

# MEASUREMENT OF ASYMMETRIC COMPONENT IN PROTON–PROTON COLLISIONS

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It is argued that a standard measurement of multiplicities in proton–proton collisions is sufficient to construct a single nucleon fragmentation function. A proposed method is based on measurement of mean values of produced particles  $\langle n \rangle$  and pairs of particles  $\langle n(n-1) \rangle$  in symmetric and asymmetric rapidity bins.

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## 1. Introduction

It is commonly believed that the spectra of particles produced in inelastic nucleon–nucleon collisions originate not only from a projectile and a target fragmentations but also from a central production component [1]. On the other hand, there are many phenomenological and experimental evidences supporting the idea of a two-component picture (independent target and projectile contributions) of soft particle production in hadronic collisions. One of these is the success of the wounded nucleon model [2] in description of pseudorapidity spectra of charged particles in  $dAu$  collisions measured by the PHOBOS Collaboration at  $\sqrt{s} = 200$  GeV [3]. Indeed, in this calculation Bialas and Czyz [4] explicitly assumed that all particles are produced independently from the left- and right-moving wounded nucleons. As a result the wounded nucleon fragmentation function was extracted. Similar analysis was performed in the wounded quark–diquark model [5] which resulted in simultaneous description of the PHOBOS  $pp$ ,  $dAu$ ,  $CuCu$  and  $AuAu$  collisions data at  $\sqrt{s} = 200$  GeV in almost full pseudorapidity range.

The two-component picture of pion (and baryon) production in hadronic collisions was also extensively studied at SPS energies [6]. It was proved to be consistent with many experimental findings which include (*i*) the absence

of long-range two-particle correlations in  $pp$  collisions at  $|x_F| > 0.2$ , (ii) the presence of forward-backward multiplicity correlations in  $pp$  collisions at  $|x_F| < 0.2$ , and (iii) the  $x_F$  dependence of the  $\pi^+/\pi^-$  ratio in averaged  $\pi^+p$  and  $\pi^-p$  collisions. The wounded nucleon fragmentation function deduced from the above was found to be in a good qualitative agreement with the one extracted from the analysis of  $dAu$  collisions data in the wounded nucleon (quark-diquark) model [4, 5]. Indeed, both functions are peaked in the forward direction and substantially feed into the opposite hemispheres. This fact also speaks for the validity of the two-component picture of soft particle production.

Unfortunately, the above-mentioned procedures rely on rarely available precise experimental data either in nucleon-nucleus (deuteron-nucleus) collisions or in  $\pi^+p$ ,  $\pi^-p$  and  $pp$  collisions. Moreover, in the former case we also rely on the specific model of particle production in such reactions.

In the present paper we show that the particle density from one wounded nucleon can be extracted solely from appropriate multiplicity measurements in  $pp$  collisions. Our method is based on measurement of average numbers of produced particles  $\langle n \rangle$  and pairs of particles  $\langle n(n-1) \rangle$  in different symmetric and asymmetric bins.

In the next section we describe our method in detail. In Section 3 some comments are included. All calculations are presented in the Appendix.

## 2. Measurement

The measurement of a single wounded nucleon fragmentation function can be performed as follows.

In the first step we measure in  $pp$  collisions the average numbers of produced particles  $\langle n \rangle_{B+F}$  and pairs of particles  $\langle n(n-1) \rangle_{B+F}$  in the combined interval  $B+F$ , where  $B$  and  $F$  are two symmetric (around  $y=0$  in the c.m. frame) rapidity<sup>1</sup> intervals. The schematic view of this process is shown in Fig. 1. The arrows indicate that the left- and right-moving wounded nucleons may populate particles into both intervals.  $p$  is the probability that a particle originating from the right(left)-moving wounded nucleon goes to  $F(B)$  interval, under the condition that this particle was found either in  $B$  or  $F$ <sup>2</sup>. Consequently, the probability that a particle originating from the left(right)-moving wounded nucleon goes to  $F(B)$  interval equals  $1-p$ .

In the second step we repeat the previous measurement but now only in  $F$  interval, see Fig. 2. We measure  $\langle n(n-1) \rangle_F$  and  $\langle n \rangle_F$ , where, of course,  $\langle n \rangle_F = \langle n \rangle_{B+F} / 2$ .

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<sup>1</sup> Our discussion is valid for any longitudinal variable, not necessarily rapidity.

<sup>2</sup> Obviously  $p$  depends on both position and width of  $F$  interval ( $B$  is symmetric around  $y=0$ ).

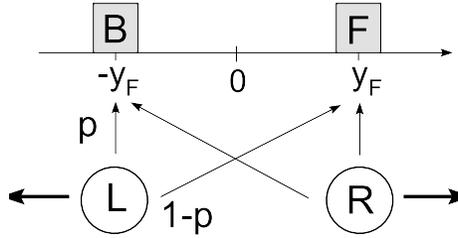


Fig. 1. In the first step we measure  $\langle n \rangle_{B+F}$  and  $\langle n(n-1) \rangle_{B+F}$  in the combined interval  $B + F$ . The arrows indicate that each wounded nucleon may populate particles into both intervals.

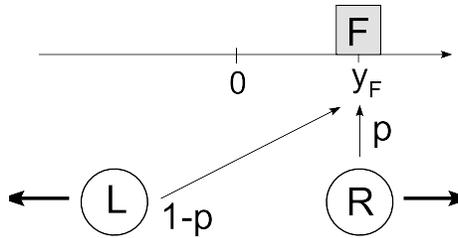


Fig. 2. In the second step we measure  $\langle n(n-1) \rangle_F$  in  $F$  interval. Both nucleons may populate to this interval with an appropriate probabilities  $p$  and  $1 - p$ .

Assuming that both nucleons fragment independently we proved, see the Appendix, that the probability  $p$  can be expressed by the previously measured quantities

$$p = \frac{1}{2} + \frac{1}{2} \sqrt{\frac{4 \langle n(n-1) \rangle_F - \langle n(n-1) \rangle_{B+F}}{\langle n(n-1) \rangle_{B+F} - \langle n \rangle_{B+F}^2}}. \tag{1}$$

Let  $\rho_R(y)$  be the right-moving wounded nucleon fragmentation function<sup>3</sup>. It is obvious that the probability  $p$  can be expressed as

$$p = \frac{\int_F \rho_R(y) dy}{\int_{B+F} \rho_R(y) dy} = \frac{\int_F \rho_R(y) dy}{\langle n \rangle_F}, \tag{2}$$

where the numerator represents the number of particles in  $F$  interval originating from the right-moving wounded nucleon.

Combining (1) and (2) we obtain the relation between unknown density of produced particles from a single wounded nucleon (integrated over  $F$ ) and previously measured  $\langle n \rangle_{B+F} = 2 \langle n \rangle_F$ ,  $\langle n(n-1) \rangle_{B+F}$  and  $\langle n(n-1) \rangle_F$ .

<sup>3</sup> In the c.m. frame a contribution from the left-moving nucleon  $\rho_L(y) = \rho_R(-y)$ .

Finally, let us notice that if  $F$  is sufficiently narrow around  $y_F$  we obtain

$$p = \frac{\int_F \rho_R(y) dy}{\int_F N(y) dy} \approx \frac{\rho_R(y_F)}{N(y_F)}, \quad (3)$$

which directly relates the fragmentation function  $\rho_R(y)$  with the rapidity multiplicity distribution measured in  $pp$  collisions  $N(y) \equiv dN(y)/dy$ .

### 3. Comments

Following comments are in order.

- (i) It is interesting to note that the result (1) does not depend on the number of active sources of particles (constituents) inside the proton. Our method rely only on the assumption that both nucleons fragment independently.
- (ii) Suppose that the multiplicity distributions in  $B + F$  and  $F$  intervals measured in  $pp$  collisions can be approximated by the negative binomial distribution [7] with appropriate  $k_{B+F}$  and  $k_F$ , where  $1/k$  measures the deviation from Poisson distribution. In this case Eq. (1) has a particularly simple form

$$p = \frac{1}{2} + \frac{1}{2} \sqrt{\frac{k_{B+F}}{k_F} - 1}. \quad (4)$$

- (iii) It is enough to perform measurement for different bins up to  $y = y_F^* > 0$  in which  $p = 1$ . Here  $F$  is populated only by the right-moving nucleon and  $B$  only be the left-moving one, thus  $\rho_R(y) = N(y)$  for  $y \geq y_F^*$ . Direct application of Eq. (1) will be disturbed in the far fragmentation region ( $y \approx y_{\text{beam}}$ ), where energy conservation effects will play an important role.
- (iv) Once a single nucleon fragmentation function is measured it can be directly tested in proton–nucleus collisions (or any other asymmetric nucleus–nucleus collisions).
- (v) Finally, we would like to emphasize that our method apply to any longitudinal variable, not necessarily rapidity.

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### Appendix A

Let  $P_{B+F}(n)$  be the multiplicity distribution from both wounded nucleons in the combined interval  $B + F$ , where  $B$  and  $F$  are two symmetric rapidity bins. It is convenient to construct the generating function

$$H_{B+F}(z) = \sum_n P_{B+F}(n) z^n. \quad (5)$$

Let  $P_{B+F}^L(n)$  and  $H_{B+F}^L(z)$  be the multiplicity distribution and corresponding generating function in  $B + F$  from the left-moving L wounded nucleon (and analogous  $P_{B+F}^R(n)$  and  $H_{B+F}^R(z)$  for the right-mover R).

Assuming that both nucleons fragment independently we obtain

$$H_{B+F}(z) = H_{B+F}^L(z) H_{B+F}^R(z). \quad (6)$$

For collision of two identical nucleons we obtain

$$H_{B+F}^L(z) = H_{B+F}^R(z) = \sqrt{H_{B+F}(z)}. \quad (7)$$

Let  $P_F^L(n)$  be the multiplicity distribution in  $F$  interval from the left-moving nucleon. It can be easily expressed by  $P_{B+F}^L$

$$P_F^L(n) = \sum_{n' \geq n} P_{B+F}^L(n') \frac{n!}{n! (n' - n)!} (1 - p)^n p^{n' - n}, \quad (8)$$

where  $p$  is the conditional probability that a particle originating from the left-moving wounded nucleon goes to  $B$  interval rather than to  $F$ . Consequently  $1 - p$  is the probability that a particle originating from the left-moving wounded nucleon goes to  $F$  interval rather than to  $B$ . The corresponding generating function

$$\begin{aligned} H_F^L(z) &= \sum_n P_F^L(n) z^n = \sum_{n'} P_{B+F}^L(n') [p + z - pz]^{n'} \\ &= H_{B+F}^L(p + z - pz). \end{aligned} \quad (9)$$

Performing similar calculations for the right-moving nucleon we obtain

$$H_F^R(z) = H_{B+F}^R(1 - p + pz). \quad (10)$$

Let  $P_F(n)$  be the multiplicity distribution from both wounded nucleons in  $F$ . Taking Eqs (7), (9) and (10) into account the corresponding generating function  $H_F$  can be expressed by  $H_{B+F}$

$$\begin{aligned} H_F(z) &= H_F^L(z) H_F^R(z) \\ &= H_{B+F}^L(p + z - pz) H_{B+F}^R(1 - p + pz) \\ &= \sqrt{H_{B+F}(p + z - pz) H_{B+F}(1 - p + pz)}. \end{aligned} \quad (11)$$

Finally, let us calculate the second derivative of Eq. (11) with respect to  $z$  at  $z = 1$ . Taking the following relations into account

$$\begin{aligned} \left. \frac{dH_{B+F}(z)}{dz} \right|_{z=1} &= \langle n \rangle_{B+F} , \\ \left. \frac{d^2 H_{B+F}(z)}{dz^2} \right|_{z=1} &= \langle n(n-1) \rangle_{B+F} , \\ \left. \frac{d^2 H_F(z)}{dz^2} \right|_{z=1} &= \langle n(n-1) \rangle_F , \end{aligned} \quad (12)$$

we obtain

$$4 \langle n(n-1) \rangle_F = \langle n(n-1) \rangle_{B+F} + [1-4p(1-p)] \left[ \langle n(n-1) \rangle_{B+F} - \langle n \rangle_{B+F}^2 \right] , \quad (13)$$

which allows to extract<sup>4</sup> the probability  $p$  given by Eq. (1).

## REFERENCES

- [1] K. Fialkowski, W. Kittel, *Rep. Prog. Phys.* **46**, 1283 (1983).
- [2] A. Bialas, M. Bleszynski, W. Czyz, *Nucl. Phys.* **B111**, 461 (1976).
- [3] B.B. Back *et al.* [PHOBOS Collaboration], *Phys. Rev.* **C72**, 031901 (2005).
- [4] A. Bialas, W. Czyz, *Acta Phys. Pol. B* **36**, 905 (2005).
- [5] A. Bialas, A. Bzdak, *Phys. Rev.* **C77**, 034908 (2008); *Phys. Lett.* **B649**, 263 (2007); *Acta Phys. Pol. B* **38**, 159 (2007); For a review, see A. Bialas, *J. Phys. G* **35**, 044053 (2008).
- [6] G. Barr *et al.*, *Eur. Phys. J.* **C49**, 919 (2007); A. Rybicki, *Acta Phys. Pol. B* **33**, 1483 (2002).
- [7] See, *e.g.*, I.M. Dremin, J.W. Gary, *Phys. Rep.* **349**, 301 (2001).

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<sup>4</sup> We assume that  $p \geq 1/2$ . It corresponds to the natural assumption that the right-moving nucleon fragmentation function is peaked in the right hemisphere.