BARYONS IN DIQUARK–QUARK MODEL

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In the framework of the quasi-particle diquark model, we explore the validation of describing the baryon as made up of a cluster of quark and diquark. The mass of the diquark is reduced in this quasi-particle approximation. The scalar and vector diquark masses are calculated for various quark combinations and these are used to estimate ground state and excited state baryon masses. The results are found to be in good agreement with experimental data.

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1. Introduction

The concept of diquarks is almost as old as that of the quarks. In the ordinary quark model, hadrons are $q\bar{q}$ or qqq states of mesons and baryons, respectively. However, Gell-Mann [1] himself suggested the idea that q–q bound states or diquarks can be formed within the hadron. The idea of a diquark within the baryon allows us to reduce a 3-body (involving q–q–q) problem to a 2-body (involving q–qq) one. The simplification has resulted in a vast volume of work on various baryon properties using the concept of diquark. Exotic hadron masses have been predicted by Lichtenberg [2] using a quark–diquark model. Recently, Gutierrez *et al.* [3] have suggested a relativistic quark–diquark model for the nucleon, where the interaction potential is taken to be of harmonic oscillator type. Assuming the proton to be a quark–diquark system, Bialas *et al.* [4] have investigated small momentum transfer elastic p–p cross-section at high energies. Maris [5] has calculated charge radius for scalar ud diquark and compared it to a corresponding pion radius. A number of studies on the ud diquark radius have

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commented on its largeness, finding its size comparable to that of the proton [2,6]. In this context, Ahlig *et al.* [7] have described baryons as bound states of a quark–diquark interacting through quark exchange. A simple model has been suggested by Santopinto [8] with direct and exchange interactions for studying baryon spectrum and form factors. Recently, we [9] have successfully calculated mass, binding energy, compressibility and excitation energy for Roper resonance of the proton assuming it to be quark–diquark system. Jaffe and Wilczek [10] have proposed the diquark model to explain the pentaquark structure. Diquark masses have been calculated from lattice QCD [11,12]. Hess *et al.* [11] have presented results for diquark correlation functions calculated in the Landau gauge on the lattice and the masses have been extracted from the long distance behaviour of the corelation functions. Ram *et al.* [13] and de Castro *et al.* [14] have investigated diquark mass and radius and used them to reproduce hadron properties successfully further strengthening the diquark theory.

In the present study we have considered the baryon to be a two-body system made up of the quark–diquark. The diquark has been described in analogy with the quasi-particle picture in many-body systems. The effective mass of the diquark has been calculated in this approach, which has already been successfully employed by us in studying hadrons and multiquark states [9,15,16]. These calculated diquark masses have been used to calculate baryon masses and the results compared to experimental data.

The paper begins with Section 1, Introduction, following with the approach and calculation clearly outlined in Section 2, Formalism. In the same section a tabular representation of the obtained data has been provided. In Section 3, Conclusions, the results have been discussed and compared with other available estimates.

2. Formalism

The aim of the present work is to estimate diquark mass, and use the same to calculate baryon mass, using the concept of a quark bound to a localized cluster of two quarks simulating a diquark. We have proposed a quasi-particle picture of the diquark in the baryon [17] in analogy to the quasiparticle scenario in many body systems. The diquark is a strongly correlated quark pair. We assume the formation of the diquark in the presence of a background quark field inside the particle due to the gluon exchange interaction, and a harmonic oscillator type of confinement interaction. The interaction between the two quarks forming a diquark is assumed to be [18],

$$V_{ij} = -\frac{\alpha}{r} + (\boldsymbol{F}_i \cdot \boldsymbol{F}_j)\left(-\tfrac{1}{2}Kr^2\right) , \tag{1}$$

where α is the coupling constant, $\boldsymbol{F}_i \cdot \boldsymbol{F}_j = -\frac{2}{3}$ for qq interaction [18] and K is the strength parameter. Hence,

$$V_{ij} = -\frac{\alpha}{r} + ar^2 \,, \tag{2}$$

where $a = K/3$. The effective mass of the diquark in the background of the quark field gets modified in the quasiparticle approximation as it happens in the crystal lattice [17]. Due to the external force F which is responsible for the confinement of quarks and V the interaction potential between the quarks, the effective mass of the diquark can be written as [17],

$$\frac{m_{q_i} + m_{q_j}}{m_D} = 1 - \frac{1}{F}\frac{\delta V}{\delta r} \,, \tag{3}$$

where m_{q_i} are the constituent quark masses, m_D is the effective mass of the diquark. The effective mass reflects the inertia of the diquark subject to the external force F. The diquark system is affected by external force $-\overrightarrow{\nabla} V$. While retrieving the $\frac{\delta V}{\delta r}$ part we have taken the one-gluon exchange potential, since it is responsible for interaction between the constituent quarks and in F we use the confinement force. This leads us to,

$$\frac{m_{q_i} + m_{q_j}}{m_D} = 1 + \frac{\alpha}{2ar_D^3} \,, \tag{4}$$

where $\alpha = \frac{2}{3}\alpha_{\mathrm{s}}$ and α_{s} is taken as 0.59 for light sector and 0.2 for heavy sector [19], $a = 0.02$ GeV^3 and r_D is the effective radius of the diquark which is taken as input. We use the scalar radii $r_{ud} = 0.5$ fm [20], $r_{us/ds} = 0.6$ fm [5], $r_{cs} = 0.767$ fm, $r_{uc/dc} = 0.835$ fm, $r_{bs} = 0.717$ fm [14]. Vector diquark radii are $r_{ud} = 0.8$ fm [20], $r_{us/ds} = 1.006$ fm, $r_{cs} = 0.785$ fm, $r_{uc/dc} = 0.861$ fm [14]. Constituent quark masses are taken as $m_u = m_d = 360$ MeV, $m_s = 540$ MeV, $m_c = 1710$ MeV and $m_b = 5050$ MeV as suggested by Karliner and Lipkin [21]. Scalar and vector diquark masses are calculated with these inputs and have been displayed in Table I.

The baryon is considered to be a cluster of the diquark and a single quark. The mass of the baryon M_B is given by

$$M_B = m_q + m_D + E_{\mathrm{BE}} + E_{\mathrm{S}} \,, \tag{5}$$

where m_q is quark mass and m_D is diquark mass. $E_{\mathrm{BE}} = \langle \Psi | V_l | \Psi \rangle$ is the binding energy for the baryon, calculated by assuming a linear potential type of interaction between quark and diquark in the baryon. The potential is expressed as

$$V_l = \lambda r_B \,, \tag{6}$$

TABLE I

Mass of diquarks.

Diquark	Mass	
	Spin-0 [GeV]	Spin-1 [GeV]
ud	0.441	0.624
us/ds	0.659	0.835
cs	2.12	2.13
uc/dc	1.98	1.99
ub/db	5.14	5.15
bs	5.21	5.22

where λ is a suitable constant and r_B is the radius of the baryon. The spin interaction $E_{\rm S}$ is taken to be given by [22],

$$E_{\rm S} = \frac{8}{9}\frac{\alpha_{\rm s}}{m_q m_D}\overrightarrow{S_q}\cdot\overrightarrow{S_D}|\Phi(0)|^2\,, \tag{7}$$

where $\overrightarrow{S_q}$ and $\overrightarrow{S_D}$ are the spins of the single quark and diquark, respectively.

The wave function for the baryon is taken to be that proposed by the Statistical Model [23–25] for hadrons which is found to be very successful in describing the different properties of hadrons. In this model, the hadron is assumed to be constituted of a virtual $q\overline{q}$ in addition to the valence partners which determines the quantum number of the colourless hadron. The quarks, real and virtual, are assumed to be of same colour and flavour so that they may be regarded as identical and indistinguishable, and are treated on the same footing. The indistinguishability of the valence quark with the virtual partner calls for the existence of quantum mechanical uncertainty in its available phase space. The valence quarks are assumed to be non-interacting with each other and considered to be moving almost independently in conformity with the experimental finding of asymptotic freedom. However, the valence quarks are considered to be moving in an average smooth background potential due to their interaction with the virtual sea. With the above consideration we arrive at an expression for the probability density of the baryon in the ground state as

$$|\phi(r)|^2 = \frac{315}{64\pi r_B^{9/2}}(r_B - r)^{3/2}\Theta(r_B - r)\,, \tag{8}$$

where r_B is the radius parameter of the meson and $\Theta(r_B - r)$ represents the step function. Using this wave function, the spin contribution and binding energy of the baryon are calculated and substituted in the mass formula in

Eq. (5). The baryon radii taken are those calculated using the Statistical Model [26] except for the radius of Ω_b^- which has been taken from the work of Brac [27]. We have chosen $\lambda = 0.11$ GeV^2 [28]. The values of ground state and excited state baryon masses are calculated by substituting scalar and vector diquark masses, respectively, in the mass formula of the baryon. The spin contribution is also modified accordingly. Mass difference between excited and ground states are calculated and the entire results have been presented alongside current experimental values in Table II. The comparison between our calculation and experiment is found to be favourable.

TABLE II

Ground state (Gr. st.) and excited state (Exc. st.) masses from theory and experiment [30,31] along with their differences (Diff.).

Baryon	Exp. mass Gr. st. [GeV]	Th. mass Gr. st. [GeV]	Diff. [MeV]	Exp. mass Exc. st. [GeV]	Th. mass Exc. st. [GeV]	Diff. [MeV]
N	0.939	1.021	81.5	1.232	1.232	0.0
Λ_c^+	2.286	2.466	180.0	2.628	2.649	20.9
Σ_c^+	2.452	2.330	123.0	2.517	2.518	0.5
Ξ_c^0	2.471	2.502	31.0	2.646	2.684	37.9
Λ_b^0	5.620	5.570	50.0	—	5.770	—
Σ_b^+	5.807	5.560	247.0	5.829	5,765	64.0
Ξ_b^0	5.7924 ± 0.003	5.770	22.4	—	5.970	—
Ω_b^-	6.0544 ± 0.007	5.950	104.4	—	5.960	—

3. Conclusion

The paper presents an estimate of diquark mass, using which masses of certain heavy baryons have been calculated. The quasi-particle approach to studying diquarks [15–17] allows us to predict diquark mass, with input of diquark radius. Experimental results are not available for comparison of diquark mass. However, a comparison with calculations performed by others is found to be satisfactory. It may be noted that Jaffe and Wilczek [29] have calculated mass of *ud* diquark to be 420 MeV, whereas our calculation is 429.6 MeV. Others have also calculated *ud* diquark mass but with varying results [13,14]. The baryon masses calculated using these diquark masses are found to compare favourably with the experiment [30]. We find that the excited state Λ_c^+ baryon mass is 2.64 GeV which experiment puts at 2.62 GeV. Similarly, our result for the higher state of the Σ_c^+ baryon mass is 2.518 GeV, whereas experimental value is 2.5175 GeV. Excellent agreement is found for both Ξ_c^0 and Ξ_b^0 states as well. Though the Σ_b^+ masses do not agree so well with the experiment, the deviation from experimental results is less than 10 percent for all the baryons. However, we have calculated the value of Ω_b^- baryon mass as 5.95 GeV, whereas the recently reported mass of it is $6054.4 \pm 6.8 \pm 0.9$ MeV/c^2 [31] by CDF Collaboration. In this case we get the deviation from the measured mass as 104.4 MeV.

The spin-0 and spin-1 diquark masses are found to differ appreciably in our study, as proposed in the findings of Wilczek [32]. The calculation of diquark masses involves the input of diquark radii and constituent quark masses. Any errors in these will also be reflected in the diquark mass and hence in the baryon masses. We have also calculated the mass differences between the experimental and theoretical masses for the ground states and excited states of the baryons and they have been displayed in Table II. However, the comparison is not possible for the all the b baryons, except Σ_b^+, the higher states not being very stable, making experimental masses unavailable. The comparison is found to be in good agreement in most cases with the exception of Λ_c^+ and Σ_b^+.

A significant mass difference between baryons in the ground state and excited state is also seen, as expected. The mass difference, between these particles is found to arise from scalar–axial diquark mass difference, and difference in spin contribution. However, these factors are not the only ones responsible-contribution of some other factors, like quark–antiquark annihilation [33] between the single quark and diquark can also affect the mass of the excited state baryon as compared to the ground state. The calculations in the present work have been performed using the non-relativistic model, widely used in hadron physics [34–36]. It has been found that the non-relativistic model works well even when studying light quarks [37]. Moreover, the spurious excitation of the centre of mass motion can be eliminated in the non-relativistic framework. The restriction to the non-relativistic framework cannot prevent us from providing a validation of diquark clustering in the baryon.

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