighted'* and *'tweak_generated'* samples.

We now show the results of such a validation. In Fig. 1(a) the cross-section parameter M_A^{QEL} has been set to +15% above its nominal value.

[8] As long as they have not been selected using hadronic system characteristics.

The outgoing lepton spectrum is plotted and, as expected, increasing M_A^{QEL} increases the total rate as well as changing the shape of the distribution. In Fig. 1(b) the intranuclear rescattering parameters x^{π}_{abs} and x^{N}_{abs} have been set to +10% above their nominal values. The effect on the momentum distributions for the final state nucleons is shown. As expected increasing the cross-section for nuclear absorption increases the number of nucleons in the final state[9]. Both of these figures show very good agreement between *'tweak_reweighted'* and *'tweak_generated'* samples.

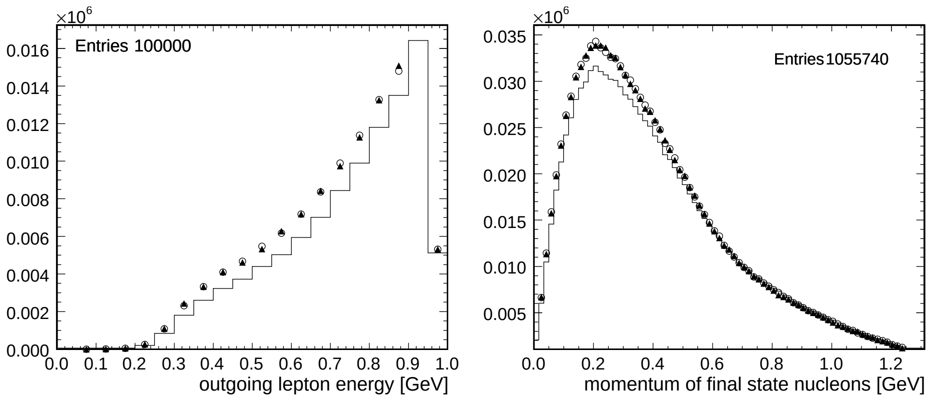

Fig. 1. Validation plots showing nominal (line), regenerated (filled triangle) and reweighted (circle) samples. Errors are statistical and similar in size to markers. (a) Lepton energy with M_A^{QEL} at +15% of nominal. (b) Momentum of final state nucleons with x^{N}_{abs} and x^{π}_{abs} at +10% of nominal.

Other important validation tests for the hadron transport reweighting scheme included checking that the unitarity constraint was met to $\sim 1/1000$ and showing that the outgoing leptonic distributions were unaffected.

3. An application: neutral current $1\pi^0$ error envelope

An example application of the reweighting scheme is shown in Fig. 2. Here the reweighting scheme has been used to generate an error envelope, due to intranuclear rescattering effects, for neutral current events with a single π^0 in the final state hadronic system. An original event sample of 200 000 events was used. Of these only 12 538 had the required topology.

To generate the error envelope the INTRANUKE/hA parameter space was scanned using the reweighting scheme. Pion and nucleon intranuke parameters were treated separately. In total ~ 170 parameter configurations

[9] This is because nuclear absorption, followed by emission of nucleons is a dominant source of nucleons in the final state.

were scanned, which without reweighting would be equivalent to generating a total event sample of 34×10^6. This highlights the power of an effective reweighting scheme.

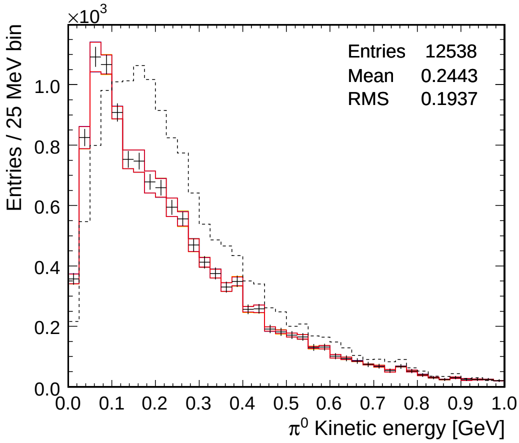

Fig. 2. Error envelope for outgoing kinetic energy of single π^0's due to intranuclear parameter uncertainties. Light dashed line shows pion spectrum before rescattering, black crosses are pion spectra after intranuclear rescattering effects, and the black line envelope around these is the systematic uncertainty from intranuclear rescattering.

4. Conclusions

We have presented an event reweighting scheme for quantifying neutrino cross-section and intranuclear hadron transport systematics. The reweighting methods were outlined and we detailed the validation process. The usefulness of the scheme was demonstrated in an example application relevant to current neutrino experiments. We believe there are many possible applications for this scheme.

Although developed within the context of a T2K analysis much of this work stems from experience gained within the MINOS experiment. We would like to acknowledge contributions made by non-T2K colleagues, especially by our GENIE collaborators Steve Dytman and Hugh Gallagher.

REFERENCES

[1] [GENIE Collaboration] C. Andreopoulos *et al.*, `arXiv:0905.2517[hep-ph]`.

[2] The T2K Experiment: `http://jnusrv01.kek.jp/public/t2k/`.