

$\eta$ - $\eta'$  MIXING IN  $\eta$ -MESIC NUCLEI\*

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*(Received July 27, 2010)*

$\eta$  bound states in nuclei are sensitive to the flavour-singlet component in the  $\eta$ . The bigger the singlet component, the more attraction and the greater the binding.  $\eta$ - $\eta'$  mixing plays an important role in understanding the value of the  $\eta$ -nucleon scattering length  $a_{\eta N}$ . Working with the Quark Meson Coupling model, we find a factor of two enhancement from mixing relative to the prediction with a pure octet  $\eta$ .

PACS numbers: 11.30.Rd, 14.40.Be, 21.65.Jk

**1. Introduction**

Measurements of the pion, kaon and  $\eta$ -meson masses and their interactions in finite nuclei provide new constraints on our understanding of dynamical symmetry breaking in low energy QCD [1]. The  $\eta$  nucleon interaction is attractive suggesting that  $\eta$  mesons may form strong-interaction bound-states in nuclei. There is presently a vigorous experimental programme to search for evidence of these bound states [2]. Here we explain that for the  $\eta$  the in-medium mass  $m_{\eta}^*$  is sensitive to the flavour-singlet component in the  $\eta$ , and hence to the non-perturbative glue associated with axial U(1) dynamics. An important source of the in-medium mass modification comes from light-quarks coupling to the scalar  $\sigma$  mean-field in the nucleus [3, 4]. Increasing the flavour-singlet component in the  $\eta$  at the expense of the octet component gives more attraction, more binding and a larger value of the

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\* Presented by Steven D. Bass at the International Symposium on Mesic Nuclei, Kraków, Poland, June 16, 2010.

$\eta$  nucleon scattering length,  $a_{\eta N}$  [5]. Since the mass shift is approximately proportional to the  $\eta$  nucleon scattering length, it follows that the physical value of  $a_{\eta N}$  should be larger than if the  $\eta$  were a pure octet state.

## 2. QCD considerations

Spontaneous chiral symmetry breaking suggests an octet of would-be Goldstone bosons: the octet associated with chiral  $SU(3)_L \otimes SU(3)_R$  plus a singlet boson associated with axial  $U(1)$  — each with mass squared  $m_{\text{Goldstone}}^2 \sim m_q$ . The physical  $\eta$  and  $\eta'$  masses are about 300–400 MeV too big to fit in this picture. One needs extra mass in the singlet channel associated with non-perturbative topological gluon configurations and the QCD axial anomaly; for reviews and related phenomenology see Refs. [6–8]<sup>1</sup>. The strange quark mass induces considerable  $\eta$ – $\eta'$  mixing. For free mesons the  $\eta$ – $\eta'$  mass matrix (at leading order in the chiral expansion) is

$$M^2 = \begin{pmatrix} \frac{4}{3}m_K^2 - \frac{1}{3}m_\pi^2 & -\frac{2}{3}\sqrt{2}(m_K^2 - m_\pi^2) \\ -\frac{2}{3}\sqrt{2}(m_K^2 - m_\pi^2) & [\frac{2}{3}m_K^2 + \frac{1}{3}m_\pi^2 + \tilde{m}_{\eta_0}^2] \end{pmatrix}. \quad (1)$$

Here  $\tilde{m}_{\eta_0}^2$  is the gluonic mass term which has a rigorous interpretation through the Witten–Veneziano mass formula [10, 11] and which is associated with non-perturbative gluon topology, related perhaps to confinement [12] or instantons [13]. The masses of the physical  $\eta$  and  $\eta'$  mesons are found by diagonalizing this matrix, *viz.*

$$\begin{aligned} |\eta\rangle &= \cos\theta |\eta_8\rangle - \sin\theta |\eta_0\rangle, \\ |\eta'\rangle &= \sin\theta |\eta_8\rangle + \cos\theta |\eta_0\rangle, \end{aligned} \quad (2)$$

where

$$\eta_0 = \frac{1}{\sqrt{3}} (u\bar{u} + d\bar{d} + s\bar{s}), \quad \eta_8 = \frac{1}{\sqrt{6}} (u\bar{u} + d\bar{d} - 2s\bar{s}). \quad (3)$$

One obtains values for the  $\eta$  and  $\eta'$  masses:

$$m_{\eta',\eta}^2 = (m_K^2 + \tilde{m}_{\eta_0}^2/2) \pm \frac{1}{2}\sqrt{(2m_K^2 - 2m_\pi^2 - \frac{1}{3}\tilde{m}_{\eta_0}^2)^2 + \frac{8}{9}\tilde{m}_{\eta_0}^4}. \quad (4)$$

The physical mass of the  $\eta$  and the octet mass  $m_{\eta_8} = \sqrt{\frac{4}{3}m_K^2 - \frac{1}{3}m_\pi^2}$  are numerically close, within a few percent. However, to build a theory of the  $\eta$  on the octet approximation risks losing essential physics associated with the singlet component.

<sup>1</sup> The QCD axial anomaly also features in discussion of the proton spin puzzle [9].

Turning off the gluonic term in Eq. (4) one finds the expressions  $m_{\eta'} \sim \sqrt{2m_K^2 - m_\pi^2}$  and  $m_\eta \sim m_\pi$ . That is, without extra input from glue, in the OZI limit, the  $\eta$  would be approximately an isosinglet light-quark state ( $\frac{1}{\sqrt{2}}|\bar{u}u + \bar{d}d\rangle$ ) degenerate with the pion and the  $\eta'$  would be a strange-quark state  $|\bar{s}s\rangle$  — mirroring the isoscalar vector  $\omega$  and  $\phi$  mesons. Taking the value  $\tilde{m}_{\eta_0}^2 = 0.73 \text{ GeV}^2$  in the leading-order mass formula, Eq. (4), gives agreement with the physical masses at the 10% level. This value is obtained by summing over the two eigenvalues in Eq. (4):  $m_\eta^2 + m_{\eta'}^2 = 2m_K^2 + \tilde{m}_{\eta_0}^2$  and substituting the physical values of  $m_\eta$ ,  $m_{\eta'}$  and  $m_K$  [11]. The corresponding  $\eta$ - $\eta'$  mixing angle  $\theta \simeq -18^\circ$  is within the range  $-17^\circ$  to  $-20^\circ$  obtained from a study of various decay processes in [14, 15]. The key point of Eq. (4) is that mixing and gluon dynamics play a crucial role in both the  $\eta$  and  $\eta'$  masses and that treating the  $\eta$  as an octet pure would-be Goldstone boson risks losing essential physics.

### 3. The axial anomaly and $\tilde{m}_{\eta_0}^2$

What can QCD tell us about the behaviour of the gluonic mass contribution in the nuclear medium?

The physics of axial U(1) degrees of freedom is described by the U(1)-extended low-energy effective Lagrangian [11]. In its simplest form this reads

$$\begin{aligned} \mathcal{L} = & \frac{F_\pi^2}{4} \text{Tr} \left( \partial^\mu U \partial_\mu U^\dagger \right) + \frac{F_\pi^2}{4} \text{Tr} M \left( U + U^\dagger \right) \\ & + \frac{1}{2} iQ \text{Tr} \left[ \log U - \log U^\dagger \right] + \frac{3}{\tilde{m}_{\eta_0}^2 F_0^2} Q^2. \end{aligned} \quad (5)$$

Here  $U = \exp i \left( \Phi/F_\pi + \sqrt{\frac{2}{3}}\eta_0/F_0 \right)$  is the unitary meson matrix where  $\Phi = \sum \pi_a \lambda_a$  denotes the octet of would-be Goldstone bosons associated with spontaneous chiral  $SU(3)_L \otimes SU(3)_R$  breaking and  $\eta_0$  is the singlet boson. In Eq. (5)  $Q$  denotes the topological charge density ( $Q = \frac{\alpha_s}{4\pi} G_{\mu\nu} \tilde{G}^{\mu\nu}$ );  $M = \text{diag}[m_\pi^2, m_\pi^2, 2m_K^2 - m_\pi^2]$  is the quark-mass induced meson mass matrix. The pion decay constant  $F_\pi = 92.4 \text{ MeV}$  and  $F_0$  is the flavour-singlet decay constant,  $F_0 \sim F_\pi \sim 100 \text{ MeV}$  [14].

The flavour-singlet potential involving  $Q$  is introduced to generate the gluonic contribution to the  $\eta$  and  $\eta'$  masses and to reproduce the anomaly in the divergence of the gauge-invariantly renormalised flavour-singlet axial-vector current. The gluonic term  $Q$  is treated as a background field with no kinetic term. It may be eliminated through its equation of motion to

generate a gluonic mass term for the singlet boson, *viz.*

$$\frac{1}{2}iQ\text{Tr}[\log U - \log U^\dagger] + \frac{3}{\tilde{m}_{\eta_0}^2 F_0^2} Q^2 \mapsto -\frac{1}{2}\tilde{m}_{\eta_0}^2 \eta_0^2. \quad (6)$$

The interactions of the  $\eta$  and  $\eta'$  with other mesons and with nucleons can be studied by coupling the Lagrangian Eq. (5) to other particles [16,17]. For example, the OZI violating interaction  $\lambda Q^2 \partial_\mu \pi_a \partial^\mu \pi_a$  is needed to generate the leading (tree-level) contribution to the decay  $\eta' \rightarrow \eta \pi \pi$  [17]. When iterated in the Bethe–Salpeter equation for meson–meson rescattering this interaction yields a dynamically generated exotic state with quantum numbers  $J^{PC} = 1^{-+}$  and mass about 1400 MeV [18]. This suggests a dynamical interpretation of the lightest-mass  $1^{-+}$  exotic observed at BNL and CERN.

To investigate what happens to  $\tilde{m}_{\eta_0}^2$  in the medium we first couple the  $\sigma$  (correlated two-pion) mean-field in nuclei to the topological charge density  $Q$  through adding the Lagrangian term

$$\mathcal{L}_{\sigma Q} = Q^2 g_\sigma^Q \sigma. \quad (7)$$

Here  $g_\sigma^Q$  denotes coupling to the  $\sigma$  mean field — that is, we consider an in-medium renormalization of the coefficient of  $Q^2$  in the effective chiral Lagrangian. Following the treatment in Eq. (6) we eliminate  $Q$  through its equation of motion. The gluonic mass term for the singlet boson then becomes

$$\tilde{m}_{\eta_0}^2 \mapsto \tilde{m}_{\eta_0}^{*2} = \tilde{m}_{\eta_0}^2 \frac{1 + 2x}{(1 + x)^2} < \tilde{m}_{\eta_0}^2, \quad (8)$$

where

$$x = \frac{1}{3} g_\sigma^Q \sigma \tilde{m}_{\eta_0}^2 F_0^2. \quad (9)$$

That is, *the gluonic mass term decreases in-medium* independent of the sign of  $g_\sigma^Q$  and the medium acts to partially neutralize axial U(1) symmetry breaking by gluonic effects.

This discussion motivates the *existence* of medium modifications to  $\tilde{m}_{\eta_0}^2$  in QCD<sup>2</sup>. However, a rigorous calculation of  $m_\eta^*$  from QCD is beyond present theoretical technology. Hence, one has to look to QCD motivated models and phenomenology for guidance about the numerical size of the effect. The physics described in Eqs. (1)–(4) tells us that the simple octet approximation may not suffice.

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<sup>2</sup> In the chiral limit the singlet analogy to the Weinberg–Tomozawa term does not vanish because of the anomalous glue terms. Starting from the simple Born term one finds anomalous gluonic contributions to the singlet-meson nucleon scattering length proportional to  $\tilde{m}_{\eta_0}^2$  and  $\tilde{m}_{\eta_0}^4$  [19].

## 4. The $\eta$ in nuclei

### 4.1. QCD inspired models

Meson masses in nuclei are determined from the scalar induced contribution to the meson propagator evaluated at zero three-momentum,  $\vec{k} = 0$ , in the nuclear medium. Let  $k = (E, \vec{k})$  and  $m$  denote the four-momentum and mass of the meson in free space. Then, one solves the equation

$$k^2 - m^2 = \text{Re } \Pi \left( E, \vec{k}, \rho \right) \quad (10)$$

for  $\vec{k} = 0$  where  $\Pi$  is the in-medium  $s$ -wave meson self-energy. Contributions to the in medium mass come from coupling to the scalar  $\sigma$  field in the nucleus in mean-field approximation, nucleon-hole and resonance-hole excitations in the medium. The  $s$ -wave self-energy can be written as [20]

$$\Pi \left( E, \vec{k}, \rho \right) \Big|_{\{\vec{k}=0\}} = -4\pi\rho \left( \frac{b}{1 + b \langle \frac{1}{r} \rangle} \right). \quad (11)$$

Here  $\rho$  is the nuclear density,  $b = a(1 + \frac{m}{M})$  where  $a$  is the meson-nucleon scattering length,  $M$  is the nucleon mass and  $\langle \frac{1}{r} \rangle$  is the inverse correlation length,  $\langle \frac{1}{r} \rangle \simeq m_\pi$  for nuclear matter density [20]. ( $m_\pi$  is the pion mass.) Attraction corresponds to positive values of  $a$ . The denominator in Eq. (11) is the Ericson-Ericson-Lorentz-Lorenz double scattering correction.

What should we expect for the  $\eta$  and  $\eta'$ ?

This physics with  $\eta$ - $\eta'$  mixing has been investigated by Bass and Thomas [5]. Phenomenology is used to estimate the size of the effect in the  $\eta$  using the Quark Meson Coupling model (QMC) of hadron properties in the nuclear medium [4]. Here one uses the large  $\eta$  mass (which in QCD is induced by mixing and the gluonic mass term) to motivate taking an MIT Bag description for the  $\eta$  wavefunction, and then coupling the light (up and down) quark and antiquark fields in the  $\eta$  to the scalar  $\sigma$  field in the nucleus working in mean-field approximation [4]. The coupling constants in the model for the coupling of light-quarks to the  $\sigma$  (and  $\omega$  and  $\rho$ ) mean-fields in the nucleus are adjusted to fit the saturation energy and density of symmetric nuclear matter and the bulk symmetry energy. The strange-quark component of the wavefunction does not couple to the  $\sigma$  field and  $\eta$ - $\eta'$  mixing is readily built into the model.

Increasing the mixing angle increases the amount of singlet relative to octet components in the  $\eta$ . This produces greater attraction through increasing the amount of light-quark compared to strange-quark components in the  $\eta$  and a reduced effective mass. Through Eq. (11), increasing the mixing angle also increases the  $\eta$  nucleon scattering length  $a_{\eta N}$ . The model

results are shown in Table I. The values of  $\text{Re } a_\eta$  quoted in Table I are obtained from substituting the in-medium and free masses into Eq. (11) with the Ericson–Ericson denominator turned-off (since we choose to work in mean-field approximation), and using the free mass  $m = m_\eta$  in the expression for  $b$ <sup>3</sup>. The QMC model makes no claim about the imaginary part of the scattering length. The key observation is that  $\eta$ – $\eta'$  mixing with the phenomenological mixing angle  $-20^\circ$  leads to a factor of two increase in the mass-shift and in the scattering length obtained in the model relative to the prediction for a pure octet  $\eta_8$ . This result may explain why values of  $a_{\eta N}$  extracted from phenomenological fits to experimental data where the  $\eta$ – $\eta'$  mixing angle is unconstrained give larger values than those predicted in theoretical models where the  $\eta$  is treated as a pure octet state — see below.

TABLE I

Physical masses fitted in free space, the bag masses in medium at normal nuclear-matter density,  $\rho_0 = 0.15 \text{ fm}^{-3}$ , and corresponding meson-nucleon scattering lengths (calculated at the mean-field level with the Ericson–Ericson–Lorentz–Lorenz factor switched off).

	$m$ (MeV)	$m^*$ (MeV)	$\text{Re } a$ (fm)
$\eta_8$	547.75	500.0	0.43
$\eta$ ( $-10^\circ$ )	547.75	474.7	0.64
$\eta$ ( $-20^\circ$ )	547.75	449.3	0.85
$\eta_0$	958	878.6	0.99
$\eta'$ ( $-10^\circ$ )	958	899.2	0.74
$\eta'$ ( $-20^\circ$ )	958	921.3	0.47

The density dependence of the mass-shifts in the QMC model is discussed in Ref. [4]. Neglecting the Ericson–Ericson term, the mass-shift is approximately linear for densities  $\rho$  between 0.5 and 1 times  $\rho_0$  (nuclear matter density) we find

$$m_\eta^*/m_\eta \simeq 1 - 0.17\rho/\rho_0 \quad (12)$$

for the mixing angle  $-20^\circ$ . The scattering lengths extracted from this analysis are density independent to within a few percent over the same range of densities.

Present experiments [2] are focused on searches for  $\eta$ -mesic helium. QMC model calculations for finite nuclei are reported in [4]. For an octet  $\eta$ ,  $\eta_8$ , one finds a binding energy of 10.7 MeV in  ${}^6\text{He}$ . (This binding energy is expected to double with  $\eta$ – $\eta'$  mixing included.) Calculations of the

<sup>3</sup> The effect of exchanging  $m$  for  $m^*$  in  $b$  is a 5% increase in the quoted scattering length.

$\rho$ -meson mass in  ${}^3\text{He}$  and  ${}^4\text{He}$  are reported in [21]. One finds that the average mass for a  $\rho$  meson formed in  ${}^3\text{He}$  and  ${}^4\text{He}$  is expected to be around 730 and 690 MeV.

#### 4.2. Comparison with $\eta$ phenomenology and other models

It is interesting to compare these results with other studies and the values of  $a_{\eta N}$  and  $a_{\eta' N}$  extracted from phenomenological fits to experimental data.

The  $\eta$ -nucleon interaction is characterised by a strong coupling to the  $S_{11}(1535)$  nucleon resonance. For example,  $\eta$ -meson production in proton–nucleon collisions close to threshold is known to proceed via a strong isovector exchange contribution with excitation of the  $S_{11}(1535)$ . Recent measurements of  $\eta'$  production suggest a different mechanism for this meson [22]. Different model procedures lead to different values of the  $\eta$  nucleon scattering length with real part between about 0.2 fm and 0.9 fm.

In quark models the  $S_{11}$  is interpreted as a 3-quark state:  $(1s)^2(1p)$ . This interpretation has support from quenched lattice calculations [23] which also suggest that the  $\Lambda(1405)$  resonance has a significant non 3-quark component. In the Cloudy Bag Model the  $\Lambda(1405)$  is dynamically generated in the kaon–nucleon system [24].

*Phenomenological determinations of  $a_{\eta N}$  and  $a_{\eta' N}$ :* Green and Wycech [25] have performed phenomenological  $K$ -matrix fits to a variety of near-threshold processes ( $\pi N \rightarrow \pi N$ ,  $\pi N \rightarrow \eta N$ ,  $\gamma N \rightarrow \pi N$  and  $\gamma N \rightarrow \eta N$ ) to extract a value for the  $\eta$ -nucleon scattering. In these fits the  $S_{11}(1535)$  is introduced as an explicit degree of freedom — that is, it is treated like a 3-quark state — and the  $\eta$ - $\eta'$  mixing angle is taken as a free parameter. The real part of  $a_{\eta N}$  extracted from these fits is 0.91(6) fm for the on-shell scattering amplitude.

From measurements of  $\eta$  production in proton–proton collisions close to threshold, COSY-11 have extracted a scattering length  $a_{\eta N} \simeq 0.7 + i 0.4$  fm from the final state interaction (FSI) based on the effective range approximation [26]. For the  $\eta'$ , COSY-11 have deduced a conservative upper bound on the  $\eta'$ -nucleon scattering length  $|\text{Re } a_{\eta' N}| < 0.8$  fm [27] with a preferred a value between 0 and 0.1 fm [28] obtained by comparing the FSI in  $\pi^0$  and  $\eta'$  production in proton–proton collisions close to threshold.

*Chiral Models:* Chiral models involve performing a coupled channels analysis of  $\eta$  production after multiple rescattering in the nucleus which is calculated using the Lippmann–Schwinger [29] or Bethe–Salpeter [30] equations with potentials taken from the SU(3) chiral Lagrangian for low-energy QCD. In these chiral model calculations the  $\eta$  is taken as pure octet state ( $\eta = \eta_8$ ) with no mixing and the singlet sector turned off. These calcula-

tions yield a small mass shift in nuclear matter  $m_{\eta}^*/m_{\eta} \simeq 1 - 0.05\rho/\rho_0$ . The values of the  $\eta$ -nucleon scattering length extracted from these chiral model calculations are  $0.2 + i 0.26$  fm [29] and  $0.26 + i 0.24$  fm [30] with slightly different treatment of the intermediate state mesons. Chiral coupled channels models with an octet  $\eta = \eta_8$  agree with lattice and Cloudy Bag model predictions for the  $\Lambda(1405)$  and differ for the  $S_{11}(1535)$ , which is interpreted as a  $K\Sigma$  quasi-bound state in these coupled channel calculations [31].

## 5. Conclusions

$\eta$ - $\eta'$  mixing plays a vital role in the  $\eta$ -nucleon and -nucleus interactions. The greater the flavour-singlet component in the  $\eta$ , the greater the  $\eta$  binding energy in nuclei through increased attraction and the smaller the value of  $m_{\eta}^*$ . Through Eq. (11), this corresponds to an increased  $\eta$ -nucleon scattering length  $a_{\eta N}$ , greater than the value one would expect if the  $\eta$  were a pure octet state. Measurements of  $\eta$  bound-states in nuclei are therefore a probe of singlet axial U(1) dynamics in the  $\eta$ .

We thank K. Tsushima for helpful communications. S.D.B. thanks P. Moskal for the invitation to talk at this stimulating meeting. The research of S.D.B. is supported by the Austrian Science Fund, FWF, through grant P20436, while A.W.T. is supported by the Australian Research Council through an Australian Laureate Fellowship and by the University of Adelaide.

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