Vol. 41 (2010) *ACTA PHYSICA POLONICA B* No 11

THREE-PARTY CONTROLLED QUANTUM TELEPORTATION WITH SIX-PHOTON ENTANGLED STATES VIA COLLECTIVE NOISE CHANNEL

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(Received July 5, 2010; revised version received August 16, 2010)

Two three-party controlled quantum teleportation protocols using six-photon entangled states are proposed for circumventing collective noise. It can be performed in collective-dephasing noise or collective-rotation noise with unitary successful probability. Due to the symmetry of the quantum channel, each participant can act as a sender, a receiver or a controller. Moreover, it can be generalized to multiparty controlled teleportation protocols.

PACS numbers: 03.67.Hk, 03.67.Dd, 03.65.Ud

1. Introduction

Quantum teleportation is a protocol for transmitting the state of a quantum system from some place to other place without transmitting the system itself. In 1993, Bennett *et al.* [1] first showed that quantum entanglement can be used to teleport an unknown quantum state with the help of classical information. Since then, the theory [2–12] and experiment [13] with respect to quantum teleportation is always an interesting topic based on its important applications in quantum computation and quantum communication.

Controlled quantum teleportation can be realized when a third one (a controller) participates [14–18]. A receiver cannot obtain the original state without the cooperation of a controller. Controlled teleportation can be used to complete many important assignments, such as quantum communication network, joint computation, quantum voting, quantum conference.

In the practical process of teleportation, channel noise increases the error rate of the teleported state. Especially, in optical communications, one kind of main noise results from fluctuation of birefringence of optical fiber, where the adjacent photons are affected by the same noise, collective noise, so some applicable methods must be adopted to conquer the collective noise.

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Generally speaking, there are two ways to solve the problem of noise. One is a quantum repeater [19, 20]. Placing some repeaters in spots on quantum channel can increase the length of communication. The other is channel coding. Error tolerance coding [21–27] is one of the great channel coding in collective noise channel. Using it, quantum key distribution and quantum secret communication can be realized effectively, and it does not need a complex system.

Multi-partite entangled state has many applications in quantum information, and it has been studied extensively by many researchers [28–42]. Recently multi-partite entangled state has been generated in experiment [43–45], such as GHZ state, W state, cluster state. Within them, GHZ state has attracted more attention in fulfilling various quantum tasks for their properties of symmetry and long-range order.

In this paper, using one six-photon GHZ state and another six-photon entangled state which can be generated from GHZ state, we present two controlled quantum teleportation protocols against collective-dephasing noise and collective-rotation noise in Sec. 2 and Sec. 3, respectively. In Sec. 4, we concluded with discussion and summary.

2. Three-party controlled quantum teleportation via collective-dephasing noise channel

Collective-dephasing noise is one kind of collective noise. When a polarization beam $|H\rangle$ (horizonal polarization) or $|V\rangle$ (vertical polarization) passes through the channel, it will be influenced by the noise as

$$U_d|H\rangle = |H\rangle, U_d|V\rangle = e^{i\phi}|V\rangle \,. \tag{1}$$

ϕ is the parameter of the noise and it fluctuates with time.

A sender, Alice, wants to send an unknown single photon state to a receiver, Bob,

$$|\phi\rangle = (\alpha\,|H\rangle + \beta\,|V\rangle)_a \,. \tag{2}$$

As a controller, Charlie prepares a six-photon maximally entangled GHZ state

$$|\Psi_1\rangle = \frac{1}{\sqrt{2}}(|HVHVHV\rangle + |VHVHVH\rangle)_{A_1A_2B_1B_2C_1C_2}\,, \tag{3}$$

which has the property that it is unchanged when any two of the six photons pass through equal collective-dephasing noise channel.

And then, Charlie sends photons (A_1, A_2) to Alice, photons (B_1, B_2) to Bob. In the period of transmitting process, the six-photon GHZ state influenced by the collective dephasing noise can be described by

$$|\Psi_1\rangle \xrightarrow{U_d} |\Psi_1'\rangle = \frac{1}{\sqrt{2}} e^{i(\theta_1+\theta_2)}(|HVHVHV\rangle + |VHVHVH\rangle)_{A_1A_2B_1B_2C_1C_2} \,. \tag{4}$$

The six-photon GHZ state gains a global phase flip. That is to say, it can circumvent the influence of the noise in the distribution process.

The whole system can be expressed as

$$\begin{aligned} |\Psi\rangle &= |\phi\rangle\,|\Psi_1\rangle \\ &= \frac{1}{\sqrt{2}}(\alpha\,|H\rangle + \beta\,|V\rangle)_a(|HVHVHV\rangle + |VHVHVH\rangle)_{A_1A_2B_1B_2C_1C_2} \\ &= \tfrac{1}{2}\big|\phi^+\big\rangle_{aA_1}(\alpha\,|HVVHV\rangle + \beta\,|VHHVH\rangle)_{B_1A_2B_2C_1C_2} \\ &\quad +\tfrac{1}{2}\big|\phi^-\big\rangle_{aA_1}(\alpha\,|HVVHV\rangle - \beta\,|VHHVH\rangle)_{B_1A_2B_2C_1C_2} \\ &\quad +\tfrac{1}{2}\big|\psi^+\big\rangle_{aA_1}(\alpha\,|VHHVH\rangle + \beta\,|HVVHV\rangle)_{B_1A_2B_2C_1C_2} \\ &\quad +\tfrac{1}{2}\big|\psi^-\big\rangle_{aA_1}(\alpha\,|VHHVH\rangle - \beta\,|HVVHV\rangle)_{B_1A_2B_2C_1C_2} \,, \end{aligned} \tag{5}$$

where the Bell basis can be denoted as

$$\big|\phi^\pm\big\rangle = \frac{1}{\sqrt{2}}(|HH\rangle \pm |VV\rangle), \big|\psi^\pm\big\rangle = \frac{1}{\sqrt{2}}(|HV\rangle \pm |VH\rangle)\,. \tag{6}$$

In order to realize controlled quantum teleportation, three parties (Alice, Bob and Charlie) need to perform Hadmard operations which can be denoted as

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \tag{7}$$

on photons (A_2, B_2, C_1, C_2).

$$\begin{aligned} &|\Psi\rangle \xrightarrow{H_{A_2B_2C_1C_2}} \big|\Psi'\big\rangle \\ &= \tfrac{1}{2}\,\big|\phi^+\big\rangle_{aA_1}(\alpha\,|H\rangle\,|--+-\rangle + \beta\,|V\rangle\,|++-+\rangle)_{B_1A_2B_2C_1C_2} \\ &+\tfrac{1}{2}\,\big|\phi^-\big\rangle_{aA_1}(\alpha\,|H\rangle\,|--+-\rangle - \beta\,|V\rangle\,|++-+\rangle)_{B_1A_2B_2C_1C_2} \\ &+\tfrac{1}{2}\,\big|\psi^+\big\rangle_{aA_1}(\alpha\,|V\rangle\,|++-+\rangle + \beta\,|H\rangle\,|--+-\rangle)_{B_1A_2B_2C_1C_2} \\ &+\tfrac{1}{2}\,\big|\psi^-\big\rangle_{aA_1}(\alpha\,|V\rangle\,|++-+\rangle - \beta\,|H\rangle\,|--+-\rangle)_{B_1A_2B_2C_1C_2} \,, \end{aligned} \tag{8}$$

where $|+\rangle = \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)\,, |-\rangle = \frac{1}{\sqrt{2}}(|H\rangle - |V\rangle)\,.$

For teleporting the single photon state $|\phi\rangle_a$ denoted as Eq. (2), Alice performs a Bell-basis measurement on photons (a, A_1). Suppose the measurement result is $|\phi^+\rangle$, the whole system will collapse onto the following state

$$
\begin{aligned}
&(\alpha\,|H\rangle\,|--+-\rangle+\beta\,|V\rangle\,|++-+\rangle)_{B_1A_2B_2C_1C_2}\\
&=\tfrac{1}{4}\,(\alpha\,|H\rangle+\beta\,|V\rangle)_{B_1}\,(|HHHH\rangle-|VVVV\rangle+|HVHV\rangle-|VHVH\rangle\\
&+|HVVH\rangle-|VHHV\rangle+|HHVV\rangle-|VVHH\rangle)_{A_2B_2C_1C_2}\\
&+\tfrac{1}{4}\,(\alpha\,|H\rangle-\beta\,|V\rangle)_{B_1}\,(|HHVH\rangle-|VVHV\rangle+|HVVV\rangle-|VHHH\rangle\\
&+|VHVV\rangle-|HVHH\rangle+|VVVH\rangle-|HHHV\rangle)_{A_2B_2C_1C_2}\,. \qquad (9)
\end{aligned}
$$

Subsequently, Bob can obtain the original state by performing an appropriate unitary operation $U_i(i=1,2,3,4)$ on photon B_1 on the basis of the $\{|H\rangle,|V\rangle\}$ measurement result on photons (A_2,B_2,C_1,C_2), where

$$U_1=I\,,\qquad U_2=\sigma_z\,,\qquad U_3=\sigma_x\,,\qquad U_4=i\sigma_y\,. \qquad (10)$$

From Eq. (9) we can deduce that if the $\{|H\rangle,|V\rangle\}$ basis measurement result of photons (A_2, B_2, C_1, C_2) is one of the following ones $\{|HHHH\rangle$, $|VVVV\rangle$, $|HVHV\rangle$, $|VHVH\rangle$, $|HVVH\rangle$, $|VHHV\rangle$, $|HHVV\rangle$, $|VVHH\rangle\}$, that is, the number of measurement result $|V\rangle$ on photons (A_2,B_2,C_1,C_2) is even, Bob's photon B_1 is in the state $\langle\psi\rangle_{B_1}=\alpha\,|H\rangle+\beta\,|V\rangle$. Bob needs to perform a unitary operation U_1 on photon B_1 to recover the original state. On the contrary, if the number of measurement result $|V\rangle$ on photons (A_2,B_2,C_1,C_2) is odd, Bob's photon B_1 is in the state $\langle\psi\rangle_{B_1}=\alpha\,|H\rangle-\beta\,|V\rangle$. Bob needs to perform a unitary operation U_2 on photon B_1 to recover the original state. The similar discussions are expressed in Table I if the other Bell states are attained by Alice.

TABLE I

The relation among the $\{|H\rangle,|V\rangle\}$ basis measurement result on photons (A_2,B_2,C_1,C_2), the Bell-basis measurement result on photons (a,A_1), and the unitary operation U_i should be performed by Bob to recover the original state.

$A_2B_2C_1C_2$	$\|\phi^+\rangle_{aA_1}$	$\|\phi^-\rangle_{aA_1}$	$\|\psi^+\rangle_{aA_1}$	$\|\psi^-\rangle_{aA_1}$
Even $\|V\rangle$	U_1	U_2	U_3	U_4
Odd $\|V\rangle$	U_2	U_1	U_4	U_3

3. Three-party controlled quantum teleportation via collective-rotation noise channel

Collective-rotation noise is another kind of collective noise. When a polarization beam $|H\rangle$ or $|V\rangle$ passes through the channel, it will be affected by the noise as

$$
\begin{aligned}
U_{\mathrm{r}}|H\rangle &= \cos\theta|H\rangle + \sin\theta|V\rangle\,, \\
U_{\mathrm{r}}|V\rangle &= -\sin\theta|H\rangle + \cos\theta|V\rangle\,,
\end{aligned} \tag{11}
$$

where θ is the parameter of the noise.

At first, Charlie prepares the following six-photon entangled state

$$
|\Psi_2\rangle = \frac{1}{\sqrt{2}}\Big(\left|\phi^+\right\rangle_{A_1A_2}\left|\phi^+\right\rangle_{B_1B_2}\left|\phi^+\right\rangle_{C_1C_2} + \left|\psi^-\right\rangle_{A_1A_2}\left|\psi^-\right\rangle_{B_1B_2}\left|\psi^-\right\rangle_{C_1C_2}\Big)\,, \tag{12}
$$

which has the merit that it is unchanged under the equal rotations on the six photons when they via equal path in optical fiber. It can be achieved by performing Hadmard operations denoted by Eq. (7) on photons (A_2, B_2, C_2) and CNOT operations which can be expressed as

$$
\mathrm{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}\,, \tag{13}
$$

with photons (A_1, B_1, C_1) as controlled bits and corresponding photons (A_2, B_2, C_2) as target bits on the following GHZ state

$$
|\mathrm{GHZ}\rangle = \frac{1}{\sqrt{2}}(|HHHHHH\rangle + |VVVVVV\rangle)_{A_1A_2B_1B_2C_1C_2}\,. \tag{14}
$$

And then, Charlie sends photons (A_1, A_2) to Alice, photons (B_1, B_2) to Bob. In the transmitting process, the six-photon state can circumvent the effect of the collective-rotation noise. The whole system can be expressed as

$$
\begin{aligned}
&|\Phi\rangle = (\alpha\,|H\rangle + \beta\,|V\rangle)_a\,|\Psi_2\rangle = (\alpha\,|H\rangle + \beta\,|V\rangle)_a \\
&\times\Big(\left|\phi^+\right\rangle_{A_1A_2}\left|\phi^+\right\rangle_{B_1B_2}\left|\phi^+\right\rangle_{C_1C_2} + \left|\psi^-\right\rangle_{A_1A_2}\left|\psi^-\right\rangle_{B_1B_2}\left|\psi^-\right\rangle_{C_1C_2}\Big)\,.
\end{aligned} \tag{15}
$$

Different from the former one against collective-dephasing noise, it only needs Charlie to perform Hadmard operation on photon C_2 to realize the control function in the protocol

$$
\begin{aligned}
&|\Phi\rangle \xrightarrow{H_{C_2}} \left|\Phi'\right\rangle \\
&= \frac{1}{\sqrt{2}}(\alpha\,|H\rangle + \beta\,|V\rangle)_a\Big[\Big(\left|\phi^+\right\rangle_{A_1A_2}\left|\phi^+\right\rangle_{B_1B_2} + \left|\psi^-\right\rangle_{A_1A_2}\left|\psi^-\right\rangle_{B_1B_2}\Big) \\
&\times(|HH\rangle - |VV\rangle)_{C_1C_2} + \Big(\left|\phi^+\right\rangle_{A_1A_2}\left|\phi^+\right\rangle_{B_1B_2} - \left|\psi^-\right\rangle_{A_1A_2}\left|\psi^-\right\rangle_{B_1B_2}\Big) \\
&\times(|HV\rangle + |VH\rangle)_{C_1C_2}\Big].
\end{aligned} \tag{16}
$$

After performing Hadmard operation, Charlie performs $\{|H\rangle, |V\rangle\}$ basis measurement on photons (C_1, C_2). Subsequently, Alice performs a Bell-basis measurement on photons (a, A_1) and $\{|H\rangle, |V\rangle\}$ basis measurement on photon A_2. And thus, Bob can obtain the original state by performing an appropriate unitary operation U_i on photon B_1 according to the classical information from Alice and Charlie and the $\{|H\rangle, |V\rangle\}$ basis measurement result on photon B_2 performed by himself. The explicit details are expressed in Table II.

TABLE II

The relation among the $\{|H\rangle, |V\rangle\}$ basis measurement result on photons (C_1, C_2) and photons (A_2, B_2), the Bell-basis measurement result on photons (a, A_1), and the unitary operation U_i should be performed by Bob to recover the original state.

C_1C_2	A_2B_2	$\|\phi^+\rangle_{aA_1}$	$\|\phi^-\rangle_{aA_1}$	$\|\psi^+\rangle_{aA_1}$	$\|\psi^-\rangle_{aA_1}$
Even $\|V\rangle$	Even $\|V\rangle$	U_1	U_2	U_3	U_4
Even $\|V\rangle$	Odd $\|V\rangle$	U_4	U_3	U_2	U_1
Odd $\|V\rangle$	Even $\|V\rangle$	U_2	U_1	U_4	U_3
Odd $\|V\rangle$	Odd $\|V\rangle$	U_3	U_4	U_1	U_2

4. Discussion and summary

In this paper, we propose two controlled quantum teleportation protocols against collective-dephasing noise or collective-rotation noise. The advantage of the two protocols is that they can be performed in given collective noise channel with unitary successful probability.

As we know, controlled quantum teleportation has been used in many quantum secret communication protocols, so our protocols can be applied in quantum secret communication net protocols. The security is very important for quantum secret communication. If an eavesdropper wants to steal the secret information, she can take some attack methods, intercept and resend attack, entanglement swapping attack, or CNOT gate attack, *etc.* For preventing it, Alice and Bob must check whether the channel is secure or not by performing $\{|H\rangle, |V\rangle\}$ basis and $\{|+\rangle, |-\rangle\}$ basis measurement randomly before they teleport the secret state. In case the channel is insecure, they abandon the communication and start from beginning. Otherwise they begin to teleport, and Eve has no chance to steal the secret state any more. The eavesdropper not only steals nothing about the secret but also can be detected by the legitimate communicators for her stealing behavior.

The two protocols can be generalized to multiparty controlled quantum teleportation protocols by increasing the number of photons of the quantum channel. For conquering collective-dephasing noise or collective-rotation noise, the channel can be replaced by

$$|\Psi_1\rangle = \frac{1}{\sqrt{2}}\Big(|HVHV\rangle_{A_1A_2B_1B_2} \sum_i |HV\rangle_{C_{1i}C_{2i}} + |VHVH\rangle_{A_1A_2B_1B_2} \sum_i |VH\rangle_{C_{1i}C_{2i}} \Big), \tag{17}$$

and

$$|\Psi_2\rangle = \frac{1}{\sqrt{2}}\Big(|\phi^+\rangle_{A_1A_2} |\phi^+\rangle_{B_1B_2} \sum_i |\phi^+\rangle_{C_{1i}C_{2i}} + |\psi^-\rangle_{A_1A_2} |\psi^-\rangle_{B_1B_2} \sum_i |\psi^-\rangle_{C_{1i}C_{2i}} \Big). \tag{18}$$

The rest manipulations are similar to the above protocols. Bob can recover the original state according to Bell-basis measurement result, the number of measurement result $|V\rangle$ on photons (A_2, B_2), and the number of measurement result $|V\rangle$ from all controllers similar to Table I and Table II. Because the quantum channel is symmetric, each participant in the net can act as a sender, a receiver or a controller.

This study was supported by the Research Programs of the Educational Office of Liaoning Province of China (Grant No. 2008006).

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