TREE AND PENGUIN AMPLITUDES FROM $B \to \pi\pi, K\pi, K\bar{K}$

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The question of the relative size of tree and penguin amplitudes is analyzed using the data on $B \to \pi\pi$, $B^+ \to \pi^+ K^0$, and $B^+ \to K^+ \bar{K}^0$ decays. Our discussion involves an estimate of SU(3) breaking in the final quark-pair-creating hadronization process. The estimate is based on Regge phenomenology, which many years ago proved very successful in the description of soft hadronic physics. Accepting the Regge prediction as solid, it is then shown that the relative size and phase of the two parts of the penguin amplitude can be unambiguously extracted from the data on the decays considered. This enables fixing the C/T ratio of "true" tree amplitudes, which — on the basis of the existing data — is shown to be small (of the order of 0.2).

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1. Introduction

Rare charmless nonleptonic B meson decays provide us with a lot of information on weak interactions of quarks and the Cabibbo–Kobayashi–Maskawa (CKM) matrix. Yet, since quarks are forever confined, this information is necessarily blended with strong interaction effects. Additionally, there may be contributions from New Physics beyond the Standard Model. Since this New Physics is by definition unknown, proper disentangling of all three effects requires a thorough understanding of the strong interaction part.

Unfortunately, the calculation of low-energy quark-confining strong interaction effects from first principles is at present impossible. Consequently, the only way to achieve a reliable understanding of these effects is through their parametrization and subsequent extraction of relevant parameters from the experimental data. For example, simple quark-level arguments suggest that the ratio C/T of the so-called colour-suppressed (C) and tree (T) diagrammatic amplitudes (see *e.g.* [1]) should be small. Yet, since the time of the theoretically clean analysis of Ref. [2], it is known that the effective tree and colour-suppressed amplitudes in $B \to \pi\pi$ decays are roughly equal in absolute magnitudes, thus contradicting simple quark-level expectations. Hence, additional quark-level [3, 4] and/or hadron-level [5] effects must be important.

In fact, as various authors have argued, the effective tree ($\tilde{T}$) and colour-suppressed ($\tilde{C}$) amplitudes involve contributions from penguin diagrams as well (see *e.g.* [2,6]). Disentangling this "second" penguin contribution from the "true" C and T amplitudes is not possible on the basis of $B \to \pi\pi$ data alone. For this reason, Ref. [7] considered $B \to \pi\pi$ decays in conjunction with $B \to \pi\rho$ and $B \to \pi\omega$ processes. Since under very reasonable assumptions it may be shown that both $B \to \pi\pi$ and $B \to \pi\rho, \pi\omega$ processes depend on the *same ratio* of C/T, the number of equations constraining C/T increases, thereby enabling extraction of the latter from the data. In Ref. [7] it was then shown that the data allow a solution for C/T which is in agreement with the simple quark-level expectations.

Alternatively, one may supplement the data on $B \to \pi\pi$ decays with those on $B^+ \to K^+\bar{K}^0$. This was the route attempted in Ref. [8]. The problem with that route is that one has to know how to treat SU(3) breaking.

In this paper, we analyse $B \to \pi\pi$ and $B^+ \to K^+\bar{K}^0$ decays in conjunction with $B^+ \to \pi^+K^0$ processes, and under a specific assumption concerning the pattern of SU(3) breaking. The assumption on SU(3) breaking adopted by us has been experimentally known to be correct since the mid-seventies of the last century. We show that the data on $B \to \pi\pi, K^+\bar{K}^0$, π^+K^0 point then unambiguously toward a small $|C/T|$ ratio, of the order of 0.2.

The paper is organized as follows. In Sec. 2 our main definitions and conventions are set and the basic analysis of the $B \to \pi\pi$ sector [2, 7] is repeated with the new, more precise data. Sec. 3 contains the analysis of the $B^+ \to \pi^+K^0$, $K^+\bar{K}^0$ decays. This involves a Regge estimate of SU(3) breaking in the hadronization stage, which is crucial in bringing the extracted value of $|C/T|$ into agreement with the expectations. We conclude in Sec. 4.

2. Decays $B \to \pi\pi$

Below we employ the notation used in Ref. [7]. The diagrammatic approach gives the following expressions for the amplitudes in the $B \to \pi\pi$ decays:

$$
\begin{aligned}
-\sqrt{2}A\left(B^+ \to \pi^+\pi^0\right) &= \tilde{T} + \tilde{C}\,, \\
-A\left(B_d^0 \to \pi^+\pi^-\right) &= \tilde{T} + P_c\,, \\
-\sqrt{2}A\left(B_d^0 \to \pi^0\pi^0\right) &= \tilde{C} - P_c\,, \qquad (1)
\end{aligned}
$$

where we have kept the leading terms only, *i.e.* the contributions from effective tree $\tilde{T}$, colour-suppressed tree $\tilde{C}$, and penguin amplitudes P_c. The effective amplitudes present in Eq. (1) are related to "true" tree T and colour-suppressed C amplitudes, as well as to two contributions to the total penguin amplitude P:

$$P = -\lambda_u^{(d)}\mathcal{P}_{tu} - \lambda_c^{(d)}\mathcal{P}_{tc}\,, \tag{2}$$

where

$$\begin{aligned} \lambda_q^{(k)} &= V_{qk}V_{qb}^*\,, \\ \mathcal{P}_{tq} &= \mathcal{P}_t - \mathcal{P}_q\,, \end{aligned} \tag{3}$$

with V_{qk} being the CKM elements, and $\mathcal{P}_q$ being the contribution from quark q running inside the penguin loop.

After introducing

$$P_q \equiv -\lambda_c^{(d)}\mathcal{P}_{tq} = A\lambda^3\mathcal{P}_{tq}\,, \tag{4}$$

where A and λ are Wolfenstein parameters, the relevant formulas for the effective amplitudes are:

$$\begin{aligned} P_c &= A\lambda^3\mathcal{P}_{tc}\,, \\ \tilde{T} &= e^{i\gamma}(T - R_bP_u)\,, \\ \tilde{C} &= e^{i\gamma}(C + R_bP_u)\,, \end{aligned} \tag{5}$$

where the weak phase factor has been explicitly factored out from the "true" tree amplitudes C, T, and $R_b = \sqrt{\bar{\rho}^2 + \bar{\eta}^2} = 0.37 \pm 0.02$ (all experimental numbers are taken from HFAG [9]).

Following Refs. [2, 7], we define

$$\begin{aligned} de^{i\theta} &= -e^{i\gamma}\frac{P_c}{\tilde{T}} = \frac{P_c}{R_bP_u - T}\,, \\ xe^{i\Delta} &= \frac{\tilde{C}}{\tilde{T}} = \frac{C + R_bP_u}{T - R_bP_u}\,. \end{aligned} \tag{6}$$

Then, asymmetries $A^{\rm dir}_{\pi^+\pi^-}$, $A^{\rm mix}_{\pi^+\pi^-}$, with experimental values of

$$\begin{aligned} A^{\rm dir}_{\pi^+\pi^-} &= C_{\pi\pi} = -0.38 \pm 0.06\,, \\ A^{\rm mix}_{\pi^+\pi^-} &= -S_{\pi\pi} = +0.65 \pm 0.07 \end{aligned} \tag{7}$$

are expressed in terms of d and θ as

$$\begin{aligned} A^{\rm dir}_{\pi^+\pi^-} &= -\frac{2d\sin\theta\sin\gamma}{1 - 2d\cos\theta\cos\gamma + d^2}\,, \\ A^{\rm mix}_{\pi^+\pi^-} &= \frac{\sin(2\beta + 2\gamma) - 2d\cos\theta\sin(2\beta + \gamma) + d^2\sin(2\beta)}{1 - 2d\cos\theta\cos\gamma + d^2}\,. \end{aligned} \tag{8}$$

Using $\beta = 21.2°$, $\gamma = 65.5°$ (as in Ref. [8]), one then solves the above equations for d and θ to find:

$$\begin{aligned} d &= 0.51^{+0.10}_{-0.08}\,, \\ \theta &= 140° \pm 6°\,. \end{aligned} \tag{9}$$

When the following ratios of CP-averaged $B \to \pi\pi$ branching ratios are defined:

$$\begin{aligned} R^{\pi\pi}_{+-} &\equiv 2\frac{\left\langle \mathcal{B}\left(B^+ \to \pi^+\pi^0\right)\right\rangle_{\mathrm{CP}}}{\left\langle \mathcal{B}\left(B_d \to \pi^+\pi^-\right)\right\rangle_{\mathrm{CP}}}\frac{\tau_{B^0_d}}{\tau_{B^+}}\,, \\ R^{\pi\pi}_{00} &\equiv 2\frac{\left\langle \mathcal{B}\left(B_d \to \pi^0\pi^0\right)\right\rangle_{\mathrm{CP}}}{\left\langle \mathcal{B}\left(B_d \to \pi^+\pi^-\right)\right\rangle_{\mathrm{CP}}}\,, \end{aligned} \tag{10}$$

they may be expressed in terms of the yet undetermined parameters x, Δ as

$$\begin{aligned} R^{\pi\pi}_{+-} &= \frac{1 + 2x\cos\Delta + x^2}{1 - 2d\cos\theta\cos\gamma + d^2}\,, \\ R^{\pi\pi}_{00} &= \frac{d^2 + 2dx\cos(\Delta - \theta)\cos\gamma + x^2}{1 - 2d\cos\theta\cos\gamma + d^2}\,. \end{aligned} \tag{11}$$

For the values $\tau_{B^+}/\tau_{B^0_d} = 1.073 \pm 0.008$, $\langle\mathcal{B}(B^\pm \to \pi^\pm\pi^0)\rangle_{\mathrm{CP}} = 5.59^{+0.41}_{-0.40}$, $\langle\mathcal{B}(B_d \to \pi^+\pi^-)\rangle_{\mathrm{CP}} = 5.16 \pm 0.22$, and $\langle\mathcal{B}(B_d \to \pi^0\pi^0)\rangle_{\mathrm{CP}} = 1.55 \pm 0.19$ (in units of 10^{-6}) one finds

$$\begin{aligned} R^{\pi\pi}_{+-} &= 2.02 \pm 0.17\,, \\ R^{\pi\pi}_{00} &= 0.60 \pm 0.08\,. \end{aligned} \tag{12}$$

Using the central values for d and θ, Eqs. (11) then yield

$$\begin{aligned} x &= 1.06^{+0.06}_{-0.07}\,, \\ \Delta &= -59°^{+12^0}_{-11°}\,. \end{aligned} \tag{13}$$

A large (close to 1) value of x constitutes a problem in those approaches in which the contribution of P_u is neglected since for small P_u we have $|C/T| \approx x$ (Eq. (6)).

If P_u is not neglected, then — as proposed in Ref. [7] — using information on $B \to \pi\rho$ and $B \to \pi\omega$ decays and making a very reasonable physical assumption concerning the creation of the $q\bar{q}$ pair in the final hadronization process, one can try to extract the true ratio C/T. In fact, it was shown in Ref. [7] that there exists a solution with a small value of $|C/T|$ (of the order of 0.3). Extraction of C/T is thus possible with the help of additional information available from other data.

The method of Ref. [8] uses for that purpose the data on $B \to K\bar{K}$. Under the assumption of exact SU(3) it shows that the true C/T may indeed be small. However, when a specific way of SU(3) breaking is considered, Ref. [8] predicts significantly increased central values of C/T (albeit the corresponding errors also increase).

3. Decays $B \to \pi K, K\bar{K}$

The problem with the method of Ref. [8] is that it assumes either exact SU(3), or — as we will show — an inadequate estimate of SU(3) breaking. In the following, we show how SU(3) breaking in the hadronization stage should be included, and how this affects the discussion of the tree and penguin amplitudes (other studies of $B \to K\bar{K}$ decays may be found in Refs. [10,11]). In order to present the problem with SU(3) breaking clearly, it is appropriate to consider two different but closely related pure penguin processes, *i.e.* the decays $B^+ \to K^0\pi^+$ and $B^+ \to K^+\bar{K}^0$, and to discuss them in conjunction.

3.1. $B^+ \to \pi^+ K^0$

When compared with the $B \to \pi\pi$ amplitudes, the $B^+ \to \pi^+ K^0$ amplitude differs in that it is now an s quark and not a u or d quark that is being produced. The relevant difference enters through the CKM factors $\lambda_q^{(k)}$ only. When $\lambda_q^{(k)}$s are factored out, the remaining factors in penguin amplitudes (*i.e.* $\mathcal{P}_{tu}$, $\mathcal{P}_{tc}$) should be the same in both $B \to \pi\pi$ and $B \to \pi K$ (*cf.* [7]). Thus, the $B^+ \to \pi^+ K^0$ amplitude is given by

$$A\left(B^+ \to \pi^+ K^0\right) \;=\; P' = -\lambda_u^{(s)}\mathcal{P}_{tu} - \lambda_c^{(s)}\mathcal{P}_{tc}\,. \tag{14}$$

When the ratio of $\mathcal{P}_{tu}/\mathcal{P}_{tc}$ is expressed in terms of P_u/P_c, one obtains

$$P' \;=\; -\frac{1}{\sqrt{\varepsilon}}\,P_c\left(1+\varepsilon R_b\frac{P_u}{P_c}\,e^{i\gamma}\right), \tag{15}$$

where

$$\varepsilon = \frac{\lambda^2}{1-\lambda^2} \approx 0.05 \tag{16}$$

and the factor of $1/\sqrt{\varepsilon}$ takes care of the suppression of the term $-\lambda_c^{(s)}\mathcal{P}_{tc}$ when compared to $P_c = -\lambda_c^{(d)}\mathcal{P}_{tc}$ (Eq. (4)).

It is convenient to introduce

$$ze^{i\zeta} \equiv R_b\frac{P_u}{P_c} = -\,\frac{xe^{i\Delta}-C/T}{de^{i\theta}(1+C/T)}\,, \tag{17}$$

so that

$$P' = -\frac{1}{\sqrt{\varepsilon}}\, P_c \left(1 + \varepsilon z e^{i(\zeta+\gamma)}\right) . \tag{18}$$

For $C/T \approx 0$ one then expects

$$z e^{i\zeta} \approx \frac{x}{d}\, e^{i(\pi+\Delta-\theta)} = (2.08 \pm 0.38)\, e^{i(-19\pm13)} . \tag{19}$$

3.2. $B^+ \to K^+\bar{K}^0$

In the SU(3) symmetric case, the $B^+ \to K^+\bar{K}^0$ amplitude is given by P of Eq. (2):

$$\begin{aligned} A\left(B^+ \to K^+\bar{K}^0\right) = P_{KK} &= -\lambda_u^{(d)}\mathcal{P}_{tu} - \lambda_c^{(d)}\mathcal{P}_{tc} \\ &= P_c \left(1 - R_b \frac{P_u}{P_c} e^{i\gamma}\right) \\ &= P_c \left(1 - z e^{i(\zeta+\gamma)}\right) . \end{aligned} \tag{20}$$

We know, however, that SU(3) is broken. The main difference between the $B^+ \to \pi^+ K^0$ and $B^+ \to K^+\bar{K}^0$ amplitudes stems then from the fact that in the first decay the $q\bar{q}$ pair created after the weak decay is composed of light quarks, while in the other decay it is an $s\bar{s}$ pair. Now, it is known that processes, in which such newly produced quarks q and $\bar{q}$ end up in different (and separated by large rapidity gap) hadrons, are suppressed for strange quarks more than for the light ones. Let us therefore write the $B^+ \to K^+\bar{K}^0$ amplitude for the case of SU(3) breaking (parametrized by $\kappa < 1$) as

$$P_{KK} = \kappa P_c \left(1 - z e^{i(\zeta+\gamma)}\right) . \tag{21}$$

3.3. Estimate of SU(3) breaking

In order to estimate the size of κ consider quark line diagrams corresponding to decays $B^+ \to K^+\bar{K}^0, \pi^+ K^0$ as visualized in Fig. 1. The decay process starts from the short-distance penguin transition $\bar{b} \to \bar{s}$ (or $\bar{b} \to \bar{d}$), in Fig. 1 denoted by crosses. As the highly energetic $\bar{s}$ or $\bar{d}$ quark recedes from the spectator u quark, a complicated hadronization process sets in. The additional $q\bar{q}$ pair observed in the final state emerges only at the very end of this soft process. The complicated nature of the latter is visualized in the diagrams of Fig. 1 with the help of closed quark loops, which symbolize the fact that the creation of the final $q\bar{q}$ pair may in general go through various many-body intermediate states. Thus, the diagrams depict a general situation with *all* final state interactions included. The difference between the

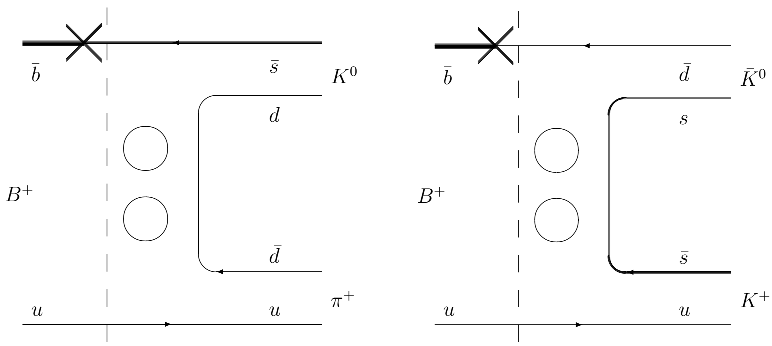

Fig. 1. B decays to πK and $K\bar{K}$ final states. Penguin $\bar{b} \to \bar{s}$ transitions are denoted by crosses. Vertical dashed line symbolizes onset of long range dynamics. Loops symbolize transition through many-body intermediate states.

two soft processes shown in Fig. 1 is induced by the difference in the flavour of the produced $q\bar{q}$ pair only. The question is: can we establish the relative size of these two complicated processes from the *experimental* knowledge of other processes? This can be done if we can find other, *purely soft*, processes (*i.e.* not involving B decays at all), which differ just like the two processes shown in Fig. 1, and on which experimental data exist.

Imagine now that the $\bar{s}u$ or $\bar{d}u$ states — from which the soft hadronization processes of Fig. 1 start — are generated not via a B-decay, but by a different initial process, nearly identical for both $\bar{q}u$ states, as shown in Fig. 2(b) (the following ideas lie at the foundations of the Regge-based estimates of strong decay widths as performed in Ref. [12]). This is a process in which two mesons collide and form an intermediate $\bar{s}u$ or $\bar{d}u$ state. The initial process, leading to the $\bar{s}u$ or $\bar{d}u$ state, and the final process, leading to $K^0\pi^+$ or $\bar{K}^0K^+$, may be redrawn together as shown in Fig. 2(a). In this figure, the complicated nature of soft interactions, visualized in Fig. 2(b) by quark loops, is not shown explicitly at all. Obviously, however, if we extract *from experiment* the amplitudes corresponding to the topology of the diagrams shown in Fig. 2(a), the effect of *all* such soft interactions will be included in our extracted amplitudes. At energy $s = m_B^2$, which is relevant for our case, these amplitudes are dominated by ρ and K^* Regge exchanges in the t-channel, and the experimental amplitudes may be expressed in terms of the corresponding Regge parameters. Since these parameters are extracted directly from high energy scattering experiments, their values take into account *all* final state interactions in the s-channel, even those generated by Pomeron exchange (the so-called Reggeon–Pomeron cuts) — the only strong interaction allowed after the final $d\bar{d}$ or $s\bar{s}$ pair is produced.

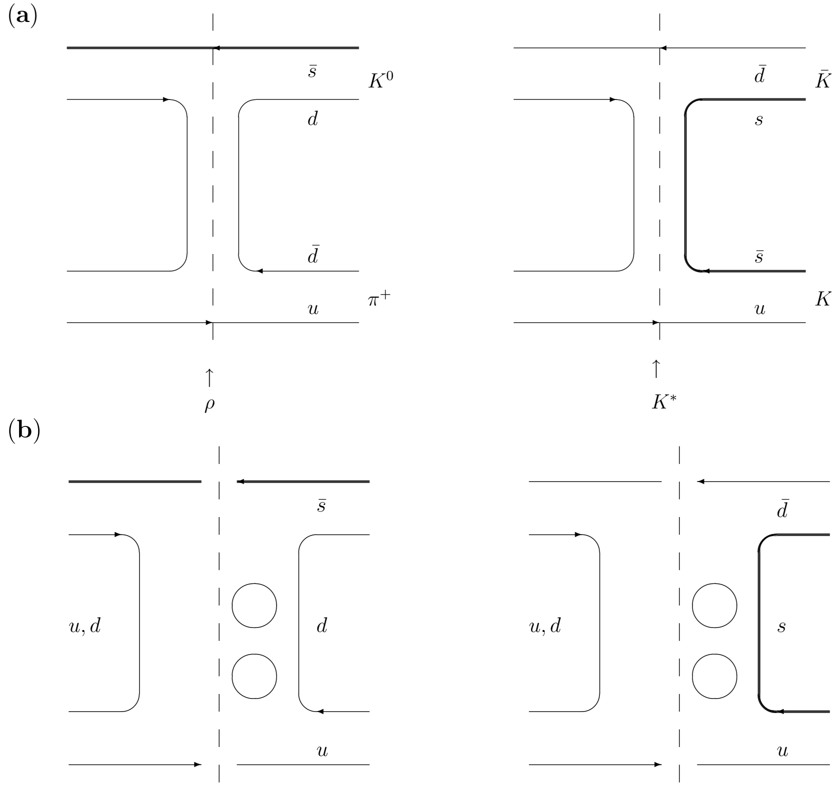

Fig. 2. (a) Reggeon exchanges leading to πK and $K\bar{K}$ final states. (b) Corresponding product structure of initial and final soft hadron interactions. Loops symbolize some of complicated intermediate hadronic states not shown in upper diagrams explicitly.

The experimental amplitudes corresponding to Fig. 2(a) may be parameterized in terms of the product of Regge couplings and Regge propagators. Thus, we have (for the left and right diagrams, respectively):

$$g_{\rho KK}\, g_{\rho\pi\pi}\,(s/s_0)^{\alpha_\rho(t)}\;, \tag{22}$$

$$g_{K^*\pi K}\, g_{K^*\pi K}\,(s/s_0)^{\alpha_{K^*}(t)}\;, \tag{23}$$

where $\alpha_M(t)$ is the Regge trajectory for meson M, given in terms of its intercept $\alpha_0(M)$ and the universal slope α' by:

$$\alpha_M(t) \;=\; \alpha_0(M) + \alpha' t \tag{24}$$

with $s_0 = (\alpha')^{-1} \approx 1\,\mathrm{GeV}^2$ being the scale parameter relevant for soft processes. When one takes into account that the intercepts $\alpha_0(M)$ are determined in soft processes, consistent application of Eq. (24) requires that one cannot use any other value for the scale parameter (like *e.g.* $s_0 = m_B^2$). The scale $s_0 = 1\,\mathrm{GeV}^2$ is fixed by the Regge behaviour as experimentally observed in soft processes. It is irrelevant here that the true Regge behaviour sets in at an energy much higher than s_0 (in fact the Regge formula describes also the region of low energies, albeit only in an average way).

Let us now discuss the issue of SU(3) symmetry breaking. Regge amplitudes result from the summation over exchanged resonances (the sums being performed either in the s- or in the t-channel). In principle, the couplings of external π, K mesons to these individual resonances may break SU(3). One may then wonder if such SU(3) breaking effects could not add up in the summation procedure in an uncontrolled way and lead to unknown SU(3) breaking in Reggeon couplings. Yet, please note that in fact we are talking not about the calculation of SU(3) properties of Regge amplitudes from those of the resonances, but about the parametrization of *experimental* flavour-exchange amplitudes at such energies at which Regge behaviour is observed. It is the experimentally observed energy dependence of these amplitudes as well as their *absolute* and *relative* magnitudes that determine Regge parametrization. These things are known from the fits to the cross-sections' data: the observed energy dependence fixes the intercepts, while their absolute size fixes Reggeon couplings. In fact, from the relative magnitude of the amplitudes it is known that the extracted couplings of the leading non-Pomeron Reggeons to the external particles (*i.e.* π, K) satisfy SU(3) symmetry well [13] (see also [14]). An analogous statement is true for various hadronic couplings (*c.f.* the successes of SU(3)-symmetric parametrization of the MBB' couplings of ground-state mesons and baryons in terms of SU(3) parameters F and D, and many other similar examples). In fact, SU(3) breaking in hadronic couplings at low energies is much weaker than in hadron masses. A corresponding statement in Regge phenomenology is that SU(3) breaking in Regge residues is much less important than in the intercepts. In the following we shall therefore accept SU(3) of Reggeon couplings which means that $g_{\rho KK} g_{\rho\pi\pi} = g_{K^*\pi K} g_{K^*\pi K}$[1].

At $s = m_B^2$ the size of the K^*-exchange Regge amplitude relative to that of the ρ exchange is then clearly given by

$$(m_B^2/s_0)^{\alpha_0(K^*)-\alpha_0(\rho)}. \tag{25}$$

The fact that the initial stage of the collision process, leading to the intermediate $\bar{s}u$ or $\bar{d}u$ state, is the same in both cases (see Fig. 2(b)) means that Eq. (25) provides also a good estimate of κ in B meson decays. The

[1] Although SU(3) symmetry is not satisfied by Pomeron couplings, this does not affect our estimates at all, as our scheme does not use these couplings, but only those of the non-leading Reggeons, as required by the topology of the diagrams under consideration and extracted from experiments relevant for these topologies.

irrelevance of the initial process leading to $\bar{s}u$ or $\bar{d}u$ may be seen in a yet another way. Namely, the intercepts of the leading Regge trajectories depend on the flavour of the exchanged quarks, and it is known that this dependence is approximately additive, *i.e.*

$$\begin{aligned} \alpha_0(\rho) &= \alpha_{0n} + \alpha_{0n}\,, & (26)\\ \alpha_0(K^*) &= \alpha_{0n} + \alpha_{0s}\,, & (27)\\ \alpha_0(\phi) &= \alpha_{0s} + \alpha_{0s}\,, & (28) \end{aligned}$$

where subscripts n, s correspond to nonstrange and strange quarks. The difference $\alpha_0(K^*) - \alpha_0(\rho)) = \alpha_{0s} - \alpha_{0n}$ originates from the difference in the $\bar{q}u$ decay phase only. The dependence on the factor describing the production phase cancels out. With $\alpha_{0s} < \alpha_{0n}$ we obtain suppression of strange quark exchange in the decay phase relative to that of the nonstrange quark.

In conclusion, the ratio of the two amplitudes in Fig. 1 is given by

$$\kappa = (m_B^2/s_0)^{\alpha_0(K^*)-\alpha_0(\rho)}\,. \tag{29}$$

Obviously, even though in the above formula there appears an expression resembling the K^* and ρ Reggeon propagators, no K^* or ρ mesons are actually exchanged. The formula simply provides an estimate of the relative size of effective quark exchanges, all soft interactions included.

In Refs. [13, 15] it was estimated that

$$\alpha_0(K^*) - \alpha_0(\rho) \;\approx\; -0.20\,. \tag{30}$$

Thus, at $m_B^2 = 27.9\,\mathrm{GeV}^2$ one expects

$$\kappa \approx 0.50\,. \tag{31}$$

If one accepts $\alpha_0(K^*) - \alpha_0(\rho) = -0.15$ as sometimes used, one gets $\kappa \approx 0.60$.

One may wonder why the above method of estimating SU(3)-breaking should be preferred to calculations based on effective field theories (and the factorization approach in particular). Indeed, it is known that QCD factorization and hadron-level S-matrix predictions in general do not lead to the same asymptotics as $m_b \to \infty$ (see, *e.g.* [16]). Yet, in Ref. [16] Donoghue *et al.* give preference to the arguments based on S-matrix theory, as stated in their concluding section: "For large m_b, there is hope that one can directly calculate the weak matrix elements through variants of the factorization hypothesis or by pertubative QCD. Final state interactions will impose limits on the accuracy of such methods, as no existing technique includes the effect of inelastic scattering. There must exist, in every valid theoretical calculation, a region of the parameter space where the nonperturbative Regge physics is manifest".

While the S-matrix approach imposes theoretical requirements on any calculation performed at quark level, these requirements may or may not be satisfied by existing quark-level techniques. For example, in [17] it was argued, in a QCD-based model, that factorization in B decays to two pseudoscalars holds exactly at the leading order. These arguments do not apply to our case, however. The point is that we have receding colour triplets, while Ref. [17] deals with receding colour octets. While gluon exchanges — as discussed in [17] — may turn octets into singlets, they cannot change colour triplets into singlets. The only way to turn colour triplet into a singlet is through an exchange of a colour triplet, *i.e.* a quark (not considered in [17]). Such an exchange necessarily involves flavour exchange as well. In the abstract of [16] we read: "flavour off-diagonal FSI are suppressed by a power of m_B, but are likely to be significant at $m_b \approx 5\,\mathrm{GeV}$". It is the SU(3) breaking in such flavour exchanges that is estimated in our approach with the help of Regge arguments.

3.4. Constraint from branching ratios

The CP-averaged branching ratios for the $B^+ \to \pi^+ K^0, K^+\bar{K}^0$ decays are given by

$$\begin{aligned}\left\langle \mathcal{B}\left(B^+ \to \pi^+ K^0\right)\right\rangle_{\mathrm{CP}} &\approx \frac{1}{\varepsilon}|P_c|^2\,\left(1 + 2\,\varepsilon\, z\,\cos\zeta\cos\gamma\right)\,,\\ \left\langle \mathcal{B}\left(B^+ \to K^+\bar{K}^0\right)\right\rangle_{\mathrm{CP}} &= \kappa^2|P_c|^2\,\left(1 + z^2 - 2z\cos\zeta\cos\gamma\right)\,. \end{aligned} \tag{32}$$

Thus, we find that

$$R_{\pi K}^{KK} \equiv \frac{\left\langle \mathcal{B}\left(B^+ \to K^+\bar{K}^0\right)\right\rangle_{\mathrm{CP}}}{\left\langle \mathcal{B}\left(B^+ \to \pi^+ K^0\right)\right\rangle_{\mathrm{CP}}} = \varepsilon\,\kappa^2\,\frac{1 + z^2 - 2z\cos\zeta\cos\gamma}{1 + 2\,\varepsilon\, z\,\cos\zeta\cos\gamma}\,. \tag{33}$$

The experimental branching ratios for $B^+ \to \pi^+ K^0, K^+\bar{K}^0$, and $B^0 \to K^0\bar{K}^0$ decays are (in our approximation the amplitudes for $B^+ \to K^+\bar{K}^0$ and $B^0 \to K^0\bar{K}^0$ are identical):

$$\begin{aligned}\left\langle \mathcal{B}\left(B^+ \to \pi^+ K^0\right)\right\rangle_{\mathrm{CP}} &= 23.1 \pm 1.0\,,\\ \left\langle \mathcal{B}\left(B^+ \to K^+\bar{K}^0\right)\right\rangle_{\mathrm{CP}} &= 1.36^{+0.29}_{-0.27}\,,\\ \left\langle \mathcal{B}\left(B_d \to K^0\bar{K}^0\right)\right\rangle_{\mathrm{CP}} &= 0.96^{+0.21}_{-0.19}\,. \end{aligned} \tag{34}$$

Thus, one finds

$$R_{\pi K}^{KK} = 0.059 \pm 0.012\,, \tag{35}$$

or

$$R_{\pi K}^{KK} = 0.049 \pm 0.008\,, \tag{36}$$

where the first (second) line is obtained if the $B^+ \to K^+\bar{K}^0$ (the average of the $B^+ \to K^+\bar{K}^0$ and $B_d \to K^0\bar{K}^0$) branching ratio(s) is used. For illustration purposes, let us accept $R^{KK}_{\pi K} \approx 0.055$. Solving Eq. (33) for this value of $R^{KK}_{\pi K}$ yields one positive solution only, a constraint on z and ζ:

$$z = (1+\varepsilon r)\cos\gamma\cos\zeta + \sqrt{r - 1 + ((1+\varepsilon r)\cos\gamma\cos\zeta)^2}\,, \tag{37}$$

where

$$r = \frac{R^{KK}_{\pi K}}{\varepsilon\kappa^2} \approx \begin{cases} 1.1 & \text{for exact SU(3)}\,, \\ 4.4 & \text{for } \kappa = 0.50\,. \end{cases} \tag{38}$$

The analysis of [7] indicates that for small C/T the value of ζ is of the order of -15° to -45° (see also Eq. (19)). If, as might be expected, the relative strong phase of $\mathcal{P}_{tu}$ with respect to $\mathcal{P}_{tc}$ is indeed that small (say $|\zeta| < 30^\circ$), one estimates that for exact SU(3)

$$z \approx 0.9 - 1\,, \tag{39}$$

while if SU(3) is broken, one gets (for $\kappa = 0.50$)

$$z \approx 2.3\,. \tag{40}$$

In both cases, for larger values of $|\zeta|$ one obtains smaller z, with the minima (0.1 for exact SU(3), 1.4 for $\kappa = 0.50$) achieved for $|\zeta| = 180^\circ$.

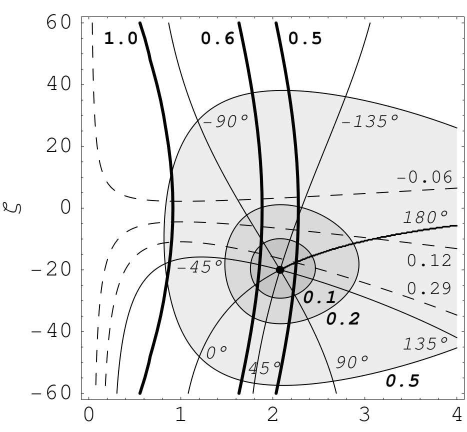

Fig. 3. Constraints on C/T and $P_u/P_c = ze^{i\zeta}/R_b$. Thick lines: branching ratio constraint from Eq. (33) for $\kappa = 0.5, 0.6, 1.0$. Dashed lines: curves of constant asymmetry $A_{\rm CP}(B^+ \to K^+\bar{K}^0)$ equal to $-0.06, 0.12, 0.29$. Borders of shadowed areas: $|C/T| = 0.1, 0.2, 0.5$. Lines of constant $\mathrm{Arg}(C/T)$ are also shown.

The constraint on z and ζ (Eqs. (33), (37)) depends on SU(3) breaking and is shown in Fig. 3 with thick lines (for $\kappa = 0.5, 0.6, 1.0$). Their positioning depends also somewhat on the size of the $B^+ \to K^+\bar{K}^0$ experimental branching ratio. An increase (decrease) in the value of the latter by 0.10 corresponds roughly to a decrease (increase) in the size of κ by 0.02 (*cf.* Eq. (38)).

3.5. Constraint from asymmetry

While the $A_{\rm CP}(B^+ \to \pi^+ K^0)$ asymmetry should be very small, its $B^+ \to K^+\bar{K}^0$ counterpart may be significant, as it results from an interference of two penguin contributions of comparable sizes (see Eq. (20)). Thus, the analysis of this asymmetry, given by

$$A_{\rm CP}\left(B^+ \to K^+\bar{K}^0\right) \;=\; -\,\frac{2z\sin\zeta\sin\gamma}{1+z^2-2z\cos\zeta\cos\gamma}\,, \tag{41}$$

may provide us with important information on z and ζ.

The experimental data on the $A_{\rm CP}(B^+ \to K^+\bar{K}^0)$ asymmetry impose another condition (via Eq. (41)) on the allowed values of z and ζ. The curves along which the asymmetry is constant (for the experimental values of $0.12^{+0.17}_{-0.18}$) are shown in Fig. 3 as dashed lines. Their positioning is independent of SU(3) breaking.

In the SU(3)-breaking case the two constraints cross in the region:

$$z \;\approx\; 1.8 \text{ to } 2.3 \quad (\text{for } \kappa = 0.6 \text{ to } 0.5)\,, \qquad \zeta \approx -15^\circ \text{ to } 0^\circ\,. \tag{42}$$

3.6. Implications for C/T

Let us now return to $B \to \pi\pi$. In the following we accept the central values of x, d, Δ, and θ. From Eqs. (9),(13),(17), one then gets (independently of θ)

$$z = 2.08\left|\frac{1-0.94\,e^{i59^\circ}C/T}{1+C/T}\right|\,. \tag{43}$$

For real C/T, approaching $z \approx 1$ (which for small ζ is equivalent to the SU(3) case), requires large positive values of C/T (around 0.8–1.0). On the other hand, the SU(3) breaking case (for small ζ corresponding to $z \approx 1.8$–2.3) clearly needs C/T close to zero. Thus, inclusion of SU(3) breaking in the hadronization stage is very important.

The fit of Ref. [8] does not produce large $|C/T|$ in the case of exact SU(3) since it does not take into account the condition of Eq. (33) (imposed by the relative size of the $B^+ \to \pi^+ K^0$ and $B^+ \to K^+\bar{K}^0$ branching ratios), which forces $z \leq 1$ for any ζ. Clearly, the fit should be reconsidered with the SU(3)-breaking expression of Eq. (21) and $\kappa \approx 0.5$.

For any z and ζ, the value of C/T may be evaluated from Eq. (17), provided the $B \to \pi\pi$ parameters are known sufficiently well. For central values of x, d, Δ, and θ, the relevant contour plot of $|C/T|$ is presented in Fig. 3, with the borders of the shadowed areas coresponding to $|C/T| = 0.1, 0.2, 0.5$. Lines of constant $\mathrm{Arg}(C/T)$ are also shown.

Fig. 3 explicitly demonstrates that for central values of $B \to \pi\pi$ parameters, the data on the $B^+ \to K^+\bar{K}^0$ branching ratio and asymmetry indicate that $|C/T|$ is small. If present errors in $B \to \pi\pi$ parameters are taken into account, the point $C/T = 0$ (small central blob in the figure) is shifted by $\Delta z = \pm 0.38$, $\Delta\zeta = \pm 13°$ (Eq. (19)). The qualitative conclusion is not changed. Further improvement in the accuracy of the measurement of the $B^+ \to K^+\bar{K}^0$ and $B \to \pi\pi$ decay parameters could provide us with more detailed information on C/T.

Although our analysis of $B^+ \to \pi^+K^0, K^+\bar{K}^0$ does not need or use the size of P_c, the latter may be estimated from the $B^+ \to \pi^+K^0$ branching ratio. Assuming small C/T, one gets

$$P_c = 1.03 \pm 0.02\,, \tag{44}$$

an update on the estimate given in [7].

4. Conclusions

We have adopted the old Regge model for high-energy soft processes in the description of SU(3) breaking in the hadronization stage of B decays. We have pointed out that in an analysis of the relative size of C, T and penguin amplitudes the decay $B^+ \to \pi^+K^0$ provides important information which has to be taken into account in addition to that obtained from $B \to \pi\pi$ and $B \to K\bar{K}$. We have shown that the data on $B \to \pi\pi$, π^+K^0, and $K^+\bar{K}^0$ consistently point to a small value of $|C/T|$. Further improvement in the accuracy of the relevant measurements could tell us more about the actual value of C/T.

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