# EVOLUTION OF PERTURBATIONS IN NONCOMMUTATIVE INFLATION

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Using the recently proposed approach to noncommutative inflation based on the coherent state picture of noncommutativity, we study the evolution of scalar density perturbations and calculate the spectrum of perturbations and scalar spectral index in this setup. As an important result we show that noncommutativity may be responsible for the deviations from the scale invariant spectrum predicted in usual inflationary scenarios.

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## 1. Introduction

It has been an old idea to assume a non-zero commutation relation between space-time coordinates [1]. This idea was developed by many authors (see for instance [2]). As a result of the existence of a fundamental minimum length scale in the system the (non)-commutation relation between coordinate operators takes the following form [3,4]

$$[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu}\,, \tag{1}$$

where the $\theta^{\mu\nu}$ is a real, antisymmetric matrix with the dimension of length squared which describes the fundamental discreteness of space-time

$$\theta^{\mu\nu} = \begin{pmatrix} 0 & \theta & 0 & 0 \\ -\theta & 0 & \theta & 0 \\ 0 & -\theta & 0 & \theta \\ 0 & 0 & -\theta & 0 \end{pmatrix}.$$

This change of view renders space-time point a meaningless notion and introduces a fundamental uncertainty relation which in a sense leads to fundamental space-time cells with the linear size of $\sqrt{\theta}$ instead of points.

Generally, it is believed that cosmic microwave background embodies the effects of trans-Planckian physics [5-11]. Noncommutative inflation is a process which imprints these effects to CMB, so there have been various attempts at constructing noncommutative inflationary models to test the results against observations of CMB. One approach is to use relation (1) for space–space coordinates [12,13] and to construct a noncommutative field theory on the space-time manifold by replacing ordinary product of fields by Weyl–Wigner–Moyal $*$-product. Another approach is to incorporate the fundamental noncommutativity of space-time into inflationary models via a generalized uncertainty principle (GUP) [14].

In a recent approach Rinaldi [15] has proposed the application of coherent state picture of noncommutativity [16] to inflation. Contrary to the other approaches which are based on $*$-product, this model is free from problems like unexpected divergences and UV/IR mixing (see [17] for a full review). The idea is that the non-vanishing commutator (1) excludes a common basis in coordinate representation. The best one can do is to define mean values between appropriately chosen states, *i.e.* coherent states. These mean values are the closest one can get to the classical commuting coordinates, since coherent states are minimum uncertainty states. The use of mean values of quantum position operators as classical coordinates leads to the emergence of a quasi-classical space-time manifold, where the position of any physical object is intrinsically uncertain. This uncertainty can be seen either as a Gaussian cut-off in momentum space Green functions, or as a substitution of position Dirac delta with minimal width Gaussian function [16].

As it is shown by Nocolini *et al.* [18], the smearing is mathematically equivalent to a substitution rule: position Dirac-delta function is replaced everywhere with a Gaussian distribution of minimal width $\sqrt{\theta}$. In this framework, the mass density of a static, spherically symmetric, smeared, particle-like gravitational source takes the following form

$$\rho(r) = \frac{1}{(4\pi\theta)^{3/2}}\, e^{-r^2/4\theta}\,. \tag{2}$$

This density expresses the diffusion of the particle with mass $M$ in a region of linear size $\sqrt{\theta}$ due to noncommutativity, instead of being perfectly localized in a point, as it is in an ordinary space-time.

To use this approach to noncommutativity in a cosmological background and construct an inflationary model, we assume that the initial singularity is smeared due to the noncommutativity of space-time and an exponential form like (2) is taken for its density. As we will see, the negative pressure which arises in this noncommutative mechanism [18] is the cause of an inflationary phase in the Universe without the need for any additional fields like inflation. In this model, there will be an smooth transition between pre and post Big Bang eras via an accelerated expansion derived by noncommutativity. This new approach to noncommutative inflation has been used in studying the inflationary dynamics and cosmological perturbations in extra-dimensional scenarios in [19]. Here we analyze the evolution of perturbations in usual 4D inflationary scenario and calculate the noncommutative modifications to the amplitude of density perturbations and the evolution of the scalar spectral index.

## 2. Time evolution of the perturbations

To obtain a set of equations for computing time evolution of density and expansion perturbations, we begin with continuity, Euler and Raychaudhuri equations [20]

$$\begin{aligned} \frac{d\rho}{dt_{\rm pr}} &= -3H(\rho+P)\,, & (3)\\ \vec{a} &= -\frac{\nabla P}{\rho+P}\,, & (4)\\ \dot{H}(\vec{x},t)+H^2(\vec{x},t) &= -\frac{4\pi G}{3}\rho(\vec{x},t)+\frac{1}{3}\nabla\cdot\vec{a}\,, & (5)\end{aligned}$$

where $t_{\rm pr}$ is the proper time. Local functions of $\rho$, $P$ and $H$ are taken to be the sum of a homogeneous part and a perturbation

$$\begin{aligned} \rho(\vec{x},t) &= \rho(t)+\delta\rho(\vec{x},t)\,,\\ P(\vec{x},t) &= P(t)+\delta P(\vec{x},t)\,,\\ H(\vec{x},t) &= H(t)+\delta H(\vec{x},t)\,. \qquad (6)\end{aligned}$$

Putting the relation for $P$ in the Euler equation (4) we obtain

$$a_i(\vec{x},t) = -\frac{1}{a}\frac{\partial_i\delta P(\vec{x},t)}{\rho(t)+P(t)}\,, \qquad (7)$$

where $a$ is the scale factor. Inserting this result in the Raychaudhuri equation (5) gives

$$\frac{dH}{dt_{\rm pr}}+H^2 = -\frac{4\pi G}{3}(\rho+3P)-\frac{1}{3}\frac{\nabla^2\delta P}{\rho+P}\,. \qquad (8)$$

It is known that $\nabla(dt/dt_{\rm pr}) = -\vec{a}(dt/dt_{\rm pr})$ [20]. Inserting $\vec{a}$ from Euler equation and taking the divergence, one obtains to first order

$$\nabla^2\left(\frac{dt}{dt_{\rm pr}}\right) = \nabla^2\left(\frac{\delta P}{P+\rho}\right). \tag{9}$$

To zeroth order $dt/dt_{\rm pr} = 1$, and $\nabla^2$ acts only on $[dt/dt_{\rm pr} - 1]$; so we have

$$\frac{dt}{dt_{\rm pr}} = 1 + \frac{\delta P}{\rho + P}, \tag{10}$$

which leads to

$$\delta\left(\frac{df}{dt_{\rm pr}}\right) = (\delta f)^{\dot{}} + \frac{\delta P}{\rho+P}\dot{f} \tag{11}$$

for any function $f$. Using equation (11) we reach the desired equations for perturbations from continuity and Raychaudhuri equations:

$$\begin{aligned} \delta\dot{\rho}_k &= -3(\rho+P)\delta H_k - 3H\delta\rho_k\,, && (12)\\ \delta\dot{H}_k &= -2H\delta H_k - \frac{4\pi G}{3}\,\delta\rho_k + \frac{1}{3}\left(\frac{k}{a}\right)^2\frac{\delta P_k}{\rho+P}\,. && (13) \end{aligned}$$

These equations describe the evolution of the Fourier modes of the perturbations. We work in the semi-classical framework of diffusion of the initial singularity due to noncommutativity of space-time coordinates. This inspires a Gaussian form for the energy density of the Universe at the initial singularity

$$\rho = \rho_0\, e^{-|\tau|^2/4\theta} e^{-|\vec{X}|^2/4\theta}\,, \tag{14}$$

where $R^2 = \tau^2 + |\vec{X}|^2$ and $\tau = it$ is the Euclidean time. In cosmology, isotropy implies that the energy density depends on time only so from one hypersurface to another, the $\vec{X}$-dependent part of $\rho$ does not change and it can be included into $\rho_0$. The constant $\rho_0$ is dependent on the noncommutative parameter $\theta$ and the exact form of it has been obtained in [16]. So as a result of above arguments, one can write the energy density of the Universe at the time of initial (smeared) singularity as

$$\rho(t) = \frac{1}{32\pi^2\theta^2}\, e^{-t^2/4\theta}\,. \tag{15}$$

The pressure can be directly calculated from the density by the conservation equation

$$p = -\rho + \frac{t}{6\,\theta}\, e^{-t^2/8\theta}\,. \tag{16}$$

So a full expression is achieved for $\rho + P$ present in the equations (3), (4). The Friedmann equation is

$$H^2 = \frac{8\pi}{3M_4^2}\,\rho(t)\,. \tag{17}$$

Inserting the energy density (15) in this equation gives

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi}{3M_4^2}\,\rho_0 e^{-t^2/4\theta} \equiv H_0^2\, e^{-t^2/4\theta}\,. \tag{18}$$

This equation could be solved to obtain the scale factor which is no longer singular at $t = 0$

$$a(t) = a_0\left[H_0\sqrt{2\pi\theta}\ \mathrm{erf}\left(\frac{\mathrm{t}}{2\sqrt{2\theta}}\right)\right], \tag{19}$$

where $\mathrm{erf}(x) = \int_0^x e^{-x^2}dx$.

The number of e-folds is given by

$$N = \ln\left(\frac{a_f}{a_i}\right), \tag{20}$$

where $a_{f,i} = a(t = \pm\infty)$. Using the asymptotic behavior of the error function at infinities, we find that the number of e-folds is given by

$$N = 8\pi\sqrt{\frac{G\rho_0\theta}{3}}\,. \tag{21}$$

Using the above results, the scale factor and Hubble parameter in this model are given by the following expressions, respectively

$$a(t) = a(0)\; e^{Nt/\sqrt{8\pi\theta}}\,, \tag{22}$$

$$H(t) = \sqrt{\frac{8\pi G\rho_0}{3}}\; e^{-t^2/8\theta}\,. \tag{23}$$

Inserting the above functions, the equations (3)–(5) will take the following form

$$\delta\dot{\rho}_k = 6\rho_0\; te^{-t^2/8\theta}\delta H_k - 3\sqrt{\frac{8\pi G\rho_0}{3}}\; e^{-t^2/8\theta}\delta\rho_k\,, \tag{24}$$

$$\delta\dot{H}_k = -2\sqrt{\frac{8\pi G\rho_0}{3}}\; e^{-t^2/8\theta}\delta H_k + \left[\frac{k^2}{2a^2(0)\rho_0}\; te^{t^2/8\theta - \frac{2Nt}{\sqrt{8\pi\theta}}} - \frac{4\pi G}{3}\right]\delta\rho_k\,. \tag{25}$$

From here on, for simplicity we assign the following letters to the coefficients of the equations

$$a = 6\rho_0\,, \qquad b = \sqrt{\frac{8\pi G\rho_0}{3}}\,, \qquad c = \frac{1}{2a^2(0)\rho_0}\,, \qquad d = \frac{4\pi G}{3}\,. \tag{26}$$

Finding an exact solution to these coupled equations is not feasible. However, some considerations will lead to an approximate solution: making the terms and coefficients of the same dimension and making an order of magnitude estimation of the coefficients, one reaches the conclusion that the coefficients of the last two terms of the second equation are very much smaller than the coefficients of the other terms; assuming forms like $\delta\rho_k = \delta\rho_{0k} + c\delta\rho_{1k}$ and $\delta H_k = \delta H_{0k} + c\delta H_{1k}$ for the approximate solution to the original equations (as $c$ is the smallest of all the coefficients), one arrives at these set of equations which are now solvable

$$\dot{\delta\rho_{0k}} = a\,te^{-t^2/4\theta}\delta H_{0k} - 3b\,e^{-t^2/8\theta}\delta\rho_{0k}\,, \tag{27}$$

$$\delta\dot{H}_{0k} = -2b\,te^{-t^2/8\theta}\delta H_{0k}\,, \tag{28}$$

$$\dot{\delta\rho_{1k}} = a\,e^{-t^2/4\theta}\delta H_{1k} - 3b\,e^{-t^2/8\theta}\delta\rho_{1k}\,, \tag{29}$$

$$\delta\dot{H}_{1k} = -2b\,e^{-t^2/8\theta}\delta H_{1k} + \left[k^2\,te^{t^2/4\theta - \frac{2Nt}{\sqrt{8\pi\theta}}} - \frac{d}{c}\right]\delta\rho_{0k}\,. \tag{30}$$

Everywhere in the solutions erf $(\alpha t)$ is replaced with $2\alpha t/\sqrt{\pi}$, *i.e.* $t$ is assumed to be small. The complete simplified solutions appear in the appendix.

## 3. Spectrum of scalar density perturbations

To be a realistic model of the early Universe and also to test whether or not this model is consistent with recent observational data, a scale invariant spectrum of scalar perturbations should be generated after inflation. Here scalar invariance means independence of $k$.

The curvature perturbations are given by

$$\zeta = \mathcal{R}_k - \frac{H}{\dot{\rho}}\,\delta\rho \tag{31}$$

which reduces to $\mathcal{R}$ for uniform density hypersurfaces. $\mathcal{R}$ is calculated by integrating

$$\dot{\mathcal{R}}_k = -H\frac{\delta\rho_k}{\rho + P} \tag{32}$$

with respect to $t$ [20].

The curvature perturbation $\zeta$ can be related to the density perturbations when modes re-enter the Hubble scale during the matter dominated era which is given by $\mathcal{P}_{\mathcal{R}} = (4\zeta^2)/25$. It can be useful to compare the spectrum of perturbations in commutative and noncommutative cases. Spectrum of density perturbations in commutative chaotic inflation with a single scalar field in a quadratic potential has been calculated [20] with the following result

$$\mathcal{P}_{\mathcal{R}} = A_{\mathcal{R}}^2 \left(\frac{k}{aH}\right)^{n_{\mathcal{R}}-1}, \tag{33}$$

where $a$ is the scale factor with the dynamics of the $a$ factor given by

$$a \approx a_i \;\exp\left[2\sqrt{\frac{\pi}{3}}\frac{m}{M_{\mathrm{PL}}}\left(\Phi_i t - \frac{mM_4}{4\sqrt{3\pi}}\right)\right]. \tag{34}$$

Figure 1 shows the qualitative behavior of perturbation spectra for the commutative and the noncommutative cases as functions of $t$ for a mode with a fixed $k$. Finally, spectral index is an observational quantity compared to which the theoretical results can be validated. The following relations are used to calculate the spectral index

$$n(k) - 1 = \frac{d\;\ln(\mathcal{P}_R)}{d\;\ln(k)}. \tag{35}$$

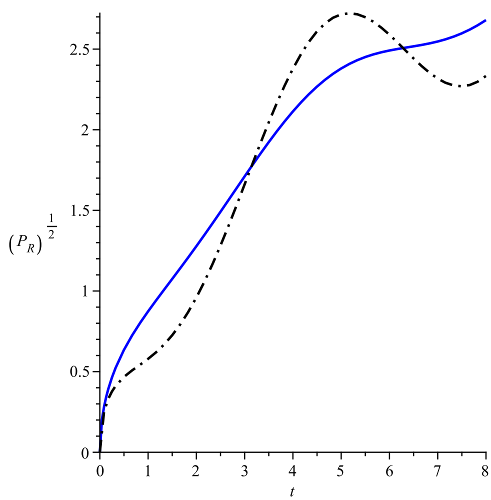


Fig. 1. Evolution of $\mathcal{P}_{\mathcal{R}}$ *versus* the cosmic time for a mode with a fixed $k$ for both noncommutative (solid line) and commutative (dashed line) inflation models. The commutative spectrum is for a quadratic chaotic type potential.

$H$, $\rho$, $\dot{\rho}$ and $P$ appeared in the above relations are the background unperturbed quantities. Figure 2 shows the spectral index as a function of time and $k$. One can see from this figure that the spectrum shows small deviation from scale invariance. This deviation is larger than what is obtained in standard approaches, but is relatively small nevertheless. This behavior was predicted in some earlier works using different approaches to noncommutativity [10,12,13].

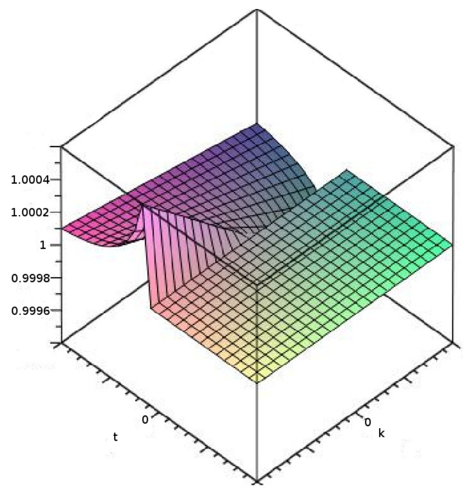


Fig. 2. Evolution of scalar spectral index, $n_{\mathrm{s}}$ *versus* $t$ and $k$.

## 4. Conclusion and discussion

Space-time noncommutativity as a trans-Planckian effect, essentially could have some observable effects on the cosmic microwave background radiation. In this respect, it is desirable to study an inflation scenario within a noncommutative background. Recently the possibility of the existence of a non-singular, bouncing, early time cosmology in a noncommutative universe has been proposed [15,19]. This model realizes an inflationary, bouncing solution without the need for any axillary scalar or vector fields. Due to noncommutative structure of the space-time which admits the existence of a fundamental length scale, there is no initial singularity in this model. Here, we have studied the time evolution of the perturbations in this noncommutative setup. Our analysis of the perturbations shows that the amplitude of scalar perturbations in the noncommutative regime evolve differently than a typical 4D inflationary model (here we used chaotic type potential for comparing the results). The spectrum of scalar perturbations in the noncommutative case shows a small deviation from scale invariance.

## Appendix A

*Perturbation functions*

$$\begin{aligned}
\delta\rho_k &= f(C_1,C_2)e^{-3bt}+c\left(\frac{a}{\sqrt{-1/\theta}\,bc^2}\left(-f(C_1,C_2)k^2\pi\,\mathrm{erf}\left(\frac{b\sqrt{8\pi\theta}+2N}{\sqrt{8\pi\theta}\sqrt{-1/\theta}}\right)\right.\right.\\
&\times\ be^{-\frac{\theta^2N^2+2\theta^3b^2\pi+2\theta^2b\sqrt{2\pi\theta}N}{2\pi\theta^2}+b^2\theta}\sqrt{\theta}\,\mathrm{erf}(b\sqrt{\theta})+f(C_1,C_2)dc^2\sqrt{-1/\theta}\,t\\
&\times\ \left.\left.-C_3c^2\,b\sqrt{-\pi}\,e^{b^2\theta}\mathrm{erf}\left(b\sqrt{\theta}\right)\right)+C_4\right)e^{-3bt}\,, \qquad (A.1)
\end{aligned}$$

$$\begin{aligned}
\delta H_k &= C_2\,e^{-2bt}+c\left(f(C_1,C_2)\right.\\
&\times\ \left.\left(\frac{k^2\sqrt{\pi}\,e^{-\frac{(N\theta+b\theta\sqrt{2\pi\theta})^2}{\pi\theta^2}}\mathrm{erf}\left(\frac{N\theta+b\theta\sqrt{2\pi\theta}}{\theta\sqrt{-2\pi}}\right)}{c^2\sqrt{-1/\theta}}+\frac{de^{-bt}}{b}\right)+C_3\right)e^{-2bt}\,, \qquad (A.2)
\end{aligned}$$

in which $f(C_1,C_2)$ is

$$f(C_1,C_2)=-aC_2\sqrt{\pi\theta}\,e^{b^2\theta}\mathrm{erf}(b\sqrt{\theta})+C_1\,. \qquad (A.3)$$

The scalar spectral index is given by

$$\begin{aligned}
n_s &= \left(cA_1\,kt-cA_2\,ke^{-1/8\,\frac{t^2}{\theta}-3\,bt}A_3^{-1}t^{-1}\left(e^{-1/4\,\frac{t^2}{\theta}}\right)^{-1}\right)\\
&\quad\times\left[A_4\,t+cA_5\,k^2t-cA_6\,\left(e^{-1/8\,\frac{t^2}{\theta}+3\,bt+b^2\theta}+A_7\right)\right.\\
&\quad+c\left(A_8\,e^{-1/8\,\frac{t^2}{\theta}-3\,bt}+A_9\,t\right)+cA_{10}\,t+e^{-1/8\,\frac{t^2}{\theta}}\\
&\quad\times\left(A_{11}e^{-3\,bt}+c\left(A_{12}k^2+A_{13}t-A_{14}+A_{15}\right)e^{-3\,bt}\right)\\
&\quad\times\left.\left(e^{-1/4\,\frac{t^2}{\theta}}\right)^{-1}\right]^{-1}+1\,, \qquad (A.4)
\end{aligned}$$

where $A_i$s are some complicated functions of coefficients defined in (26). In the above results, the terms involving $k$ are the origin of the mentioned deviation from scale invariance. Considering the definition of these coefficients, one can see that the coefficient $c$ is a very small one and it is behind every term that contains $k$, so it is natural that the deviation from scale invariance is small.

## REFERENCES


[1] H.S. Snyders, *Phys. Rev.* **71**, 38 (1947).

[2] S. Doplicher, K. Fredenhagen, J.E. Roberts, *Commun. Math. Phys.* **172**, 187 (1995); D. Bahns, S. Doplicher, K. Fredenhagen, G. Piacitelli, *Phys. Lett.* **B533**, 178 (2002); D. Bahns, S. Doplicher, K. Fredenhagen, G. Piacitelli, *Commun. Math. Phys.* **237**, 221 (2003).

[3] N. Seiberg, E. Witten, *J. High Energy Phys.* **9909**, 032 (1999); F. Ardalan, H. Arfaei, M.M. Sheikh-Jabbari, *J. High Energy Phys.* **9902**, 016 (1999); A. Connes, M. Marcolli, `arXiv:math.QA/0601054`; A. Connes, *J. Math. Phys.* **41**, 3832 (2000); M.R. Douglas, N.A. Nekrasov, *Rev. Mod. Phys.* **73**, 977 (2001); A. Micu, M.M. Sheikh-Jabbari, *J. High Energy Phys.* **0101**, 025 (2001); A. Konechny, A. Schwarz, *Phys. Rep.* **360**, 353 (2002); R.J. Szabo, *Phys. Rep.* **378**, 207 (2003); M. Chaichian *et al., Eur. Phys. J.* **C29**, 413 (2003).

[4] G. Veneziano, *Europhys. Lett.* **2**, 199 (1986); D. Amati, M. Ciafaloni, G. Veneziano, *Phys. Lett.* **B197**, 81 (1987); *Int. J. Mod. Phys.* **A3**, 1615 (1988); D.J. Gross, P.F. Mende, *Nucl. Phys.* **B303**, 407 (1988); D. Amati, M. Ciafaloni, G. Veneziano, *Phys. Lett.* **B216**, 41 (1989); *Nucl. Phys.* **B347**, 530 (1990).

[5] R. Easther, B.R. Green, W.H. Kinney, G. Shiu, *Phys. Rev.* **D64**, 103502 (2001).

[6] R. Easther, B.R. Green, W.H. Kinney, G. Shiu, *Phys. Rev.* **D67**, 063508 (2003).

[7] R. Easther, B.R. Green, W.H. Kinney, G. Shiu, *Phys. Rev.* **D66**, 023518 (2002).

[8] N. Kaloper, M. Kleban, A.E. Lawrence, S. Shenker, *Phys. Rev.* **D66**, 123510 (2002).

[9] L. Bergstrom, U.H. Danielsson, *J. High Energy Phys.* **07**, 038 (2002).

[10] J. Martin, R. Brandenberger, *Phys. Rev.* **D68**, 0305161 (2003).

[11] O. Elgaroy, S. Hannestad, *Phys. Rev.* **D62**, 041301 (2000).

[12] S. Chu, B.R. Greene, G. Shiu, *Mod. Phys. Lett.* **A16**, 2231 (2001) `[arXiv:hep-th/0011241]`; R. Easther, B.R. Greene, W.H. Kinney, G. Shiu, *Phys. Rev.* **D64**, 103502 (2001) `[arXiv:hep-th/0104102]`; F. Lizzi, G. Mangano, G. Miele, M. Peloso, *J. High Energy Phys.* **0206**, 049 (2002) `[arXiv:hep-th/0203099]`; R. Easther, B.R. Greene, W.H. Kinney, G. Shiu, *Phys. Rev.* **D67**, 063508 (2003) `[arXiv:hep-th/0110226]`; S.F. Hassan, M.S. Sloth, *Nucl. Phys.* **B674**, 434 (2003) `[arXiv:hep-th/0204110]`.

[13] R. Brandenberger, P.M. Ho, *Phys. Rev.* **D66**, 023517 (2002).

[14] S. Alexander, J. Magueijo, arXiv:hep-th/0104093; S. Alexander, R. Brandenberger, J. Magueijo, *Phys. Rev.* **D67**, 081301 (2003) [arXiv:hep-th/0108190]; S. Koh, R.H. Brandenberger, *J. Cosmol. Astropart. Phys.* **0706**, 021 (2007) [arXiv:hep-th/0702217]; S. Koh, arXiv:hep-th/0802.0355.

[15] M. Rinaldi, arXiv:0908.1949[gr-qc].

[16] A. Smailagic, E. Spallucci, *J. Phys.* **A36**, L467 (2003); *J. Phys.* **A36**, L517 (2003); *J. Phys.* **A37**, 1 (2004), [Erratum-ibid. **A37**, 7169 (2004)].

[17] P. Nicolini, *Int. J. Mod. Phys.* **A24**, 1229 (2009) [arXiv:0807.1939[hep-th]].

[18] P. Nicollini, *J. Phys.* **A38**, L631 (2005); P. Nicolini *et al., Phys. Lett.* **B632**, 547 (2006); E. Spallucci, A. Smailagic, P. Nicolini, *Phys. Rev.* **D73**, 084004 (2006).

[19] K. Nozari, S. Akhshabi, *Phys. Lett.* **B683**, 186 (2010) [arXiv:0911.4418[hep-th]]; arXiv:1004.5007[hep-th].

[20] A.R. Liddle, D.H. Lyth, *Cosmological Inflation and Large-scale Structure*, Cambridge University Press, Cambridge 2000.