

LOW DENSITY SYMMETRY ENERGY EFFECTS AND THE NEUTRON STAR CRUST PROPERTIES

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The form of the nuclear symmetry energy E_s around saturation point density leads to a different crust-core transition point in the neutron star and affects the crust properties. We show that the knowledge of E_s close to the saturation point is not sufficient to determine the position of the transition point and the very low density behaviour is required. We also claim that crust properties are strongly influenced by the very high density behaviour of E_s , so in order to conclude about the form of low density part of the symmetry energy from astrophysical data one must isolate properly the high density part.

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1. Introduction

One of the most intriguing quantity in the description of nuclear matter in neutron stars is the symmetry energy E_s , which is defined as follows:

$$E_{\text{nuc}}(n, \alpha) = V(n) + E_s(n) \alpha^2 + \mathcal{O}(\alpha^4), \quad (1)$$

where $E_{\text{nuc}}(n, \alpha)$ represents the energy of nucleonic matter per baryon as a function of baryon number density n and the isospin asymmetry α , where $\alpha = (n_n - n_p)/n$ and n_n, n_p are the neutron and proton densities. At the saturation point density, $n_0 = 0.16 \text{ fm}^{-3}$, the value of symmetry energy corresponds to the a_4 parameter in the Bethe–Weizsäcker mass formula, and is $E_s(n_0) = 30 \pm 1 \text{ MeV}$. Isoscalar part of interactions is represented by the isoscalar potential $V(n)$ which is mainly responsible for the stiffness of the Equation of State (EoS).

Density dependence of E_s is, however, highly uncertain both below and above saturation point n_0 . This dependence is one of the goals of the experimental investigations carried on the radioactive beam colliders [1, 2]. This kind of facilities allow to study nuclear matter with large isospin asymmetry. The analysis shown in [3–5] put some constraints on the slope and curvature of $E_s(n)$ around n_0 , however, we are still far from the final conclusion about the global shape of the symmetry energy.

The role played by the E_s in the context of various neutron star observables was emphasized in [6] and more detailed analysis was made in [7]. One of the first studies of the crust-core transition was presented in [8], where the stability problem was investigated. This kind of analysis with an improved nuclear model was later used in [9]. In this work authors suggested the different critical density in different nuclear models results from the symmetry energy form. The direct connection of E_s to the crust-core transition point was shown in [10] and possible phase separation in the inner core was analysed in [11]. In this work we would like to go along this line to emphasize that the very low density behaviour of the symmetry energy is essential for the crust-core transition point. It is especially interesting in view of the recent experimental results [12, 13] which show the symmetry energy to be large at densities very much below n_0 . This result is in contrast to common conviction coming from various theoretical approaches that the E_s goes almost linearly to zero for low densities. So, the consequences of the symmetry energy with such unusual feature seem to be worth investigation.

2. The crust-core transition point

In order to estimate the crust-core transition point the bulk instability conditions are applied. They are based on the vanishing compressibility which signals the one-phase system instability against the density fluctuations. The system must split into two phases. It was shown in [10] that the compressibility under constant charge chemical potential — K_μ is the proper quantity in the case of matter under beta equilibrium. It is defined as follows:

$$K_\mu = \left(\frac{\partial P}{\partial n} \right)_\mu, \quad (2)$$

where P is the total pressure (nucleons + leptons). When nuclear matter contribution to the total energy is described by Eq. (1) then the stability condition takes the form

$$K_\mu = n^2 (E_s'' \alpha^2 + V'') + 2 n (E_s' \alpha^2 + V') - \frac{2 \alpha^2 E_s'^2 n^2}{E_s} \geq 0, \quad (3)$$

where the role played by the symmetry energy E_s is apparent. For densities typical for the neutron star core the compressibility K_μ is positive, but decreases as density decreases. For some density n_c , below n_0 , the compressibility vanishes. It means the charge fluctuations are not stable — below n_c the matter cannot exist in one phase and must separate into two phases which may form a crystal lattice. However, the answer, where the inner edge of the crust is situated, is much more complicated.

There is no unique method to determine at what density the solid crust starts to form. Here we want to discuss this issue. The critical density for vanishing compressibility $n_c(K_\mu)$ represents the absolute limit for the homogeneous and neutral system. It means the phase splitting must occur before reaching n_c . Indeed, the energy for the two-phase system becomes smaller than for the one-phase system before the density approaches the critical value $n_c(K_\mu)$ [9, 18]. The boundary for two phase coexistence, $n_c(1 \leftrightarrow 2)$ is then located slightly above $n_c(K_\mu)$ as it is shown in Fig. 1. Both $n_c(K_\mu)$ and $n_c(1 \leftrightarrow 2)$ are quantities derived in the bulk approximation, *i.e.* without the finite size effects included. The finite size structures require inclusion of the additional forms of energy like the Coulomb energy and the surface energy. Competition between them leads to structures with various dimensionality (rods, plates) usually called pasta phases [22]. The presence of these exotic structures is, however, model dependent, moreover mechanical properties of rods and plates resemble rather liquid crystals than solid state [23]. The shear tensions in all spatial directions can be supported only by a 3-dimensional lattice of 0-dimensional structures, *e.g.* the lattice of almost point-like nuclei immersed in the less dense medium, *e.i.* the gas of electrons and dripped neutrons. So, for the most reliable position of the crust edge one should take the point where the nuclei start to deform into very long structures or, keeping their spherical form, dissolve in the surrounding medium.

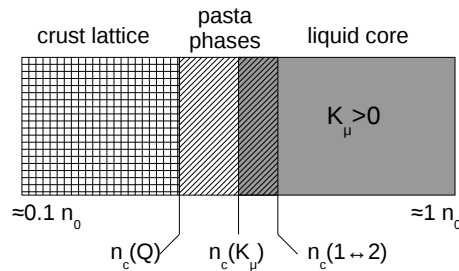


Fig. 1. Schematic picture of the crust-core transition in the neutron star. The grey region corresponds to $K_\mu < 0$, the uncertain hatched range indicates region where pasta phases may occur.

The finite size effects may also be included in the stability consideration of one-phase system as was done in [9]. The most important ingredient in this analysis appeared to be the Coulomb energy. Being repulsive, the Coulomb force stabilizes fluctuations of charge and makes matter more resistive to formation of charged clusters. The corresponding critical density, $n_c(Q)$, is then moved to lower densities, always below $n_c(K_\mu)$ (see Fig. 1). For some kind of interactions (*e.g.* Skyrme-like in [18]) the $n_c(Q)$ almost coincides with the point where nuclei disappear, so it may be treated as the very likely estimation of the crust-core transition.

In this work we stay with the simplest choice $n_c(K_\mu)$, what means we get upper bound on the crust mass and thickness. The difference between $n_c(K_\mu)$ and $n_c(Q)$ is not so large, usually smaller than 10%. The comparison between these quantities was already done in [9] and for modern nuclear models in [19, 21]. The condition (3) corresponds to the cross-point of the border of spinodal region at zero temperature with beta equilibrium condition in stellar matter as it was considered in [20].

3. Nuclear models

Different models of nuclear interactions, while being consistent at saturation point $n_0 = 0.16 \text{ fm}^{-3}$, lead to large discrepancies for EoS at higher densities. In order to estimate the role played by E_s we used a model which has the same isoscalar part of interactions $V(n)$ and different form of the isovector part $E_s(n)$. The $V(n)$, taken from Prakash *et al.* [15], corresponds to the compressibility at saturation point $K_0 = 240 \text{ MeV}$.

Various shapes of symmetry energy, are taken from MDI model [4] and named a, b, c, d as in [10]. The symmetry energy in MDI family presents large discrepancies both in the low and in the high density region. Also the slope $E'_s(n_0)$ and curvature $E''_s(n_0)$ at the saturation point take different values. The only common property is the value of the symmetry energy at n_0 . In particular the large differences at high densities lead to differences in the global stellar parameters like radius and compactness. Those global parameters indirectly affect the crust thickness and its contribution to the total star mass what obscures the picture. In order to isolate the low density properties of E_s from the rest, we use a specific parametrization which presents the same shape of E_s at saturation point and very tiny differences at high density. It has the following form

$$E_s = \frac{\alpha u^2 + \beta u + \gamma \sqrt{u} + E_s(0)}{u^2 + 1}, \quad (4)$$

where $u = n/n_\infty$. The n_∞ controls the behaviour at very high density and is $n_\infty = 1 \text{ fm}^{-3}$ giving intermediate values of E_s between extremal cases

(*a* and *d*) in the MDI family. The remaining parameters α, β, γ were derived to ensure the same slope $L = 80$ MeV and the stiffness $K_{\text{asy}} = -450$ MeV at n_0 while E_s at $n \rightarrow 0$ approaches three different values 0, 5 and 10 MeV. The form of symmetry energy for all models is shown in Fig. 2.

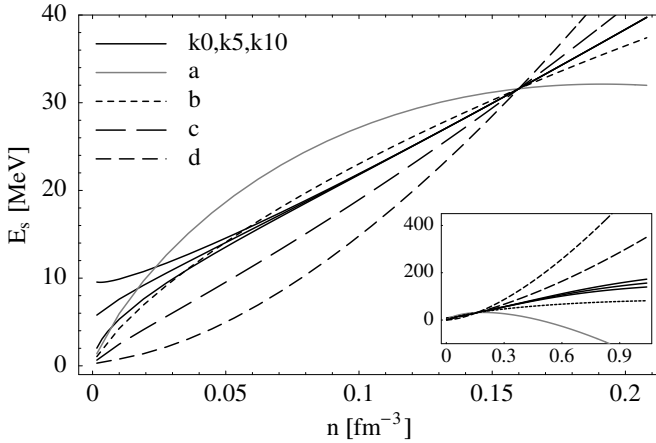
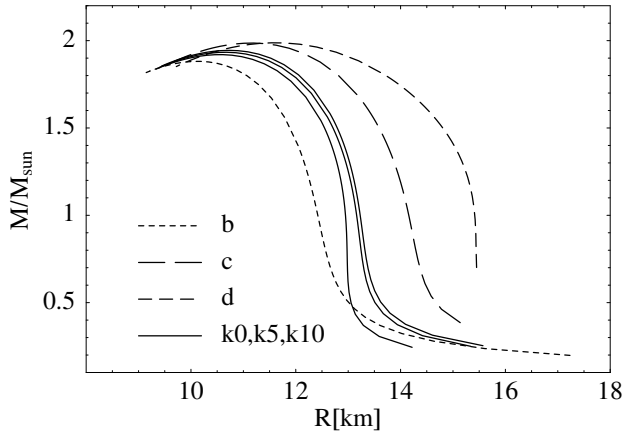


Fig. 2. The symmetry energy for different models in the low and high density region (the inset).

The above models of nuclear interactions cannot be used to obtain EoS for very low densities, in the crust region. In order to obtain the EoS in the full range, up to the edge of the star, they have to be complemented with the EoS for the crust itself. Here we used well established results from [16] and [17].

4. Results

In this section the results are presented. In Fig. 3 the mass-radius relation is shown. One may observe large differences in the MDI family. These differences are caused only by the symmetry energy, because the isoscalar part is the same for all EoSs. The symmetry energies in the MDI family diverge enormously at higher densities. There are no results for the *a* model because its E_s takes so large negative values at high density that makes the pressure bounded from above and leads to unstable EoS. The other models, *b–d*, present large differences in the compactness of neutron star giving difference in the surface gravitation. Hence any crustal parameters like thickness, mass *etc.* are changed not only by the form of E_s at the crust region, but also by its values in deep core region. For the *k* family E_s is almost the same at high density and the M – R relations for *k0–k10* models are similar to each other. The differences between them slightly

Fig. 3. The total star mass *versus* star radius.

increase with the dropping mass, what is natural as the crust occupies more an more volume of the star and differences in the EoS at low density are more pronounced. The effect of the high density E_s form is apparent when one compares critical densities for different models. In the MDI family, for b, c, d one gets $n_c = 0.092, 0.095, 0.160$, respectively (see [10]), whereas for $k0, k5, k10$: $n_c = 0.080, 0.073, 0.065$, respectively (see Table I). The differ-

TABLE I

The $E_s(n)$ parametrization for k models: 0, 5, 10. The last columns presents the critical density.

α	β	γ	$E_s(0)$ [MeV]	n_c
247.08	40.93	48.836	0	0.080
181.98	103.43	15.502	5	0.073
116.87	165.93	-17.831	10	0.065

ence in critical density for b and c is less than the k family, but the difference in the crust thickness (see Fig. 4) is much larger than one could expect from the n_c position. Surely it comes from the different compactness of the star as a whole. In the c case the star is more compact and its crust is more squeezed by the gravity. The effect is even better seen for the crustal fraction of the moment of inertia, Fig. 5. As one can notice for the typical $1.5M_\odot$ star the relative differences among b, c, d models reach hundreds of percents, whereas for the k family are not so large. At the end of the discussion we focus on the k models themselves. The symmetry energy for this family was constructed to ensure the same behaviour at high density and keep the same

slope and curvature at the saturation point in agreement with recent results from experiment. The only difference was the asymptotic value of the E_s at vanishing density $n \rightarrow 0$. We have probed values 0, 5 and 10 MeV. Considering the shapes of E_s in the Fig. 2 one may conclude that the $E_s(0)$ is very essential quantity for the crust properties. The symmetry energies for the k family are almost overlapping in the n_0 region and below, and their discrepancies seems to be negligible in comparison to b - d lines, however, they effect in quite large differences for the crust thickness and moment of inertia. We also observe that higher $E_s(0)$ systematically makes the crust thinner and its contribution to total moment of inertia becomes very low.

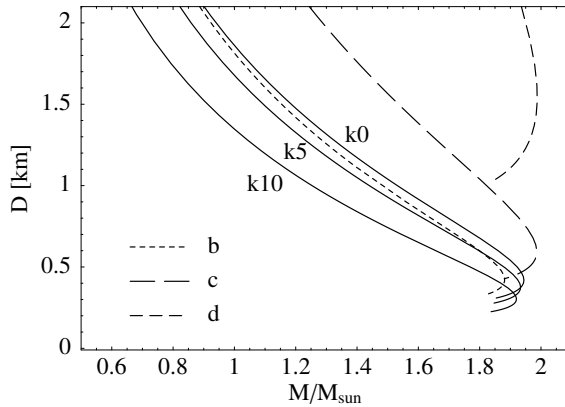


Fig. 4. The star crust thickness *versus* the total star mass.

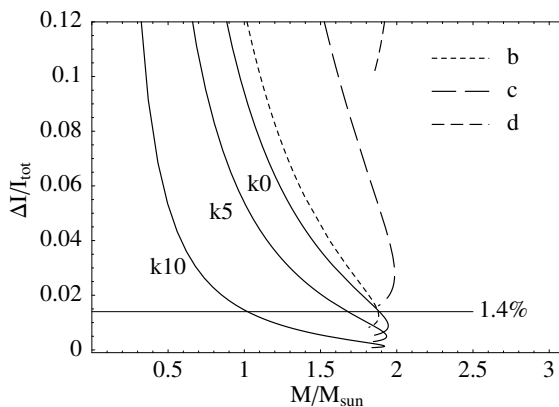


Fig. 5. The crustal fraction moment of inertia *versus* the total star mass. The lower bound, 1.4%, for I_{crust}/I coming from observations is indicated by the horizontal line.

In order to refer our results to observations we indicated the upper bound on the crustal moment of inertia for the Vela pulsar coming from analysis performed by Link in [24]. The line for $k10$ model goes below 1.4% in very wide range of stellar masses. We do not know the Vela pulsar mass, so in that sense the model cannot be verified, but if we trust experimental results that $E_s(0) \approx 10 \text{ MeV}$ it would mean the Vela pulsar mass should have its mass not greater than $1M_\odot$.

5. Summary

We perform the analysis how the very low symmetry energy behaviour affects the basic neutron star crust parameters. Although the crust encompasses the densities below n_0 , also the high density behaviour of the symmetry energy influences its properties. In order to be able to conclude about the low density part we constructed a special family of symmetry energies. They presented the same shape at saturation point and above, but differ when density goes to zero. It was shown that unusual property of the non-vanishing symmetry energy at zero density leads to a very thin crust with very small contribution to the total moment of inertia.

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