MINIMAL FLAVOUR VIOLATION AND BEYOND: TOWARDS A FLAVOUR CODE FOR SHORT DISTANCE DYNAMICS*

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This decade should provide the first definitive signals of New Physics beyond the Standard Model (SM) and the goal of these lectures is a review of flavour physics in various extensions of the SM that have been popular in the last ten years. After an overture, two pilot sections and a brief summary of the structure of flavour violation and CP violation in the SM, we will present the theoretical framework for weak decays that will allow us to distinguish between different New Physics (NP) scenarios. Subsequently we will present eleven concrete BSM models summarizing the patterns of flavour violation characteristic for each model. In addition to models with minimal flavour violation (MFV) accompanied by flavour-blind phases we will discuss a number of extensions containing non-MFV sources of flavour and CP violation and, in particular, new local operators originating in right-handed charged currents and scalar currents. Next, we will address various anomalies in the data as seen from the point of view of the SM that appear very natural in certain extensions of the SM. In this presentation selected superstars of this field will play very important role. These are processes that are very sensitive to NP effects and which are theoretically clean. Particular emphasis will be put on correlations between various observables that could allow us to distinguish between various NP scenarios. Armed with this knowledge we will propose a coding system in a form of a 3×3 matrix which helps to distinguish between various extensions of the SM. Finding which *flavour code* is chosen by nature would be an important step towards the fundamental theory of flavour. We give several examples of flavour codes representing specific models. We believe that such studies combined with new results from the Tevatron, the LHC, Belle II, Super–Flavour–Facility in Rome and dedicated Kaon and lepton flavour violation experiments should allow to improve significantly our knowledge about the dynamics at the shortest distance scales.

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Overture

The year 1676 was a very important year for the humanity. In this year Antoni van Leeuvenhoek (1632–1723) discovered the empire of bacteria. He called these small creatures *animalcula* (small animals). This discovery was a mile stone in our civilization for at least two reasons:

- He discovered invisible to us creatures which over thousands of years had been systematically killing the humans, and were often responsible for millions of death in one year. While Antoni van Leeuvanhoek did not know that bacteria could be dangerous for humans, his followers like Louis Pasteur (1822–1895), Robert Koch (1843–1910) and other *microbe hunters* not only realized the danger coming from this tiny creatures but also developed weapons against this empire.
- He was the first human who looked at short distance scales invisible to us, discovering thereby a new *underground world*. At that time researchers looked mainly at large distances, discovering new planets and finding laws, like Kepler laws, that Izaak Newton was able to derive from his mechanics.

While van Leeuvanhoek could reach the resolution down to roughly 10^{-6}m, over the last 334 years this resolution could be improved by twelve orders of magnitude. On the way down to shortest distance scales scientists discovered *nanouniverse* (10^{-9} m), *femtouniverse* (10^{-15} m) relevant for nuclear particle physics and low-energy elementary particle physics and finally *attouniverse* (10^{-18} m) that is the territory of contemporary high energy elementary particle physics.

In this decade we will be able to improve the resolution of the short distance scales by at least an order of magnitude, extending the picture of fundamental physics down to scales 5×10^{-20} m with the help of the LHC. Further resolution down to scales as short as 10^{-21} m or even shorter scales should be possible with the help of high precision experiments in which flavour violating processes will play a prominent role. These notes deal with the latter route to the short distance scales and new *animalcula* which hopefully will be discovered both at the LHC and through high precision experiments in the coming years.

1. Introduction

In our search for a fundamental theory of elementary particles we need to improve our understanding of flavour in [1–5]. This is clearly a very ambitious goal that requires the advances in different directions as well as continuous efforts of many experts day and night, as depicted with the help of a "Flavour Clock" in figure 1.

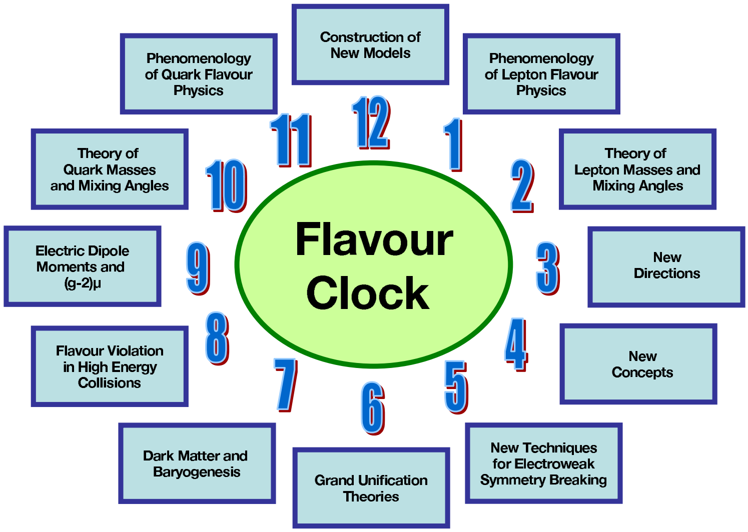

Fig. 1. Working towards the Theory of Flavour around the Flavour Clock.

Despite the impressive success of the CKM picture of flavour changing interactions [6, 7] in which also the GIM mechanism [8] for the suppression of flavour changing neutral currents (FCNC) plays a very important role, there are many open questions of theoretical and experimental nature that should be answered before we can claim to have a theory of flavour. Among the basic questions in flavour physics that could be answered in the present decade there are the following ones:

1. What is the fundamental dynamics behind the electroweak symmetry breaking that very likely plays also an important role in flavour physics?

2. Are there any new flavour symmetries that could help us to understand the existing hierarchies of fermion masses and the hierarchies in the quark and lepton flavour violating interactions?

3. Are there any flavour violating interactions that are not governed by the SM Yukawa couplings? In other words, is Minimal Flavour Violation (MFV) the whole story?

4. Are there any additional flavour violating CP-violating (CPV) phases that could explain certain anomalies present in the flavour data and simultaneously play a role in the explanation of the observed baryon–antibaryon asymmetry in the universe (BAU)?

5. Are there any flavour conserving CPV phases that could also help in explaining the flavour anomalies in question and would be signalled in this decade through enhanced electric dipole moments (EDMs) of the neutron, the electron and of other particles?

6. Are there any new sequential heavy quarks and leptons of the 4th generation and/or new fermions with exotic quantum numbers like vectorial fermions?

7. Are there any elementary neutral and charged scalar particles with masses below 1 TeV and having a significant impact on flavour physics?

8. Are there any new heavy gauge bosons representing an enlarged gauge symmetry group?

9. Are there any relevant right-handed (RH) weak currents that would help us to make our fundamental theory parity conserving at short distance scales well below those explored by the LHC?

10. How would one successfully address all these questions if the breakdown of the electroweak symmetry would turn out to be of a non-perturbative origin?

An important question is the following one: Will some of these questions be answered through the interplay of high-energy processes explored by the LHC with low-energy precision experiments or are the relevant scales of fundamental flavour well beyond the energies explored by the LHC and future colliders in this century? The existing tensions in some of the corners of the SM and still a rather big room for NP contributions in rare decays of mesons and leptons and CP-violating observables, including in particular EDMs, give us hopes that indeed several phenomena required to answer at least some of these questions could be discovered in this decade.

2. Superstars of flavour physics in 2010–2015

As far as high precision experiments are concerned a number of selected processes and observables will, in my opinion, play the leading role in learning about the NP in this new territory. This selection is based on the sensitivity to NP and theoretical cleanness. The former can be increased with the increased precision of experiments and the latter can improve with the progress in theoretical calculations, in particular the non-perturbative ones like the lattice simulations.

My superstars for the coming years are as follows:

- The mixing induced CP-asymmetry $S_{\psi\phi}(B_s)$ that is tiny in the SM: $S_{\psi\phi} \approx 0.04$. The asymmetry $S_{\phi\phi}(B_s)$ is also important. It is also very strongly suppressed in the SM and is sensitive to NP similar to the one explored through the departure of $S_{\phi K_S}(B_d)$ from $S_{\psi K_S}(B_d)$ [9].

- The rare decays $B_{s,d} \to \mu^+\mu^-$ that could be enhanced in certain NP scenarios by an order of magnitude with respect to the SM values.

- The angle γ of the unitarity triangle (UT) that will be precisely measured through tree-level decays.

- $B^+ \to \tau^+\nu_\tau$ that is sensitive to charged Higgs particles.

- The rare decays $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ that belong to the theoretically cleanest decays in flavour physics.

- The decays $B \to X_s\nu\bar{\nu}$, $B \to K^*\nu\bar{\nu}$ and $B \to K\nu\bar{\nu}$ that are theoretically rather clean and are sensitive to RH currents.

- Numerous angular symmetries and asymmetries in $B \to K^* l^- l^-$.

- Lepton flavour violating decays like $\mu \to e\gamma$, $\tau \to e\gamma$, $\tau \to \mu\gamma$, decays with three leptons in the final state and μ–e conversion in nuclei.

- Electric dipole moments of the neutron, the electron, atoms and leptons.

- Anomalous magnetic moment of the muon $(g-2)_\mu$ that indeed seems to be "anomalous" within the SM even after the inclusion of radiative corrections.

- The ratio ε'/ε in $K_L \to \pi\pi$ decays which is known experimentally within 10% and which should gain in importance in this decade due to improved lattice calculations.

Clearly, there are other stars in flavour physics but I believe that the ones above will play the crucial role in our search for the theory of flavour. Having experimental results on these decays and observables with sufficient precision accompanied by improved theoretical calculations will exclude several presently studied models reducing thereby our exploration of short distance scales to a few avenues.

In the rest of the paper I will proceed as follows. In Section 3 I will recall those ingredients of the SM that are dominantly responsible for the pattern of flavour violation and CP violation in this model. In Section 4 I will briefly recall the theoretical framework for weak decays that goes beyond the SM. In Section 5 we discuss several concrete BSM models. For each model we list the new particles and we recall the structure of their interactions with the ordinary quarks and leptons in particular paying attention to the Lorentz structure of these interactions. We summarise the patterns of flavour violation characteristic for each model. In Section 6 we address a number of anomalies present in the data from the point of view of the models of Section 5. We also illustrate how the superstars listed in Section 2, when considered together, can help in distinguishing between various NP scenarios. In this context the correlations between various observables will play a prominent role.

Armed with all this knowledge we will propose in Section 7 a new classification of various NP effects by means of a coding system in a form of a 3×3 *flavour code matrix*. Each NP model is characterised by a special code in which only some entries of the matrix in question are occupied. MFV, non-MFV sources and *flavour-blind* CP-violating phases on the one hand and LH-currents, RH-currents and scalar currents on the other hand, are the fundamental coordinates in this code. They allow to classify transparently the models discussed in Section 5. We give several examples of flavour codes corresponding to specific models discussed in the text. As we will see some models, depending on the values of parameters involved, show their presence in a number of entries of this matrix. Finally, in Section 8 we will provide a brief summary. Recent reviews on flavour physics can be found in [1–5].

3. Patterns of flavour violation and CP violation in the SM

Let us collect here those ingredients of the SM which are fundamental for the structure of flavour violating and CP-violating phenomena in this model.

- The SM contains three generations of quarks and leptons.
- The gauge interactions are described by the group $\mathrm{SU}(3)_C\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ spontaneously broken to $\mathrm{SU}(3)_C\times\mathrm{U}(1)_Q$. The strong interactions are mediated by eight gluons G_a, the electroweak interactions by $W^\pm$, Z^0 and γ.
- Concerning *Electroweak Interactions*, the left-handed leptons and quarks are put into $\mathrm{SU}(2)_L$ doublets

$$\begin{pmatrix}\nu_e\\ e^-\end{pmatrix}_L \qquad \begin{pmatrix}\nu_\mu\\ \mu^-\end{pmatrix}_L \qquad \begin{pmatrix}\nu_\tau\\ \tau^-\end{pmatrix}_L , \tag{1}$$

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L \quad \begin{pmatrix} c \\ s' \end{pmatrix}_L \quad \begin{pmatrix} t \\ b' \end{pmatrix}_L , \tag{2}$$

with the corresponding right-handed fields transforming as singlets under $\mathrm{SU}(2)_L$.

The *weak eigenstates* (d', s', b') and the corresponding *mass eigenstates* d, s, b are connected through the CKM matrix

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \equiv \hat{V}_{\mathrm{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix} . \tag{3}$$

In the leptonic sector the analogous mixing matrix is a unit matrix due to the masslessness of neutrinos in the SM. Otherwise we have the PMNS matrix.

- The charged current interactions mediated by $W^\pm$ are only between left-handed quarks in accordance with maximal parity breakdown observed in low-energy processes.

- The unitarity of the CKM matrix assures the absence of flavour changing neutral current (FCNC) transitions at the tree-level. This means that the elementary vertices involving neutral gauge bosons (G_a, Z^0,γ) and the neutral Higgs are flavour conserving. This property is known under the name of GIM mechanism [8].

- The fact that the V_{ij}s can *a priori* be complex numbers allows CP violation in the SM [7].

- The CKM matrix can be parametrized by s_{12}, s_{13}, s_{23} and a phase $\delta = \gamma$, with γ being one of the angles of the unitarity triangle. While $\gamma \approx (70 \pm 10)^\circ$, the s_{ij} exhibit a hierarchical structure

$$s_{12} = |V_{us}| \approx 0.225\,, \quad s_{13} = |V_{ub}| \approx 4\times 10^{-3}\,, \quad s_{23} = |V_{cb}| \approx 4\times 10^{-2}\,. \tag{4}$$

- This pattern of s_{ij} and the large phase γ combined with the large top quark mass and GIM mechanism imply large CP-violating effects in the B_d system ($S_{\psi K_{\mathrm{S}}} \approx 0.7$), small CP-violating effects in the B_s system ($S_{\psi\phi} \approx 0.04$) and tiny CP-violating effects in the K system ($|\varepsilon_K| \approx 0.002$).

- The EDMs predicted by the SM are basically unmeasurable in this decade.

- Lepton flavour violation in the SM is very strongly suppressed.

Presently, this global structure of flavour violating interactions works rather well but as we will see below some deviations from the SM predictions are observed in the data although most of these *anomalies* being typically 2–3 σ are certainly not conclusive.

4. Theoretical framework: beyond the SM

4.1. Preliminaries

The starting point of any serious analysis of weak decays in the framework of a given extension of the SM is the basic Lagrangian

$$\mathcal{L} = \mathcal{L}_{\rm SM}\left(g_i, m_i, V^i_{\rm CKM}\right) + \mathcal{L}_{\rm NP}\left(g_i^{\rm NP}, m_i^{\rm NP}, V^i_{\rm NP}\right) , \tag{5}$$

where $\left(g_i, m_i, V^i_{\rm CKM}\right)$ denote the parameters of the SM and $\left(g_i^{\rm NP}, m_i^{\rm NP}, V^i_{\rm NP}\right) \equiv \varrho_{\rm NP}$ the additional parameters in a given NP scenario.

Our main goal then is to identify in weak decays the effects described by $\mathcal{L}_{\rm NP}$ in the presence of the background from $\mathcal{L}_{\rm SM}$. In the first step one derives the Feynman rules following from (5), which allows to calculate Feynman diagrams. But then we have to face two challenges:

- our theory is formulated in terms of quarks, but experiments involve their bound states: $K_{\rm L}$, $K^\pm$, B_d^0, B_s^0, $B^\pm$, B_c, D, D_s, *etc.*
- NP takes place at very short distance scales 10^{-19}–10^{-18} m, while $K_{\rm L}$, $K^\pm$, B_d^0, B_s^0, $B^\pm$ and other mesons live at 10^{-16}–10^{-15} m.

The solution to these challenges is well known. One has to construct an effective theory relevant for experiments at low-energy scales. Operator Product Expansion (OPE) and Renormalization Group (RG) methods are involved here. They allow to separate the perturbative short distance (SD) effects, where NP is present, from long distance (LD) effects for which non-perturbative methods are necessary. Moreover, RG methods allow an efficient summation of large logarithms $\log(\mu_{\rm SD}/\mu_{\rm LD})$. A detailed exposition of these techniques can be found in [10, 11] and fortunately we do not have to repeat them here. At the end of the day the formal expressions involving matrix elements of local operators and their Wilson coefficients can be cast into the following *Master Formula for Weak Decays* [12].

4.2. Master Formula for Weak Decays

The master formula in question reads

$$A({\rm Decay}) = \sum_i B_i \eta^i_{\rm QCD} V^i_{\rm CKM} F_i(m_t, \varrho_{\rm NP})\,, \tag{6}$$

where B_i are non-perturbative parameters representing hadronic matrix elements of the contributing operators, $\eta^i_{\rm QCD}$ stand symbolically for the renormalization group factors, $V^i_{\rm CKM}$ denote the relevant combinations of the elements of the CKM matrix and finally $F_i(m_t, \varrho_{\rm NP})$ denote the loop functions resulting in most models from box and penguin diagrams but in some models also representing tree-level diagrams if such diagrams contribute. The internal charm contributions have been suppressed in this formula but they have to be included in particular in K decays and K^0–$\bar{K}^0$ mixing. $\varrho_{\rm NP}$ denotes symbolically all parameters beyond m_t, in particular the set $\left(g_i^{\rm NP}, m_i^{\rm NP}, V^i_{\rm NP}\right)$ in (5). It turns out to be useful to factor out $V^i_{\rm CKM}$ in all contributions in order to see transparently the deviations from MFV that will play a prominent role in these lectures.

In the SM only a particular set of parameters B_i is relevant as there are no right-handed charged current interactions, the functions F_i are *real* and the flavour and CP-violating effects enter only through the CKM factors $V^i_{\rm CKM}$. This implies that the functions F_i are universal with respect to flavour so that they are the same in the K, B_d and B_s systems. Consequently a number of observables in these different systems are strongly correlated with each other within the SM.

The simplest class of extensions of the SM are models with Constrained Minimal Flavour Violation (CMFV) [13–15]. In these models all flavour changing transitions are governed by the CKM matrix with the CKM phase being the only source of CP violation. Moreover, the B_i factors in (6) are only those that are also relevant in the SM. This implies that relative to the SM only the values of F_i are modified but their universal character remains intact. In particular they are real. This implies various correlations between different observables that we will discuss as we proceed.

In more general MFV models [16–18] new parameters B_i and $\eta^i_{\rm QCD}$, related to new operators, enter the game but if flavour blind CP-violating phases (FBP) are absent or negligible, the functions F_i still remain real quantities as in the CMFV framework and do not involve any flavour violating parameters. Consequently the CP and flavour violating effects in these models are again governed by the CKM matrix. However, the presence of new operators makes this approach less constraining than the CMFV framework. We will discuss some other aspects of this approach below.

Most general MFV models can also contain FBPs that can have profound implications for the phenomenology of weak decays because of the interplay of the CKM matrix with these phases. In fact, such models became very popular recently and we will discuss them below.

In the simplest non-MFV models, the basic operator structure of CMFV models remains but the functions F_i in addition to real SM contributions can contain new flavour parameters and new complex phases that can be both flavour violating and flavour-blind.

Finally, in the most general non-MFV models, new operators (new B_i parameters) contribute and the functions F_i in addition to real SM contributions can contain new flavour parameters and new complex phases.

In [1] we have presented a classification of different classes of models in a form of a 2×2 flavour matrix which distinguished only between models with SM operators and models with new operators on one hand and MFV and non-MFV on the other hand. From the present perspective this matrix is insufficient as it does not take into account the possible presence of FBPs and moreover, does not distinguish sufficiently between different Lorentz structures of the operators involved. In particular, it does not distinguish between right-handed currents that involve gauge bosons and scalar currents resulting primarily from Higgs exchanges. Therefore, at the end of the paper we will attempt to improve on this by proposing a flavour code in a form of a bigger matrix: the 3×3 *flavour code matrix* (FCM).

Clearly without a good knowledge of non-perturbative factors B_i no precision studies of flavour physics will be possible unless the non-perturbative uncertainties can be reduced or even removed by taking suitable ratios of observables. In certain rare cases it is also possible to measure the relevant hadronic matrix elements entering rare decays by using leading tree-level decays. Examples of such fortunate situations are certain mixing induced CP asymmetries and the branching ratios for $K \to \pi\nu\bar{\nu}$ decays. Yet, in many cases one has to face the direct evaluation of B_i. While lattice calculations, QCD-sum rules, light-cone sum rules and large-N methods have made significant progress in the last 20 years, the situation is clearly not satisfactory and one should hope that new advances in the calculation of B_i parameters will be made in the LHC era in order to adequately use improved data. Recently an impressive progress in calculating the parameter $\hat{B}_K$, relevant for CP violation in K^0–$\bar{K}^0$ mixing, has been made and we will discuss its implications in Section 6.

Concerning the factors $\eta^i_{\rm QCD}$ an impressive progress has been made during the last 20 years. The 1990s can be considered as the era of NLO QCD calculations. Basically, NLO corrections to all relevant decays and transitions have been calculated already in the last decade [10], with a few exceptions, like the width differences $\Delta\Gamma_{s,d}$ in the $B^0_{s,d}$–$\bar{B}^0_{s,d}$ systems that were completed only in 2003 [19–21]. The last decade can be considered as the era of NNLO calculations. In particular, one should mention here the NNLO calculations of QCD corrections to $B \to X_s l^+ l^-$ [22–28], $K^+ \to \pi^+\nu\bar{\nu}$ [29–31], and especially to $B_s \to X_s\gamma$ [32] with the latter one being by far the most difficult one. Also important steps towards a complete calculation of NNLO corrections to non-leptonic decays of mesons have been made in [33]. Most recently NNLO QCD corrections to the parameter $\eta^{\rm ct}_{\rm QCD}$ in ε_K [34] and two-loop electroweak corrections to $K \to \pi\nu\bar{\nu}$ [35] have been calculated.

The final ingredients of our master formula, in addition to V^i_{CKM} factors, are the loop functions F_i resulting from penguin and box diagrams with the exchanges of the top quark, $W^{\pm}$, Z^0, heavy new gauge bosons, heavy new fermions and scalars. They are known at one-loop level in several extensions of the SM, in particular in the two Higgs doublet model (2HDM), the littlest Higgs model without T-parity (LH), the ACD model with one universal extra dimension (UED) [36], the MSSM with MFV and non-MFV violating interactions, the flavour-blind MSSM (FBMSSM), the littlest Higgs model with T-parity (LHT), Z'-models, Randall–Sundrum (RS) models, left–right symmetric models, the model with the sequential fourth generation of quarks and leptons. Moreover, in the SM $\mathcal{O}(\alpha_{\mathrm{s}})$ corrections to all relevant one-loop functions are known. It should also be stressed again that in the loop functions in our master formula one can conveniently absorb tree level FCNC contributions present in particular in RS models.

4.3. Local operators in the SM

As a preparation for the construction of the new flavour matrix we have to make a closer look at the Lorentz structure of the operators involved, first in the SM and then beyond it. To this end we have to cast our master formula in (6) into the more familiar formula that results from the relevant effective Hamiltonian.

In this more formal picture an amplitude for a decay of a given meson $M = K, B, \ldots$ into a final state $F = \pi\nu\bar{\nu}, \pi\pi, DK, \ldots$ is then simply given by

$$A(M \to F) = \langle F|\mathcal{H}_{\mathrm{eff}}|M\rangle = \frac{G_{\mathrm{F}}}{\sqrt{2}} \sum_i V^i_{\mathrm{CKM}} C_i(\mu) \langle F|Q_i(\mu)|M\rangle\,, \qquad (7)$$

where $\langle F|Q_i(\mu)|M\rangle$ are the matrix elements of the local operators Q_i between M and F, evaluated at the renormalization scale μ and $C_i(\mu)$ are the Wilson coefficients that collect compactly the effects of physics above the scale μ.

4.3.1. Non-leptonic operators

Of particular interest are the operators involving quarks only. In the case of the $\Delta B = 1$ transitions the relevant set of operators is given as follows

Current–current

$$Q_1 = (\bar{c}_\alpha b_\beta)_{V-A}\,(\bar{s}_\beta c_\alpha)_{V-A}\,, \qquad Q_2 = (\bar{c}b)_{V-A}\,(\bar{s}c)_{V-A}\,, \qquad (8)$$

QCD–penguins

$$Q_3 = (\bar{s}b)_{V-A} \sum_{q=u,d,s,c,b} (\bar{q}q)_{V-A}\,, \qquad Q_4 = (\bar{s}_\alpha b_\beta)_{V-A} \sum_{q=u,d,s,c,b} (\bar{q}_\beta q_\alpha)_{V-A}\,, \tag{9}$$

$$Q_5 = (\bar{s}b)_{V-A} \sum_{q=u,d,s,c,b} (\bar{q}q)_{V+A}\,, \qquad Q_6 = (\bar{s}_\alpha b_\beta)_{V-A} \sum_{q=u,d,s,c,b} (\bar{q}_\beta q_\alpha)_{V+A}\,, \tag{10}$$

Electroweak penguins

$$Q_7 = \tfrac{3}{2}\,(\bar{s}b)_{V-A} \textstyle\sum_{q=u,d,s,c,b} e_q\,(\bar{q}q)_{V+A}\,,$$

$$Q_8 = \tfrac{3}{2}\,(\bar{s}_\alpha b_\beta)_{V-A} \textstyle\sum_{q=u,d,s,c,b} e_q(\bar{q}_\beta q_\alpha)_{V+A}\,, \tag{11}$$

$$Q_9 = \tfrac{3}{2}\,(\bar{s}b)_{V-A} \textstyle\sum_{q=u,d,s,c,b} e_q(\bar{q}q)_{V-A}\,,$$

$$Q_{10} = \tfrac{3}{2}\,(\bar{s}_\alpha b_\beta)_{V-A} \textstyle\sum_{q=u,d,s,c,b} e_q\,(\bar{q}_\beta q_\alpha)_{V-A}\,. \tag{12}$$

Here, α,β denote colours and e_q denotes the electric quark charges reflecting the electroweak origin of $Q_7,\ldots,Q_{10}$. Finally, $(\bar{c}b)_{V-A} \equiv \bar{c}_\alpha\gamma_\mu(1-\gamma_5)b_\alpha$.

These operators play a crucial role in non-leptonic decays of B_s and B_d mesons and have also, through mixing under renormalization, an impact on other processes as is evident from the treatises in [10, 11]. For non-leptonic K decays the quark flavours have to be changed appropriately. Explicit expressions can be found in [10, 11]. In particular the analogues of Q_1 and Q_2 govern the $\Delta I = 1/2$ rule in $K_{\rm L}\to\pi\pi$ decays, while the corresponding QCD penguins and electroweak penguins enter directly the ratio ε'/ε.

Before continuing one observation should be made. We have stated before that charged current weak interactions are governed by left-handed (LH) currents. This is indeed the case as seen in (8): only $V-A$ currents are present there. This is no longer the case when QCD penguins and Electroweak Penguins that govern FCNC processes are considered. Yet, also there the presence of only LH charged currents in the SM is signalled by the fact that the first currents in each operator have $V-A$ structure. The fact that $V+A$ structures appear in (10) is related to the vectorial character of gluon interactions that in the process of the renormalization group analysis have to be decomposed into $V-A$ and $V+A$ parts. Similar comments apply to (11), where the photon penguins and Z^0 penguins are involved.

This discussion implies that in the presence of right-handed (RH) charged currents also operators with $V-A$ replaced by $V+A$ in (8) and in the first factors in the remaining operators in (9–12) would contribute.

4.3.2. Magnetic penguins

In the case of $B \to X_s\gamma$ and $B \to X_s l^+l^-$ decays and corresponding exclusive decays the crucial role is played by *magnetic* penguin operators

$$\begin{aligned} Q_{7\gamma} &= \frac{e}{8\pi^2} m_b \bar{s}_\alpha \sigma^{\mu\nu}(1+\gamma_5) b_\alpha F_{\mu\nu}\,, \\ Q_{8G} &= \frac{g}{8\pi^2} m_b \bar{s}_\alpha \sigma^{\mu\nu}(1+\gamma_5) T^a_{\alpha\beta} b_\beta G^a_{\mu\nu}\,. \end{aligned} \tag{13}$$

The operator Q_{8G} can also be relevant in non-leptonic decays. The magnetic operators are often called *dipole* operators.

4.3.3. $\Delta S = 2$ and $\Delta B = 2$ operators

In the case of K^0–$\bar{K}^0$ mixing and B_d^0–$\bar{B}_d^0$ mixing the relevant operators within the SM are

$$Q(\Delta S = 2) = (\bar{s}d)_{V-A}(\bar{s}d)_{V-A}\,, \qquad Q(\Delta B = 2) = (\bar{b}d)_{V-A}(\bar{b}d)_{V-A}\,. \tag{14}$$

For B_s^0–$\bar{B}_s^0$ mixing one has to replace d by s in the last operator.

4.3.4. Semi-leptonic operators

In the case of $B \to X_s l^+l^-$ also the following operators on top of magnetic penguins contribute

$$Q_{9V} = (\bar{s}b)_{V-A}(\bar{\mu}\mu)_V\,, \qquad Q_{10A} = (\bar{s}b)_{V-A}(\bar{\mu}\mu)_A\,. \tag{15}$$

Changing appropriately flavours one obtains the corresponding operators relevant for $B \to X_d l^+l^-$ and $K_{\rm L} \to \pi^0 l^+l^-$.

The rare decays $B \to X_s\nu\bar{\nu}$, $B \to K^*\nu\bar{\nu}$, $B \to K\nu\bar{\nu}$ and $B_s \to \bar{\mu}\mu$ are governed by

$$Q_{\nu\bar{\nu}}(B) = (\bar{s}b)_{V-A}(\bar{\nu}\nu)_{V-A}\,, \qquad Q_{\mu\bar{\mu}}(B) = (\bar{s}b)_{V-A}(\bar{\mu}\mu)_{V-A}\,. \tag{16}$$

The rare decays $K \to \pi\nu\bar{\nu}$ and $K_{\rm L} \to \bar{\mu}\mu$ are governed on the other hand by

$$Q_{\nu\bar{\nu}}(K) = (\bar{s}d)_{V-A}(\bar{\nu}\nu)_{V-A}\,, \qquad Q_{\mu\bar{\mu}}(K) = (\bar{s}d)_{V-A}(\bar{\mu}\mu)_{V-A}\,. \tag{17}$$

4.4. Local operators in extensions of the SM

NP can generate new operators. Typically new operators are generated through the presence of RH currents and *scalar* currents with the latter strongly suppressed within the SM. New gauge bosons and scalar exchanges are at the origin of these operators that can have important impact on phenomenology. The two-loop anomalous dimensions of these operators have been calculated in [37, 38].

4.4.1. $\Delta F = 2$ non-leptonic operators

For definiteness, we shall consider here operators responsible for the K^0–$\bar{K}^0$ mixing and consequently relevant also for ε_K. There are 8 such operators of dimension 6. They can be split into 5 separate sectors, according to the chirality of the quark fields they contain. The operators belonging to the first three sectors (VLL, LR and SLL) read [38]

$$\begin{aligned}
Q_1^{\rm VLL} &= (\bar{s}^\alpha \gamma_\mu P_L d^\alpha)\left(\bar{s}^\beta \gamma^\mu P_L d^\beta\right), \\[2ex]
Q_1^{\rm LR} &= (\bar{s}^\alpha \gamma_\mu P_L d^\alpha)\left(\bar{s}^\beta \gamma^\mu P_R d^\beta\right), \\
Q_2^{\rm LR} &= (\bar{s}^\alpha P_L d^\alpha)\left(\bar{s}^\beta P_R d^\beta\right), \\[2ex]
Q_1^{\rm SLL} &= (\bar{s}^\alpha P_L d^\alpha)\left(\bar{s}^\beta P_L d^\beta\right), \\
Q_2^{\rm SLL} &= (\bar{s}^\alpha \sigma_{\mu\nu} P_L d^\alpha)\left(\bar{s}^\beta \sigma^{\mu\nu} P_L d^\beta\right). \qquad (18)
\end{aligned}$$

where $\sigma_{\mu\nu} = \frac{1}{2}[\gamma_\mu, \gamma_\nu]$ and $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$. The operators belonging to the two remaining sectors (VRR and SRR) are obtained from $Q_1^{\rm VLL}$ and $Q_i^{\rm SLL}$ by interchanging P_L and P_R. For $\Delta B = 2$ the flavours have to be changed appropriately.

4.4.2. $\Delta F = 1$ operators

The list of $\Delta F = 1$ operators in the extensions of the SM is much longer and will not be given here. All the dimension six four-quark operators are discussed in [38] where also their two-loop anomalous dimensions have been calculated. See also [37], where a different operator basis is used.

Concerning the semi-leptonic operators in the extensions of the SM the typical examples of operators related to the presence of *RH currents* are

$$\tilde{Q}_{9V} = (\bar{s}b)_{V+A}(\bar{\mu}\mu)_V\,, \qquad \tilde{Q}_{10A} = (\bar{s}b)_{V+A}(\bar{\mu}\mu)_A\,, \qquad (19)$$

$$\tilde{Q}_{\nu\bar{\nu}}(B) = (\bar{s}b)_{V+A}(\bar{\nu}\nu)_{V-A}\,, \qquad \tilde{Q}_{\mu\bar{\mu}}(B) = (\bar{s}b)_{V+A}(\bar{\mu}\mu)_{V-A}\,, \qquad (20)$$

$$Q_{\nu\bar{\nu}}(K) = (\bar{s}d)_{V+A}(\bar{\nu}\nu)_{V-A}\,, \qquad Q_{\mu\bar{\mu}}(K) = (\bar{s}d)_{V+A}(\bar{\mu}\mu)_{V-A}\,. \qquad (21)$$

If *scalar currents* resulting from scalar exchanges like the heavy Higgs in the 2HDM models or sparticles in the MSSM are present, scalar operators enter the game. The most prominent are the ones that govern the $B_s \to \mu^+\mu^-$ decay in 2HDMs and the MSSM at large $\tan\beta$

$$Q_{\rm S} = (\bar{s}P_L b)(\bar{\mu}\mu)\,, \qquad Q_{\rm P} = (\bar{s}P_L b)(\bar{\mu}\gamma_5\mu)\,, \qquad (22)$$

$$\tilde{Q}_{\rm S} = (\bar{s}P_R b)(\bar{\mu}\mu)\,, \qquad \tilde{Q}_{\rm P} = (\bar{s}P_R b)(\bar{\mu}\gamma_5\mu)\,. \qquad (23)$$

4.5. Penguin-box expansion

After this rather formal presentation let us just state how our master formula (6) can be obtained from the expression (7). We can start with (7) but instead of evaluating it at the low-energy scale we choose for μ the high-energy scale to be called μ_H at which heavy particles are integrated out. Then expressing the Wison coefficients $C_i(\mu_H)$ in terms of the loop functions F_i we arrive at (6). As the expansion in (6) involves basic one-loop functions from penguin and box diagrams it was naturally given the name of the *Penguin-Box Expansion* (PBE) [39].

Originally PBE was designed to expose the m_t-dependence of FCNC processes [39] which was hidden in the Wilson coefficients. In particular in the case of ε'/ε where many of these functions enter, this turned out to be very useful. After the top quark mass has been measured precisely this role of PBE is less important. On the other hand, PBE is very well suited for the study of the extensions of the SM in which new particles are exchanged in the loops and as discussed above all these effects are encoded in the functions F_i.

If new operators are present it is often useful to work first with the coefficients $C_i(\mu_H)$ rather then loop functions. Then absorbing $G_{\rm F}/\sqrt{2}$ and $V^i_{\rm CKM}$ in the Wilson coefficients $C_i(\mu_H)$ the amplitude for M–$\overline{M}$ mixing ($M = K, B_d, B_s$) is then simply given by

$$A(M \to \bar{M}) = \sum_{i,a} C_i(\mu_H) \left\langle \overline{M} \left| Q_i^a(\mu_H) \right| M \right\rangle , \tag{24}$$

where the sum runs over all the operators in (18), that is $i = 1, 2$ and $a = \mathrm{VLL}, \mathrm{VRR}, \mathrm{LR}, \ldots$. The matrix elements for B_d–$\bar{B}_d$ mixing are for instance given as follows [40]

$$\left\langle \bar{B}^0 \left| Q_i^a \right| B^0 \right\rangle = \tfrac{2}{3} M_{B_d}^2 F_{B_d}^2 P_i^a(B_d) \,, \tag{25}$$

where the coefficients $P_i^a(B_d)$ collect compactly all RG effects from scales below μ_H as well as hadronic matrix elements obtained by lattice methods at low-energy scales. Analytic formulae for these coefficients are given in [40] while the recent application of this method can be found in [41–43]. As the Wilson coefficients $C_i(\mu_H)$ depend directly on the loop functions and fundamental parameters of a given theory, this dependence can be exhibited as in PBE or (6) if necessary. Again as in the case of PBE the virtue of using high-energy scale rather than the low-energy scale is that the coefficients $P_i^a(M)$ can be evaluated once for all if the hadronic matrix elements are known. Other virtues of this approach are discussed in [41–43].

5. A closer look at selected BSM models

5.1. Three strategies in waiting for NP in flavour physics

Particle physicists are waiting eagerly for a solid evidence of NP for the last 30 years. Except for neutrino masses, the BAU and Dark Matter, no clear signal emerged so far. While waiting several strategies for finding NP have been developed. They can be divided roughly into three classes.

5.1.1. Precision calculations within the SM

Here basically the goal is to calculate precisely the background to NP coming from the known dynamics of the SM. At first sight this approach is not very exciting. Yet, in particular in flavour physics, where the signals of NP are generally indirect, this approach is very important. From my point of view, having been involved for more than one decade in calculations of higher order QCD corrections [11], I would claim that for most interesting decays these perturbative and renormalization group improved calculations reached already the desired level. See references in Section 4.

The main progress is now required from the lattice groups. Here the main goals for the coming years are more accurate values of weak decay constants $F_{B_{d,s}}$ and various $\hat{B}_i$ parameters relevant for $B_{d,s}$ physics. For K^0–$\bar{K}^0$ mixing the relevant parameter $\hat{B}_K$ is now known with an accuracy of 4% [44]. An impressive achievement. Let us hope that also the parameters B_6 and B_8, relevant for ε'/ε will be known with a similar accuracy within this decade.

Clearly further improvements on the hadronic part of two-body non-leptonic decays is mandatory in order to understand more precisely the direct CP violation in $B_{s,d}$ decays.

5.1.2. The bottom–up approach

In this approach one constructs effective field theories involving only light degrees of freedom including the top quark in which the structure of the effective Lagrangians is governed by the symmetries of the SM and often other hypothetical symmetries. This approach is rather powerful in the case of electroweak precision studies and definitely teaches us something about $\Delta F = 2$ transitions. In particular lower bounds on NP scales depending on the Lorentz structure of operators involved can be derived from the data [2, 45].

However, except for the case of MFV and closely related approaches based on flavour symmetries, the bottom–up approach ceases, in my view, to be useful in $\Delta F = 1$ decays, because of very many operators that are allowed to appear in the effective Lagrangians with coefficients that are basically unknown [46, 47]. In this approach then the correlations between various $\Delta F = 2$ and $\Delta F = 1$ observables in K, D, B_d and B_s systems are either not visible or very weak, again except MFV, CMFV or closely related approaches. Moreover, the correlations between flavour violation in low-energy processes and flavour violation in high-energy processes to be studied soon at the LHC are lost. Again MFV belongs to a few exceptions.

5.1.3. The top–down approach

My personal view shared by some of my colleagues is that the top–down approach is more useful in flavour physics. Here one constructs first a specific model with heavy degrees of freedom. For high-energy processes, where the energy scales are of the order of the masses of heavy particles one can directly use this "full theory" to calculate various processes in terms of the fundamental parameters of a given theory. For low-energy processes one again constructs the low-energy theory by integrating out heavy particles. The advantage over the previous approach is that now the coefficients of the resulting local operators are calculable in terms of the fundamental parameters of this theory. In this manner correlations between various observables belonging to different mesonic systems and correlations between low-energy and high-energy observables are possible. Such correlations are less sensitive to free parameters than separate observables and represent patterns of flavour violation characteristic for a given theory. These correlations can in some models differ strikingly from the ones of the SM and of the MFV approach.

5.2. Anatomies of explicit models

Having the last strategy in mind my group at the Technical University Munich, consisting dominantly of diploma students, PhD students and young post-docs investigated in the last decade flavour violating and CP-violating processes in the following models: CMFV, MFV, MFV–MSSM, Z'-models, general MSSM, a model with a universal flat 5th dimension, the Littlest Higgs model (LH), the Littlest Higgs model with T-parity (LHT), SUSY–GUTs, Randall–Sundrum model with custodial protection (RSc), flavour-blind MSSM (FBMSSM), four classes of supersymmetric flavour models with the dominance of LH currents (δLL model), the dominance of RH currents in an Abelian flavour model (AC model), non-Abelian model with equal strength of CKM-like LH and RH currents (RVV2) and the non-Abelian AKM model in which the CKM-like RH currents dominate. The last comments applying only to the NP part as the SM part is always there. This year we have analysed the SM4, the $2\mathrm{HDM}_{\overline{\mathrm{MFV}}}$ to be defined below and finally quark flavour mixing with RH currents in an effective theory approach RHMFV. These analyses where dominated by quark flavour physics, but in the case of the LHT, FBMSSM, supersymmetric flavour models and the SM4 also lepton flavour violation has been studied in detail. In what follows I will briefly describe several of these extensions putting emphasis on flavour violating and CP-violating processes. The order of presentation does not correspond to the chronological order in which these analyses have been performed.

5.3. Constrained Minimal Flavour Violation (CMFV)

The simplest class of extensions of the SM are models with CMFV [13–15]. They are formulated as follows:

- All flavour changing transitions are governed by the CKM matrix with the CKM phase being the only source of CP violation.
- The only relevant operators in the effective Hamiltonian below the weak scale are those that are also relevant in the SM.

This implies that in the master formula (6) relative to the SM only the values of F_i are modified but their universal character remains intact. In particular they are real.

As discussed in detail in [14] this class of models can be formulated to a very good approximation in terms of 11 parameters: 4 parameters of the CKM matrix and 7 *real* values of the *universal* master functions F_i that parametrize the short distance contributions. In a given CMFV model, F_i can be calculated in perturbation theory and are generally correlated with each other but in a model independent analysis they must be considered as free parameters.

Generally, several master functions contribute to a given decay, although decays exist which depend only on a single function. We have the following correspondence between the most interesting FCNC processes and the master functions in the CMFV models

K^0–$\bar{K}^0$ mixing (ε_K)	$S(v)$
$B^0_{d,s}$–$\bar{B}^0_{d,s}$ mixing ($\Delta M_{s,d}$)	$S(v)$
$K \to \pi\nu\bar{\nu}$, $B \to X_{d,s}\nu\bar{\nu}$	$X(v)$
$K_{\rm L} \to \mu\bar{\mu}$, $B_{d,s} \to l\bar{l}$	$Y(v)$
$K_{\rm L} \to \pi^0 e^+e^-$	$Y(v)$, $Z(v)$, $E(v)$
ε', Non-leptonic $\Delta B = 1$, $\Delta S = 1$	$X(v)$, $Y(v)$, $Z(v)$, $E(v)$
$B \to X_s\gamma$	$D'(v)$, $E'(v)$
$B \to X_s$ gluon	$E'(v)$
$B \to X_s l^+l^-$	$Y(v)$, $Z(v)$, $E(v)$, $D'(v)$, $E'(v)$,

where v denotes collectively the arguments of a given function.

This table means that the observables like branching ratios, mass differences $\Delta M_{d,s}$ in $B^0_{d,s}$–$\bar{B}^0_{d,s}$ mixing and the CP violation parameters ε_K and ε', all can be to a very good approximation entirely expressed in terms of the corresponding master functions, the relevant CKM factors, low-energy parameters like the B_i and QCD factors $\eta^i_{\rm QCD}$.

All master functions have been defined in [14]. Phenomenological studies indicate that only

$$S(v)\,, \quad X(v)\,, \quad Y(v)\,, \quad Z(v)\,, \quad D'(v)\,, \quad E'(v) \tag{26}$$

receive significant NP contributions. $S(v)$ represents the box diagrams in $\Delta F = 2$ processes, $X(v)$ and $Y(v)$ stand for gauge invariant combinations of Z^0 penguin and $\Delta F = 1$ box diagrams, $Z(v)$ for a gauge invariant combination of the Z^0 penguin and the photon penguin. Finally $D'(v)$ and $E'(v)$ stand for magnetic photon penguin and magnetic gluon penguin, respectively. The NP contributions to the ordinary gluon penguin $E(v)$ are generally irrelevant in this framework.

In [14] strategies for the determination of the values of these functions have been outlined. Moreover, in certain cases these model dependent functions can be eliminated by taking certain combinations of observables. In this manner one obtains universal correlations between these observables that are characteristic for this class of models. The most interesting are the following ones

$$\frac{\Delta M_d}{\Delta M_s} = \frac{m_{B_d}}{m_{B_s}}\,\frac{\hat{B}_d}{\hat{B}_s}\,\frac{F^2_{B_d}}{F^2_{B_s}}\left|\frac{V_{td}}{V_{ts}}\right|^2 , \tag{27}$$

$$\frac{{\rm Br}(B\to X_d\nu\bar{\nu})}{{\rm Br}(B\to X_s\nu\bar{\nu})} = \left|\frac{V_{td}}{V_{ts}}\right|^2 , \tag{28}$$

$$\frac{{\rm Br}(B_d\to \mu^+\mu^-)}{{\rm Br}(B_s\to \mu^+\mu^-)} = \frac{\tau(B_d)}{\tau(B_s)}\,\frac{m_{B_d}}{m_{B_s}}\,\frac{F^2_{B_d}}{F^2_{B_s}}\left|\frac{V_{td}}{V_{ts}}\right|^2 , \tag{29}$$

that all can be used to determine $|V_{td}/V_{ts}|$ without the knowledge of $F_i(v)$ [13]. In particular the relation (27) offers a powerful determination of the length of one side of the unitarity triangle, denoted usually by $R_{\rm t}$. Combining this determination with the experimental value of the CP asymmetry $S_{\psi K_{\rm S}} = \sin 2\beta$ allows to determine the unitarity triangle that is universal for CMFV models [13].

Out of these three ratios the cleanest is (28), which is essentially free of hadronic uncertainties [48]. Next comes (29), involving SU(3) breaking effects in the ratio of B-meson decay constants. Finally, SU(3) breaking in the ratio $\hat{B}_{B_d}/\hat{B}_{B_s}$ enters in addition in (27). These SU(3) breaking effects are already known with respectable precision from lattice QCD.

Eliminating $|V_{td}/V_{ts}|$ from the three relations above allows to obtain three relations between observables that are universal within the MFV models. In particular from (27) and (29) one finds [49]

$$\frac{\mathrm{Br}(B_s \to \mu\bar{\mu})}{\mathrm{Br}(B_d \to \mu\bar{\mu})} = \frac{\hat{B}_d}{\hat{B}_s} \frac{\tau(B_s)}{\tau(B_d)} \frac{\Delta M_s}{\Delta M_d}\,, \tag{30}$$

that does not involve F_{B_q} and consequently contains substantially smaller hadronic uncertainties than the formulae considered above. It involves only measurable quantities except for the ratio $\hat{B}_s/\hat{B}_d$ that is known already now from lattice calculations with respectable precision [52]

$$\frac{\hat{B}_s}{\hat{B}_d} = 0.95 \pm 0.03\,, \qquad \hat{B}_d = 1.26 \pm 0.11\,, \qquad \hat{B}_s = 1.33 \pm 0.06\,. \tag{31}$$

Moreover, as in the MFV models there are no flavour violating CPV phases beyond the CKM phase, we also expect [50, 51]

$$(\sin 2\beta)_{\pi\nu\bar{\nu}} = (\sin 2\beta)_{\psi K_{\mathrm{S}}}\,, \qquad (\sin 2\beta)_{\phi K_{\mathrm{S}}} \approx (\sin 2\beta)_{\psi K_{\mathrm{S}}}\,, \tag{32}$$

if flavour-blind phases are assumed to be negligible.

The confirmation of these two relations would be a very important test for the MFV idea. Indeed, in $K \to \pi\nu\bar{\nu}$ the phase β originates in the Z^0 penguin diagram, whereas in the case of $S_{\psi K_{\mathrm{S}}}$ in the B_d^0–$\bar{B}_d^0$ box diagram. In the case of the asymmetry $S_{\phi K_{\mathrm{S}}}$ it originates also in B_d^0–$\bar{B}_d^0$ box diagrams but the second relation in (32) could be spoiled by new physics contributions in the decay amplitude for $B \to \phi K_{\mathrm{S}}$ that is non-vanishing only at the one-loop level.

One can also derive the following relations between $\mathrm{Br}(B_q \to \mu\bar{\mu})$ and ΔM_q [49]

$$\mathrm{Br}(B_q \to \mu\bar{\mu}) = 4.36 \times 10^{-10} \frac{\tau(B_q)}{\hat{B}_q} \frac{Y^2(v)}{S(v)} \Delta M_q\,, \qquad (q = s, d)\,. \tag{33}$$

These relations allow to predict $\mathrm{Br}(B_{s,d} \to \mu\bar{\mu})$ in a given MFV model with substantially smaller hadronic uncertainties than found by using directly the formulae for the branching ratios in question. Using the present input parameters we find

$$\mathrm{Br}(B_d \to \mu^+\mu^-)_{\mathrm{SM}} = (1.0 \pm 0.1) \times 10^{-10}\,, \tag{34}$$

$$\mathrm{Br}(B_s \to \mu^+\mu^-)_{\mathrm{SM}} = (3.2 \pm 0.2) \times 10^{-9}\,. \tag{35}$$

5.4. Minimal flavour violation

We have already formulated what we mean by CMFV. Let us first add here that the models with CMFV generally contain only one Higgs doublet and the top Yukawa coupling dominates. On the other hand, general models with MFV contain more scalar representations, in particular two Higgs doublets. Moreover, the operator structure in these models can differ from the SM one. This is the case when bottom and top Yukawa couplings are of comparable size. A well known example is the MSSM with MFV and large $\tan\beta$.

In the more general case of MFV the formulation with the help of global symmetries present in the limit of vanishing Yukawa couplings [17, 18] as formulated in [16] is elegant and useful. See also [53] for a similar formulation that goes beyond the MFV. Recent discussions of various aspects of MFV can be found in [54–59]. In order to see what is here involved we follow a compact formulation of Isidori [60].

Let us look then at the Standard Model (SM) Lagrangian which can be divided into two main parts, the gauge and the Higgs (or symmetry breaking) sector. The gauge sector is completely specified by the local symmetry $\mathcal{G}^{\rm SM}_{\rm local} = {\rm SU}(3)_C \times {\rm SU}(2)_L \times {\rm U}(1)_Y$ and by the fermion content

$$\begin{aligned}\mathcal{L}^{\rm SM}_{\rm gauge} = & \sum_{i=1\ldots3}\ \sum_{\psi=Q^i_L\ldots E^i_R} \bar\psi i \not{D}\, \psi \\ & -\tfrac{1}{4}\textstyle\sum_{a=1\ldots8} G^a_{\mu\nu}G^{a\mu\nu} - \tfrac{1}{4}\sum_{a=1\ldots3} W^a_{\mu\nu}W^{a\mu\nu} - \tfrac{1}{4}B_{\mu\nu}B^{\mu\nu}\,. \end{aligned} \tag{36}$$

The fermion content consist of five fields with different quantum numbers under the gauge group.

$$Q^i_L(3,2)_{+1/6}\,,\quad U^i_R(3,1)_{+2/3}\,,\quad D^i_R(3,1)_{-1/3}\,,\quad L^i_L(1,2)_{-1/2}\,,\quad E^i_R(1,1)_{-1}\,, \tag{37}$$

each of them appearing in three different flavours ($i = 1, 2, 3$).

As given above $\mathcal{L}^{\rm SM}_{\rm gauge}$ has a large *global* flavour symmetry ${\rm U}(3)^5$, corresponding to the independent unitary rotations in flavour space of the five fermion fields in (37). This can be decomposed as follows

$$\mathcal{G}_{\rm flavour} = {\rm U}(3)^5 \times \mathcal{G}_q \times \mathcal{G}_\ell\,, \tag{38}$$

where

$$\mathcal{G}_q = {\rm SU}(3)_{Q_L} \times {\rm SU}(3)_{U_R} \times {\rm SU}(3)_{D_R}\,, \qquad \mathcal{G}_\ell = {\rm SU}(3)_{L_L} \otimes {\rm SU}(3)_{E_R}\,. \tag{39}$$

Three of the five U(1) subgroups can be identified with the total baryon and lepton number and the weak hypercharge. The two remaining U(1) groups can be identified with the Peccei–Quinn symmetry of 2HDMs and with a global rotation of a single $SU(2)_L$ singlet.

Both the local and the global symmetries of $\mathcal{L}^{\rm SM}_{\rm gauge}$ are broken with the introduction of a $SU(2)_L$ Higgs doublet ϕ. The local symmetry is spontaneously broken by the vacuum expectation value of the Higgs field, $\langle\phi\rangle = v = 246$ GeV, while the global flavour symmetry is *explicitly broken* by the Yukawa interaction of ϕ with the fermion fields

$$-\mathcal{L}^{\rm SM}_{\rm Yukawa} = Y_d^{ij}\bar{Q}_L^i\phi D_R^j + Y_u^{ij}\bar{Q}_L^i\tilde{\phi}U_R^j + Y_e^{ij}\bar{L}_L^i\phi E_R^j + \text{h.c.} \qquad \left(\tilde{\phi} = i\tau_2\phi^\dagger\right) . \tag{40}$$

The subgroups controlling flavour-changing dynamics, in particular flavour non-universality, are the non-Abelian groups $\mathcal{G}_q$ and $\mathcal{G}_\ell$, which are explicitly broken by $Y_{d,u,e}$ not being proportional to the identity matrix.

The hypothesis of MFV amounts to assuming that the Yukawas are the only sources of the breakdown of flavour and CP violation.

The phenomenological implications of the MFV hypothesis formulated in this more grander manner than the CMFV formulation given above can be found model independently by using an effective field theory approach (EFT) [16]. In this framework the SM Lagrangian is supplemented by all higher dimension operators consistent with the MFV hypothesis, built using the Yukawa couplings as spurion fields. The NP effects in this framework are then parametrized in terms of a few *flavour-blind* free parameters and SM Yukawa couplings that are solely responsible for flavour violation and also CP violation if these flavour-blind parameters are chosen as *real* quantities as done in [16]. This approach naturally suppresses FCNC processes to the level observed experimentally even in the presence of new particles with masses of a few hundreds GeV. It also implies specific correlations between various observables, which are not as stringent as in the CMFV but are still very powerful.

Yet, it should be stressed that the MFV symmetry principle in itself does not forbid the presence of *flavour-blind* CP-violating sources [55–59, 61–65] that make effectively the flavour-blind free parameters *complex* quantities having flavour-blind phases (FBPs). These phases can in turn enhance the electric dipole moments EDMs of various particles and atoms and in the interplay with the CKM matrix can have also profound impact on flavour violating observables, in particular the CP-violating ones.

One concrete example is MFV-MSSM which in view of these FBPs suffers from the same SUSY CP problem as the ordinary MSSM. Either an extra assumption or a mechanism accounting for a natural suppression of these CP-violating phases is then desirable. Possible solution to this problem is the following [58]: the SUSY breaking mechanism is *flavour-blind* and CP conserving and the breaking of CP only arises through the MFV compatible terms breaking the *flavour-blindness*. That is, CP is preserved by the sector responsible for SUSY breaking, while it is broken in the flavour sector. While the generalized MFV ansatz still accounts for a natural solution of the SUSY CP problem, it also leads to peculiar and testable predictions in low-energy CP-violating processes [58].

Yet, independently of Supersymmetry, that we will discuss later on, the introduction of flavour-blind CPV phases compatible with the MFV symmetry principle turns out to be a very interesting set-up [55–58, 61]. In particular, as noted in [56], a large new phase in B_s^0–$\bar{B}_s^0$ mixing could in principle be obtained in the MFV framework if additional flavour-blind phases are present. This idea cannot be realized in the ordinary MSSM with MFV, as shown in [66, 67]. The difficulty of realizing this scenario in the MSSM is due to the suppression in the MSSM of effective operators with several Yukawa insertions. Sizable couplings for these operators are necessary both to have an effective large CP-violating phase in B_s^0–$\bar{B}_s^0$ mixing and, at the same time, to evade bounds from other observables, such as $B_s \to \mu^+\mu^-$ and $B \to X_s\gamma$. However, it could be realized in different underlying models, such as the up-lifted MSSM, as recently pointed out in [68] and in the 2HDM with MFV and FBPs as we will discuss now.

5.5. $\mathrm{2HDM}_{\overline{\mathrm{MFV}}}$

5.5.1. Preliminaries

We will next discuss a specific class of 2HDM models, namely 2HDM with MFV accompanied by flavour blind CP phases that we will call for short $\mathrm{2HDM}_{\overline{\mathrm{MFV}}}$ [41] with the "bar" on MFV indicating the presence of FBPs.

Before entering the details it will be instructive to recall that the standard assignment of the $\mathrm{SU(2)}_L \times \mathrm{U(1)}_Y$ quark charges, identified long ago by Glashow, Iliopoulos, and Maiani (GIM) [8], forbids tree-level flavour-changing couplings of the quarks to the SM neutral gauge bosons. In the case of only one-Higgs doublet, namely within the SM, this structure is effective also in eliminating a possible dimension-four flavour-changing neutral-current (FCNC) coupling of the quarks to the Higgs field. While the $\mathrm{SU(2)}_L \times \mathrm{U(1)}_Y$ assignment of quarks and leptons can be considered as being well established, much less is known about the Higgs sector of the theory. In the presence of more than one-Higgs field the appearance of tree-level FCNC is not automatically forbidden by the standard assignment of the $\mathrm{SU(2)}_L \times \mathrm{U(1)}_Y$ fermion charges: additional conditions have to be imposed on the model in order to guarantee a sufficient suppression of FCNC processes [69, 70]. The absence of renormalizable couplings contributing at the tree-level to FCNC processes, in multi-Higgs models, goes under the name of Natural Flavour Conservation (NFC) hypothesis.

The idea of NFC has been with us for more than 30 years. During the last decade the mechanism for the suppression of FCNC processes with the help of MFV has been developed and it is natural to ask how NFC (and GIM) are related to MFV, and *vice versa*. Motivated by a series of recent studies about the strength of FCNCs in multi-Higgs doublet models [71–75], we have presented recently a detailed analysis of the relation between the NFC and MFV hypotheses [41]. As we have shown, while the two hypotheses are somehow equivalent at the tree-level, important differences arise when quantum corrections are included. Beyond the tree-level, or beyond the implementation of these two hypotheses in their simplest version, some FCNCs are naturally generated in both cases. In this more general framework, the MFV hypothesis in its general formulation [16] turns out to be more stable in suppressing FCNCs than the hypothesis of NFC alone.

I will not repeat here these arguments as for some readers they could appear academic. In short, it is probably not surprising that flavour-blind symmetries that are used to protect FCNCs in the context of NFC are not as powerful as flavour symmetries used in the context of the MFV hypothesis. A nice summary of our work by one of my collaborators making this point very clear appeared recently [76]. Instead, I would like to summarise the phenomenological implications of this framework that were not expected by us when we started our analysis. In particular, in our second analysis the issue of EDMs in this framework has also been considered [42]. Other recent interesting analyses of FCNC processes within 2HDMs can be found in [68, 71, 72, 75, 77–79].

Let me first list the few important points of the $2\mathrm{HDM}_{\overline{\mathrm{MFV}}}$ framework.

- The presence of FBPs in this MFV framework modifies through their interplay with the standard CKM flavour violation the usual characteristic relations for the MFV framework. In particular, the mixing induced CP asymmetries in $B_d^0 \to \psi K_{\mathrm{S}}$ and $B_s^0 \to \psi\phi$ take the form known from non-MFV frameworks like LHT, RSc and SM4

$$S_{\psi K_{\mathrm{S}}} = \sin(2\beta + 2\varphi_{B_d})\,, \qquad S_{\psi\phi} = \sin(2|\beta_s| - 2\varphi_{B_s})\,, \tag{41}$$

  where φ_{B_q} are NP phases in B_q^0–$\bar{B}_q^0$ mixings. Thus in the presence of non-vanishing φ_{B_d} and φ_{B_s} originating here in non-vanishing FBPs these two asymmetries do not measure β and β_s but $(\beta + \varphi_{B_d})$ and $(|\beta_s| - \varphi_{B_s})$, respectively.

- The FBPs in the $2\mathrm{HDM}_{\overline{\mathrm{MFV}}}$ can appear both in Yukawa interactions and in the Higgs potential. While in [41] only the case of FBPs in Yukawa interactions has been considered, in [42] these considerations have been extended to include also the FBPs in the Higgs potential.

The two flavour-blind CPV mechanisms can be distinguished through the correlation between $S_{\psi K_S}$ and $S_{\psi\phi}$ that is strikingly different if only one of them is relevant. We will see this explicitly below.

- Sizable FBPs, necessary to explain possible sizable non-standard CPV effects in B_s mixing could, in principle, be forbidden by the upper bounds on EDMs of the neutron and the atoms. This question has been addressed in [42] and the answer will be given below.

Let us then briefly consider these two cases, returning to them in more details in Section 6 in the context of a general discussion of the anomalies observed in the present data.

5.5.2. FBPs in Yukawa interactions

Integrating out the neutral Higgs fields leads to tree-level contributions to scalar FCNC operators. Working in the decoupling limit for the heavy Higgs doublet, the leading Higgs contributions to $\Delta F = 1$ and $\Delta F = 2$ Hamiltonians thus generated are ($q = d, s$)

$$\Delta\mathcal{H}^{|\Delta B|=1}_{\rm MFV} = -\frac{a_0^* + a_1^*}{M_H^2} y_\ell y_b y_t^2 V_{tb}^* V_{tq} \left(\bar{b}_R q_L\right)\left(\bar{\ell}_L \ell_R\right) + \text{h.c.}\,, \tag{42}$$

$$\Delta\mathcal{H}^{|\Delta S|=1}_{\rm MFV} = -\frac{a_0^*}{M_H^2}\, y_\ell y_s y_t^2 V_{ts}^* V_{td} \left(\bar{s}_R d_L\right)\left(\bar{\ell}_L \ell_R\right) + \text{h.c.}\,, \tag{43}$$

$$\Delta\mathcal{H}^{|\Delta B|=2}_{\rm MFV} = -\frac{(a_0^* + a_1^*)(a_0 + a_2)}{M_H^2} y_b y_q \left[y_t^2 V_{tb}^* V_{tq}\right]^2 \left(\bar{b}_R q_L\right)\left(\bar{b}_L q_R\right) + \text{h.c.}\,, \tag{44}$$

$$\Delta\mathcal{H}^{|\Delta S|=2}_{\rm MFV} = -\frac{|a_0|^2}{M_H^2}\, y_s y_d \left[y_t^2 V_{ts}^* V_{td}\right]^2 (\bar{s}_R d_L)(\bar{s}_L d_R) + \text{h.c.}\,, \tag{45}$$

where a_i are flavour-blind complex coefficients that are $\mathcal{O}$ (1). Their complex phases originate in this framework precisely from FBPs in the Yukawa interactions. y_i are Yukawa couplings. Inspecting these formulae we anticipate immediately two key properties that can be directly deduced by looking at their flavour- and CP-violating structure:

- The impact in K^0–$\bar{K}^0$, B_d^0–$\bar{B}_d^0$ and B_s^0–$\bar{B}_s^0$ mixing amplitudes scales, relative to the SM, with $m_s m_d$, $m_b m_d$ and $m_b m_s$, respectively. This fact opens the possibility of sizable non-standard contributions to the B_s system without serious constraints from K^0–$\bar{K}^0$ and B_d^0–$\bar{B}_d^0$ mixing. In particular one has the relation

$$\varphi_{B_d} \approx \frac{m_d}{m_s}\varphi_{B_s} \approx \tfrac{1}{17}\varphi_{B_s}\,. \tag{46}$$

- While the possible flavour-blind phases do not contribute to the $\Delta S = 2$ effective Hamiltonian, they could have an impact in the $\Delta B = 2$ case, offering the possibility to solve the recent experimental anomalies related to the B_s mixing phase. However, this happens only if $a_1 \neq a_2$. This requires a non-trivial underlying dynamics, which does not suppress effective operators with high powers of Yukawa insertions. While in the $2\mathrm{HDM}_{\overline{\mathrm{MFV}}}$ this is generally possible, this is not the case in the MSSM with MFV, where the supersymmetry puts these two coefficients to be approximately equal [66, 67].
- The presence of scalar operators like $Q_{\mathrm{P,S}}$ in the $\Delta B = 1$ transitions allows strong enhancements of branching ratios for $B_q \to \mu^+\mu^-$ in a correlated manner that is characteristic for models with MFV.

Let us also note that NP contributions to $\Delta F = 2$ transitions in this case are dominated by the operator Q_2^{LR} whose contributions are strongly enhanced by RG effects and in the case of ε_K by the chiral enhancement of its hadronic matrix element. Fortunately the suppression of this *direct* contribution to ε_K by the relevant CKM factor and in particular by $m_d m_s$ does not introduce any problems with satisfying the ε_K constraint and in fact this contribution can be neglected.

We are now ready to summarise the main phenomenological results obtained in [41]:

1. The pattern of NP effects in this model is governed by the quark masses relevant for the particular system considered: $m_s m_d$, $m_b m_d$ and $m_b m_s$, for the K, B_d and B_s systems, respectively.

2. If we try to accommodate a large CP-violating phase in B_s^0–$\bar{B}_s^0$ mixing in this scenario, we find a correlated shift in the relation between $S_{\psi K_{\mathrm{S}}}$ and the CKM phase β. This shift is determined unambiguously by the relation (46) and contains no free parameters. The shift is such that the prediction of $S_{\psi K_{\mathrm{S}}}$ *decreases* with respect to the SM case at fixed CKM inputs if a large positive value of $S_{\psi\phi}$ is chosen. This relaxes the existing tension between $S_{\psi K_{\mathrm{S}}}^{\mathrm{exp}}$ and its SM prediction as seen in figure 2 of [41].

3. The NP contribution to ε_K is tiny and can be neglected. However, given the modified relation between $S_{\psi K_{\mathrm{S}}}$ and the CKM phase β in (41), with $\varphi_{B_d} < 0$ the true value of β extracted in this scenario increases with respect to SM fits in which this phase is absent. As a result of this modified value of β, also the predicted value for ε_K *increases* with respect to the SM case, resulting in a better agreement with data. See figure 3 in [41].

4. The branching ratios $\mathrm{Br}(B_q \to \mu^+\mu^-)$ can be enhanced by an order of magnitude over the SM values in (34) and (35), thus reaching the upper bounds from CDF and D0. Moreover, the relation in (29) appears to be basically unaffected by the presence of possible FBPs. On the other hand, the CMFV relation gets modified as follows

$$\frac{\mathrm{Br}(B_s \to \mu^+\mu^-)}{\mathrm{Br}(B_d \to \mu^+\mu^-)} = \frac{\hat{B}_{B_d}}{\hat{B}_{B_s}} \frac{\tau(B_s)}{\tau(B_d)} \frac{\Delta M_s}{\Delta M_d} r \,, \qquad r = \frac{M^4_{B_s}}{M^4_{B_d}} \frac{|S_d|}{|S_s|} \,, \tag{47}$$

where for $r = 1$ we recover the SM and CMFV relation derived in [49]. S_q are the box functions that now are different for B_s and B_d systems. In the present model r can deviate from one; however, this deviation is at most of $\mathcal{O}$ (10%). We are looking forward to the test of (47) which will be possible once $\mathrm{Br}(B_q \to \mu^+\mu^-)$ will be known.

5. For large $S_{\psi\phi}$, the branching ratios $\mathrm{Br}(B_{s,d} \to \mu^+\mu^-)$ are strongly enhanced over SM values as seen in figure 4 of [42].

5.5.3. FBPs in the Higgs potential

If the FBPs are present only in the Higgs potential the relation between new phases φ_{B_q} changes to

$$\varphi_{B_d} = \varphi_{B_s} \,, \tag{48}$$

modifying certain aspects of the phenomenology. In particular the correlation between the CP asymmetries $S_{\psi K_S}$ and $S_{\psi\phi}$ is now very different as seen in figure 5 of [42] and in order to be consistent with the data on $S_{\psi K_S}$ the asymmetry $S_{\psi\phi}$ cannot exceed 0.3. We will return to this in Section 6.

5.5.4. Correlation between EDMs and $S_{\psi\phi}$

In [42] the correlations between EDMs and CP violation in $B_{s,d}$ mixing in $2\mathrm{HDM}_{\overline{\mathrm{MFV}}}$ including FBPs in Yukawa interactions and in the Higgs potential have been studied in detail. It has been shown that in both cases the upper bounds on EDMs of the neutron and the atoms do not forbid sizable non-standard CPV effects in B_s mixing. However, if a large CPV phase in B_s mixing will be confirmed, this will imply hadronic EDMs very close to their present experimental bounds, within the reach of the next generation of experiments, as well as $\mathrm{Br}(B_{s,d} \to \mu^+\mu^-)$ typically largely enhanced over its SM expectation. As demonstrated in figure 5 of [42] the two flavour-blind CPV mechanisms can be distinguished through the correlation between $S_{\psi K_S}$ and $S_{\psi\phi}$ that is strikingly different if only one of them is relevant. Which of these two CPV mechanisms dominates depends on the precise values of $S_{\psi\phi}$ and $S_{\psi K_S}$. Current data seems to show a mild preference for a *hybrid* scenario where both these mechanisms are at work.

5.6. Littlest Higgs model with T-parity

We will next discuss two models having the operator structure of the SM but containing new sources of flavour and CP violation. This is the Littlest Higgs Model with T-parity (LHT) and the SM4, the SM extended by a fourth sequential generation of quarks and leptons.

The Littlest Higgs model without [80] T-parity has been invented to solve the problem of the quadratic divergences in the Higgs mass without using supersymmetry. In this approach the cancellation of divergences in m_H is achieved with the help of new particles of the same spin-statistics. Basically the SM Higgs is kept light because it is a pseudo-Goldstone boson of a spontaneously broken global symmetry

$$\mathrm{SU}(5) \to \mathrm{SO}(5)\,. \tag{49}$$

Thus the Higgs is protected from acquiring a large mass by a global symmetry, although in order to achieve this the gauge group has to be extended to

$$G_{\mathrm{LHT}} = \mathrm{SU}(3)_c \times [\mathrm{SU}(2) \times \mathrm{U}(1)]_1 \times [\mathrm{SU}(2) \times \mathrm{U}(1)]_2 \tag{50}$$

and the Higgs mass generation properly arranged (*collective symmetry breaking*). The dynamical origin of the global symmetry in question and the physics behind its breakdown are not specified. But in analogy to QCD one could imagine a new strong force at scales $\mathcal{O}$ (10–20) TeV between new very heavy fermions that bind together to produce the SM Higgs. In this scenario the SM Higgs is analogous to the pion. At scales well below 5 TeV the Higgs is considered as an elementary particle but at 20 TeV its composite structure should be seen. Possibly at these high scales one will have to cope with non-perturbative strong dynamics and an unknown ultraviolet completion with some impact on low-energy predictions of Little Higgs models has to be specified. Concrete perturbative completions, albeit very complicated, have been found [81, 82]. The advantage of these models, relative to supersymmetry, is a much smaller number of free parameters but the disadvantage is that Grand Unification in this framework is rather unlikely. Excellent reviews can be found in [83, 84].

In order to make this model consistent with electroweak precision tests and simultaneously having the new particles of this model in the reach of the LHC, a discrete symmetry, T-parity, has been introduced [85,86]. Under T-parity all SM particles are *even*. Among the new particles only a heavy +2/3 charged T quark belongs to the even sector. Its role is to cancel the quadratic divergence in the Higgs mass generated by the ordinary top quark. The even sector and also the model without T-parity belong to the CMFV class if only flavour violation in the down-quark sector is considered [87,88].

More interesting from the point of view of FCNC processes is the T-odd sector. It contains three doublets of mirror quarks and three doublets of mirror leptons that communicate with the SM fermions by means of heavy $W_H^\pm$, Z_H^0 and A_H^0 gauge bosons that are also odd under T-parity. These interactions are governed by new mixing matrices V_{Hd} and V_{Hl} for down-quarks and charged leptons, respectively. The corresponding matrices in the up (V_{Hu}) and neutrino ($V_{H\nu}$) sectors are obtained by means of the relations [89, 90]

$$V_{Hu}^\dagger V_{Hd} = V_{\rm CKM}\,, \qquad V_{H\nu}^\dagger V_{Hl} = V_{\rm PMNS}^\dagger\,. \tag{51}$$

Thus we have new flavour and CP-violating contributions to decay amplitudes in this model. These new interactions can have a structure very different from the CKM and PMNS matrices.

The difference between the CMFV models and the LHT model can be transparently seen in the formulation of FCNC processes in terms of the master functions. In the LHT model the real and universal master functions in (26) become complex quantities and the property of the universality is lost. Consequently the usual CMFV relations between K, B_d and B_s systems can be strongly broken. Explicitly, the new functions are given as follows $(i = K, d, s)$

$$S_i \equiv |S_i| e^{i\theta_S^i}\,, \quad X_i^\ell \equiv \left|X_i^\ell\right| e^{i\theta_X^{i\ell}}\,, \quad Y_i \equiv |Y_i|\, e^{i\theta_Y^i}\,, \quad Z_i \equiv |Z_i|\, e^{i\theta_Z^i}\,, \tag{52}$$

$$D_i' \equiv \left|D_i'\right| e^{i\theta_{D'}^i}\,, \quad E_i' \equiv \left|E_i'\right| e^{i\theta_{E'}^i}\,. \tag{53}$$

Let us note also that in contrast to the models discussed until now the LHT model contains new heavy gauge bosons $W_H^\pm$, Z_H^0 and A_H^0. The masses of $W_H^\pm$ and Z_H^0 are typically $\mathcal{O}$ (700 GeV). A_H is significantly lighter with a mass of a few hundred GeV and, being the lightest particle with odd T-parity, it can play the role of a Dark Matter candidate. The mirror quarks and leptons can have masses typically in the range 500–1500 GeV and could be discovered at the LHC. Their impact on FCNC processes can be sometimes spectacular. A review on flavour physics in the LHT model can be found in [91] and selected papers containing details of the pattern of flavour violation in these models can be found in [92–98]. Critical discussions of the LHT model can be found in [99].

Here we only list the most interesting results from our analyses.

1. $S_{\psi\phi}$ can be much larger than its SM value but values above 0.3 are rather unlikely.

2. $\mathrm{Br}(K_{\mathrm{L}} \to \pi^0 \nu\bar{\nu})$ and $\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})$ can be enhanced up to factors of 3 and 2.5, respectively. The allowed points in the $\mathrm{Br}(K_{\mathrm{L}} \to \pi^0 \nu\bar{\nu})$ *versus* $\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})$ plot cluster around two branches. On one of them $\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})$ can reach maximal values while $\mathrm{Br}(K_{\mathrm{L}} \to \pi^0 \nu\bar{\nu})$ is SM-like. Here $\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})$ can easily reach the central experimental value of E949 Collaboration at Brookhaven [100]. On the other one $\mathrm{Br}(K_{\mathrm{L}} \to \pi^0 \nu\bar{\nu})$ can reach maximal values but $\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})$ can be enhanced by at most a factor of 1.4 and therefore not reaching the central experimental value. Some insights on this behaviour have been provided in [96].

3. Rare B-decays turn out to be SM-like but still some enhancements are possible. In particular $\mathrm{Br}(B_{s,d} \to \mu^+\mu^-)$ can be enhanced by 30% and a significant part of this enhancement comes from the T-even sector.

4. Simultaneous enhancements of $S_{\psi\phi}$ and of $\mathrm{Br}(K \to \pi\nu\bar{\nu})$ are rather unlikely.

5. $\mathrm{Br}(\mu \to e\gamma)$ can reach the upper bound of 2×10^{-11} from the MEGA Collaboration and in fact some fine tuning of the parameters is required to satisfy this bound [94, 95, 98]: either the corresponding mixing matrix in the mirror lepton sector has to be at least as hierarchical as the CKM matrix and/or the masses of mirror leptons carrying the same electric charge must be quasi-degenerate. Therefore if the MEG Collaboration does not find anything at the level of 10^{-13}, significant fine tuning of the LHT parameters will be required in order to keep $\mu \to e\gamma$ under control.

6. It is not possible to distinguish the LHT model from the supersymmetric models discussed below on the basis of $\mu \to e\gamma$ alone. On the other hand as pointed out in [94] such a distinction can be made by measuring any of the ratios $\mathrm{Br}(\mu \to 3e)/\mathrm{Br}(\mu \to e\gamma)$, $\mathrm{Br}(\tau \to 3\mu)/\mathrm{Br}(\tau \to \mu\gamma)$, *etc.* In supersymmetric models all these decays are governed by dipole operators so that these ratios are $\mathcal{O}(\alpha)$ [101–106]. In the LHT model the LFV decays with three leptons in the final state are not governed by dipole operators but by Z-penguins and box diagrams and the ratios in question turn out to be by at least one order of magnitude larger than in supersymmetric models.

7. CP violation in the D^0–$\bar{D}^0$ mixing at a level well beyond anything possible with CKM dynamics has been identified [107]. Comparisons with CP violation in K and B systems should offer an excellent test of this NP scenario and reveal the specific pattern of flavour and CP violation in the D^0–$\bar{D}^0$ system predicted by this model.

5.7. The SM with sequential fourth generation

One of the simplest extensions of the SM3 is the addition of a sequential fourth generation (4G) of quarks and leptons [108] (hereafter referred to as SM4). Therefore it is of interest to study its phenomenological implications. Beyond flavour physics possibly the most interesting implications of the presence of 4G are the following ones:

1. While being consistent with electroweak precision data (EWPT) [109–114], a 4G can remove the tension between the SM3 fit and the lower bound on the Higgs mass m_H from LEP II. Indeed, as pointed out in [110, 112, 115], a heavy Higgs boson does not contradict EWPT as soon as the 4G exists. For additional discussions see [116, 117].

2. Electroweak baryogenesis might be viable [119–121].

3. Dynamical breaking of electroweak symmetry might be triggered by the presence of 4G quarks [122–129].

However, the SM4 is also interesting for flavour physics. Several analyses of flavour physics [118, 130–141] have been performed in the last years. The SM4 introduces three new mixing angles s_{14}, s_{24}, s_{34} and two new phases in the quark sector and can still have a significant impact on flavour phenomenology. Similarly to the LHT model it does not introduce any new operators but brings in new sources of flavour and CP violation that originate in the interactions of the four generation fermions with the ordinary quarks and leptons that are mediated by the SM electroweak gauge bosons. Thus in this model, as opposed to the LHT model, the gauge group is the SM one. This implies smaller number of free parameters.

An interesting virtue of the SM4 model is the non-decoupling of new particles. Therefore, unless the model has non-perturbative Yukawa interactions, the 4G fermions are bound to be observed at the LHC with masses below 600 GeV.

Here I will only summarise the results of our analyses of quark flavour physics [137, 138] and lepton flavour violation [139]. Details can be found in these papers, in particular many correlations between various observables that are shown in numerous plots.

The most interesting patterns of flavour violation in the SM4 are the following ones:

1. All existing tensions in the UT fits can be removed in this NP scenario.

2. In particular the desire to explain the $S_{\psi\phi}$ anomaly implies uniquely the suppressions of the CP asymmetries $S_{\phi K_S}$ and $S_{\eta' K_S}$ in agreement with the data. This correlation has been pointed out in [130, 133] and

we confirmed it. However, we observed that for $S_{\psi\phi}$ significantly larger than 0.6 the values of $S_{\phi K_S}$ and $S_{\eta' K_S}$ are below their central values indicated by the data.

3. The $S_{\psi\phi}$ anomaly implies a sizable enhancement of $\mathrm{Br}(B_s \to \mu^+\mu^-)$ over the SM3 prediction although this effect is much more modest than in SUSY models where the Higgs penguin with large $\tan\beta$ is at work. Yet, values as high as 8×10^{-9} are certainly possible in the SM4, which is well beyond those attainable in the LHT model discussed previously and the RSc model discussed below. On the other hand, large values of $S_{\psi\phi}$ preclude non-SM values of $\mathrm{Br}(B_d \to \mu^+\mu^-)$. Consequently the CMFV relations in (29) and (30) can be strongly violated in this model.

4. Possible enhancements of $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$ and $\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu})$ over the SM3 values are much larger than found in the LHT and RSc models and, in particular, in SUSY flavour models discussed below, where they are SM3 like. Both branching ratios as high as several 10^{-10} are still possible in the SM4. Moreover, in this case, the two branching ratios are strongly correlated and close to the Grossman–Nir bound [142].

5. Interestingly, in contrast to the LHT and RSc models, a high value of $S_{\psi\phi}$ does not preclude a sizable enhancements of $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$, and $\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu})$.

6. NP effects in $K_L \to \pi^0\ell^+\ell^-$ and $K_L \to \mu^+\mu^-$ can be visibly larger than in the LHT and RSc models. In particular $\mathrm{Br}(K_L \to \mu^+\mu^-)_{SD}$ can easily violate the existing bound of 2.5×10^{-9}. Imposition of this bound on top of other constraints results in a characteristic shape of the correlation between $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$ and $\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu})$ that we already encountered in the LHT model.

7. Even in the presence of SM-like values for $S_{\psi\phi}$ and $\mathrm{Br}(B_s \to \mu^+\mu^-)$, large effects in the K system are possible.

8. For large positive values of $S_{\psi\phi}$ the predicted value of ε'/ε is significantly below the data, unless the hadronic matrix elements of the electroweak penguins are sufficiently suppressed with respect to the large N result and the ones of QCD penguins enhanced. We have also reemphasised [143, 144] the important role ε'/ε will play in bounding rare K decay branching ratios once the relevant hadronic matrix elements in ε'/ε will be precisely known.

9. While simultaneous large 4G effects in the K and D systems are possible, large effects in B_d generally disfavour large NP effects in the D system. Moreover, significant enhancement of $S_{\psi\phi}(B_s)$ above the

SM3 value will not allow large CP-violating effects in the D system within the 4G scenario. Additional imposition of the ε'/ε constraint significantly diminishes 4G effects in CP-violating observables in the D system.

10. The branching ratios for $\ell_i \to \ell_j\gamma$, $\tau \to \ell\pi$, $\tau \to \ell\eta^{(\prime)}$, $\mu^- \to e^-e^+e^-$, $\tau^- \to e^-e^+e^-$, $\tau^- \to \mu^-\mu^+\mu^-$, $\tau^- \to e^-\mu^+\mu^-$ and $\tau^- \to \mu^-e^+e^-$ can still all be as large as the present experimental upper bounds but not necessarily simultaneously.

11. The correlations between various branching ratios should allow to test this model. This should be contrasted with the SM3 where all these branching ratios are unmeasurable.

12. The rate for μ–e conversion in nuclei can also reach the corresponding upper bound.

13. The pattern of the LFV branching ratios in the SM4 differs significantly from the one encountered in the MSSM, allowing to distinguish these two models with the help of LFV processes in a transparent manner. Also differences from the LHT model are identified.

14. The branching ratios for $K_{\rm L} \to \mu e$, $K_{\rm L} \to \pi^0\mu e$, $B_{d,s} \to \mu e$, $B_{d,s} \to \tau e$ and $B_{d,s} \to \tau\mu$ turn out to be by several orders of magnitude smaller than the present experimental bounds.

In summary, the SM4 offers a very rich pattern of flavour violation which can be tested already in the coming years, in particular through precise measurements of $S_{\psi\phi}$, $\mathrm{Br}(B_q \to \mu^+\mu^-)$, $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$ and, later, $S_{\phi K_{\rm S}}$, $S_{\eta' K_{\rm S}}$ and $\mathrm{Br}(K_{\rm L} \to \pi^0\nu\bar{\nu})$. Also, precise measurements of the phase $\gamma \approx \delta_{13}$ and of LFV decays will be important for these investigations.

5.8. Supersymmetric Flavour (SF) models

In supersymmetric models the cancellation of divergences in m_H is achieved with the help of new particles of different spin-statistics than the SM particles: supersymmetric particles. For this approach to work, these new particles should have masses below 1 TeV, otherwise fine tuning of parameters cannot be avoided. As none of the supersymmetric particles has been seen so far, the MSSM became a rather fine tuned scenario even if much less than the SM in the presence of the GUT and Planck scales. One of the important predictions of the simplest realization of this scenario, the MSSM with R-parity, is light Higgs with $m_H \leq 130$ GeV and one of its virtues is its perturbativity up to the GUT scales. An excellent introduction to the MSSM can be found in [145].

The unpleasant feature of the MSSM is a large number of parameters residing dominantly in the soft sector that has to be introduced in the process of supersymmetry breaking. Constrained versions of the MSSM can reduce the number of parameters significantly. The same is true in the case of the MSSM with MFV.

Concerning the FCNC processes let us recall that in addition to a light Higgs, squarks, sleptons, gluinos, charginos and neutralinos, also charged Higgs particles $H^{\pm}$ and additional neutral scalars are present in this framework. All these particles can contribute to FCNC transitions through box and penguin diagrams. New sources of flavour and CP violation come from the misalignment of quark and squark mass matrices and similar new flavour and CP-violating effects are present in the lepton sector. Some of these effects can be strongly enhanced at large $\tan\beta$ and the corresponding observables provide stringent constraints on the parameters of the MSMM. In particular $B_s \to \mu^+\mu^-$ can be enhanced up to its experimental upper bound, branching ratios for $K \to \pi\nu\bar{\nu}$ can be much larger than their SM values and the CP asymmetry $S_{\psi\phi}$ can also strongly deviate from the tiny SM value.

The Higgs sector of the MSSM is at the tree-level the same as of the 2HDM II: only one Higgs doublet couples to a fermion of a given charge and there are no FCNCs mediated by Higgs particles. However, at one-loop level this is no longer true and the Higgs-penguins, in analogy to Z-penguins are born. At large $\tan\beta$ they can be very important with their "smoking gun" being the order of magnitude enhancements of $B_{s,d} \to \mu^+\mu^-$. When non-MFV sources of flavour and CP violation in the squark sector are present also the asymmetry $S_{\psi\phi}$ can be strongly enhanced. However, there is a striking difference between $2\mathrm{HDM}_{\overline{\mathrm{MFV}}}$ and the MSSM with MFV which we already mentioned before. While in the $2\mathrm{HDM}_{\overline{\mathrm{MFV}}}$ large values of $S_{\psi\phi}$ are possible, this is not the case of the MSSM with MFV, even in the presence of FBPs: supersymmetric relations between parameters do not allow for such an enhancement [66, 67].

There is a very rich literature on flavour violation in supersymmetric theories. A rather complete collection of references can be found in a paper from my group [66], where the supersymmetric flavour (SF) models have been analysed in great detail. We will now confine our discussion to these models.

The general MSSM framework with very many new flavour parameters in the soft sector is not terribly predictive and is plagued by flavour and CP problems: FCNC processes and electric dipole moments are generically well above the experimental data and upper bounds, respectively. Moreover, the MSSM framework addressing primarily the gauge hierarchy problem and the quadratic divergences in the Higgs mass does not provide automatically the hierarchical pattern of quark and lepton masses and of FCNC and CP-violating interactions.

Much more interesting from this point of view are supersymmetric models with flavour symmetries that allow for a simultaneous understanding of the flavour structures in the Yukawa couplings and in SUSY soft-breaking terms, adequately suppressing FCNC and CP violating phenomena and solving SUSY flavour and CP problems.

The SF models can be divided into two broad classes depending on whether they are based on Abelian or non-Abelian flavour symmetries. Moreover, their phenomenological output crucially depends on whether the flavour and CP violations are governed by left-handed (LH) currents or there is an important new right-handed (RH) current component [66]. They can be considered as generalisations of the Froggatt–Nielsen mechanism for generating hierarchies in fermion masses and their interactions but are phenomenologically much more successful than the original Froggatt–Nielsen model [146]. There is a rich literature on SF models and I cannot refer to all of them here. Again a rather complete list of references can be found in [66] that I will briefly summarise now. See also [147].

In [66] we have performed an extensive study of processes governed by $b \to s$ transitions in the SF models and of their correlations with processes governed by $b \to d$ transitions, $s \to d$ transitions, D^0–$\bar{D}^0$ mixing, LFV decays, electric dipole moments and $(g-2)_\mu$. Both Abelian and non-Abelian flavour models have been considered as well as the flavour-blind MSSM (FBMSSM) and the MSSM with MFV. It has been shown how the characteristic patterns of correlations among the considered flavour observables allow to distinguish between these different SUSY scenarios and also to distinguish them from RSc and LHT scenarios of NP.

Of particular importance in our study were the correlations between the CP asymmetry $S_{\psi\phi}$ and $B_s \to \mu^+\mu^-$, between the observed anomalies in $S_{\phi K_{\rm S}}$ and $S_{\psi\phi}$, between $S_{\phi K_{\rm S}}$ and d_e, between $S_{\psi\phi}$ and $(g-2)_\mu$ and also those involving LFV decays.

In the context of our study of the SF models we have analysed the following representative scenarios:

1. Dominance of RH currents (Abelian model by Agashe and Carone [148]).

2. Comparable LH and RH currents with CKM-like mixing angles represented by the special version (RVV2) of the non-Abelian SU(3) model by Ross, Velasco and Vives [149] as discussed in [150].

3. In the second non-Abelian SU(3) model by Antusch, King and Malinsky (AKM) [151] analysed by us the RH contributions are CKM-like but new LH contributions in contrast to the RVV2 model can be

suppressed arbitrarily at the high scale. Still they can be generated by RG effects at lower scales. To first approximation the version of this model considered by us can be characterised by NP being dominated by CKM-like RH currents.

4. Dominance of CKM-like LH currents in non-Abelian models [152].

In the choice of these four classes of flavour models, we were guided by our model independent analysis in Section 2 of our paper, that I cannot present here because of the lack of space. Indeed, these three scenarios predicting quite different patterns of flavour violation should give a good representation of most SF models discussed in the literature. The distinct patterns of flavour violation found in each scenario have been illustrated with several plots that can be found in figures 11–14 of [66].

The main messages from our analysis of the models in question are as follows:

1. Supersymmetric models with RH currents (AC, RVV2, AKM) and those with exclusively LH currents can be globally distinguished by the values of the CP-asymmetries $S_{\psi\phi}$ and $S_{\phi K_S}$ with the following important result: none of the models considered by us can simultaneously explain the $S_{\psi\phi}$ and $S_{\phi K_S}$ anomalies observed in the data. In the models with RH currents, $S_{\psi\phi}$ can naturally be much larger than its SM value, while $S_{\phi K_S}$ remains either SM-like or its correlation with $S_{\psi\phi}$ is inconsistent with the data. On the contrary, in the models with LH currents only, $S_{\psi\phi}$ remains SM-like, while the $S_{\phi K_S}$ anomaly can be easily explained. Thus already future precise measurements of $S_{\psi\phi}$ and $S_{\phi K_S}$ will select one of these two classes of models, if any.

2. The desire to explain the $S_{\psi\phi}$ anomaly within the models with RH currents unambiguously implies, in the case of the AC and the AKM models, values of $\mathrm{Br}(B_s \to \mu^+\mu^-)$ as high as several 10^{-8}. In the RVV2 model such values are also possible but not necessarily implied by the large value of $S_{\psi\phi}$. Interestingly, in all these models large values of $S_{\psi\phi}$ can also provide the solution to the $(g-2)_\mu$ anomaly. Moreover, the ratio $\mathrm{Br}(B_d \to \mu^+\mu^-)/\mathrm{Br}(B_s \to \mu^+\mu^-)$ in the AC and RVV2 models is dominantly below its MFV prediction and can be much smaller than the latter. In the AKM model this ratio stays much closer to the MFV value of roughly 1/33 [49, 153] and can be smaller or larger than this value with equal probability. Still, values of $\mathrm{Br}(B_d \to \mu^+\mu^-)$ as high as 1×10^{-9} are possible in all these models.

3. In the RVV2 and the AKM models, a large value of $S_{\psi\phi}$ combined with the desire to explain the $(g-2)_\mu$ anomaly implies $\mathrm{Br}(\mu \to e\gamma)$ in the reach of the MEG experiment. In the case of the RVV2 model,

$d_e \geq 10^{-29}e$ cm. is predicted, while in the AKM model it is typically smaller. Moreover, in the case of the RVV2 model, $\mathrm{Br}(\tau \to \mu\gamma) \geq 10^{-9}$ is then in the reach of Super-B machines, while this is not the case in the AKM model.

4. Next, while the Abelian AC model resolves the present UT tensions [154–157] to be discussed in Section 6 through the modification of the ratio $\Delta M_d/\Delta M_s$, the non-Abelian flavour models RVV2 and AKM provide the solution through NP contributions to ϵ_K. Moreover, while the AC model predicts sizable NP contributions to D^0–$\bar{D}^0$ mixing, such contributions are tiny in the RVV2 and AKM models.

5. The hadronic EDMs represent very sensitive probes of SUSY flavour models with RH currents. In the AC model, large values for the neutron EDM might be easily generated by both the up- and strange-quark (C)EDM. In the former case, visible CPV effects in D^0–$\bar{D}^0$ mixing are also expected while in the latter case large CPV effects in the B_s system are unavoidable. The RVV2 and AKM models predict values for the down-quark (C)EDM and, hence for the neutron EDM, above the $\approx 10^{-28}e$ cm level when a large $S_{\psi\phi}$ is generated. All the above models predict a large strange-quark (C)EDM, hence, a reliable knowledge of its contribution to the hadronic EDMs, by means of lattice QCD techniques, would be of the utmost importance to probe or to falsify flavour models embedded in a SUSY framework.

6. In the supersymmetric models with exclusively LH currents, the desire to explain the $S_{\phi K_\mathrm{S}}$ anomaly implies also the solution to the $(g-2)_\mu$ anomaly and the direct CP asymmetry in $b \to s\gamma$ much larger than its SM value. This is in contrast to the models with RH currents where this asymmetry remains SM-like.

7. Interestingly, in the LH-current-models, the ratio $\mathrm{Br}(B_d \to \mu^+\mu^-)$ over $\mathrm{Br}(B_s \to \mu^+\mu^-)$ can not only deviate significantly from its MFV value of approximately 1/33, but in contrast to the models with RH currents considered by us can also be much larger than the latter value. Consequently, $\mathrm{Br}(B_d \to \mu^+\mu^-)$ as high as $(1-2)\times 10^{-9}$ is still possible while being consistent with the bounds on all other observables, in particular the one on $\mathrm{Br}(B_s \to \mu^+\mu^-)$. Also interesting correlations between $S_{\phi K_\mathrm{S}}$ and CP asymmetries in $B \to K^*\ell^+\ell^-$ are found.

8. The branching ratios for $K \to \pi\nu\bar{\nu}$ decays in the supersymmetric models considered by us remain SM-like and can be distinguished from RSc and LHT models where they can be significantly enhanced.

9. In [160] a closer look at CP violation in D^0–$\bar{D}^0$ mixing within the SUSY alignment models has been made (see also point 5 above). Such models naturally account for large, non-standard effects in D^0–$\bar{D}^0$ mixing and within such models detectable CP-violating effects in D^0–$\bar{D}^0$ mixing would unambiguously imply a lower bound for the electric dipole moment (EDM) of hadronic systems, like the neutron EDM and the mercury EDM, in the reach of future experimental sensitivities. This correlation distinguishes the alignment models from gauge-mediated SUSY breaking models, SUSY models with MFV and non-Abelian SUSY flavour models discussed above.

5.9. The flavour-blind MSSM

The flavour-blind MSSM (FBMSSM) scenario [61–65] having new FBPs in the soft sector belongs actually to the class of MFV models but as the functions F_i become complex quantities and it is a supersymmetric framework we mention this model here. In fact our analysis of this scenario in [62] preceded our detailed analysis of SF models that we just summarised.

The FBMSSM has fewer parameters than the general MSSM implying striking correlations between various observables that we list below. These correlations originate in the fact that the SUSY contributions to $S_{\phi K_S}$, $A_{\rm CP}(b \to s\gamma)$ and the EDMs are generated by the same CP-violating invariant $A_t\mu$. On the operator level the magnetic photon penguin operator in the case of $b \to s\gamma$ and magnetic gluon penguin in the case $S_{\phi K_S}$ play here the crucial role in the NP sector and as these are dipole operators also correlations with EDMs and under mild assumptions with $(g-2)_\mu$ exist. While this framework is of MFV or even better of $\overline{\rm MFV}$ type, it does not belong to the CMFV framework as scalar exchanges can enhance $B_{s,d} \to \mu^+\mu^-$ by an order of magnitude.

The main messages from this analysis are as follows:

1. We find that $S_{\phi K_S}$ and $S_{\eta' K_S}$ can both differ from $S_{\psi K_S}$ with the effect being typically by a factor of 1.5 larger in $S_{\phi K_S}$ in agreement with the pattern observed in the data. Most interestingly, we find that the desire of reproducing the observed low values of $S_{\phi K_S}$ and $S_{\eta' K_S}$ implies uniquely.

2. Lower bounds on the electron and neutron EDMs $d_{e,n} \gtrsim 10^{-28}\, e\,$c.m.

3. Positive and sizable (non-standard) $A_{\rm CP}(b \to s\gamma)$ asymmetry in the ballpark of 1%–5%, that is having an opposite sign to the SM one.

4. The NP effects in $S_{\psi K_S}$ and $\Delta M_d/\Delta M_s$ are very small so that these observables determine the coupling V_{td}, its phase $-\beta$ and its magnitude $|V_{td}|$, without significant NP pollution. In particular we find $\gamma = 63.5^\circ \pm 4.7^\circ$ and $|V_{ub}| = (3.5 \pm 0.2) \times 10^{-3}$. We remark that the latter value differs from the one coming from inclusive determinations and consequently from the one favoured by the RHMFV scenario discussed soon.

5. $|\epsilon_K|$ turns out to be uniquely enhanced over its SM value up to a level of 15% softening the ε_K anomaly that we will discuss in the next section.

6. Only small effects in $S_{\psi\phi}$ which could, however, be still visible through the semi-leptonic asymmetry $A^s_{\rm SL}$.

7. A natural explanation of the Δa_μ anomaly (under very mild assumptions).

The FBMSSM belongs to the more general models with MFV discussed in [56] and shares several properties with supersymmetric models of the δLL type. The major difference discriminating these two scenarios regards their predictions for the leptonic and hadronic EDMs. The lower bounds on these observables are significantly stronger within the FBMSSM if one wants to eliminate the $S_{\phi K_S}$ anomaly. Thus the SUSY non-MFV models with purely left-handed currents like δLL can easier survive the future data. Yet both models will have problems if the UT-tensions and the $S_{\psi\phi}$ anomaly will be confirmed by more accurate data.

5.10. The minimal effective model with right-handed currents: RHMFV

One of the main properties of the Standard Model (SM) regarding flavour violating processes is the LH structure of the charged currents that is in accordance with the maximal violation of parity observed in low-energy processes. LH charged currents encode at the level of the Lagrangian the full information about flavour mixing and CP violation represented compactly by the CKM matrix. Due to the GIM mechanism this structure has automatically profound implications for the pattern of FCNC processes that seems to be remarkably in accordance with the present data within theoretical and experimental uncertainties, bearing in mind certain anomalies which will be discussed below and in the next section.

Yet, the SM is expected to be only the low-energy limit of a more fundamental theory in which in principle parity could be a good symmetry implying the existence of RH charged currents. Prominent examples of such fundamental theories are left–right symmetric models on which a rich literature exists. We have seen that some SF models discussed above contained RH currents as well.

Left–right symmetric models were born 35 years ago [161–166] and extensive analyses of many observables can be found in the literature (see *e.g.* [167–169] and references therein). Renewed theoretical interest in models with an underlying $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$ global symmetry has also been motivated by Higgsless models [170–173]. However, the recent phenomenological interest in making another look at the right-handed currents in general, and not necessarily in the context of a given left–right symmetric model, originated in tensions between inclusive and exclusive determinations of the elements of the CKM matrix $|V_{ub}|$ and $|V_{cb}|$. It could be that these tensions are due to the underestimate of theoretical and/or experimental uncertainties. Yet, it is a fact, as pointed out and analysed recently in particular in [174, 175], that the presence of right-handed currents could either remove or significantly weaken some of these tensions, especially in the case of $|V_{ub}|$.

Assuming that RH currents provide the solution to the problem at hand, there is an important question whether the strength of RH currents required for this purpose is consistent with other observables and whether it implies new effects somewhere else that could be used to test this idea more globally.

This question has been addressed in [43]. The starting point of our analysis is the assumption that the SM is the low-energy limit of a more fundamental theory. We do not know the exact structure of this theory, but we assume that in the high-energy limit it is left–right symmetric. The difference of LH and RH sectors observed in the SM is only a low-energy property, due to appropriate symmetry-breaking terms.

In particular, we assume that the theory has a $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$ global symmetry, explicitly broken only in the Yukawa sector and by the $\mathrm{U}(1)_Y$ gauge coupling. Under this symmetry the SM quark fields can be grouped into three sets of LH or RH doublets with $B-L$ charge $1/3$

$$Q_L^i = \begin{pmatrix} u_L^i \\ d_L^i \end{pmatrix}, \qquad Q_R^i = \begin{pmatrix} u_R^i \\ d_R^i \end{pmatrix}, \qquad i = 1\ldots 3\,. \tag{54}$$

With this assignment the SM hypercharge is given by $Y = T_{3R} + (B–L)/2$. In order to recover the SM electroweak gauge group, we assume that only the $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_Y$ subgroups of $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$ are effectively gauged below the TeV scale. In close analogy we can introduce three sets of LH and RH leptons, L_L^i and L_R^i (including three RH neutrinos), with $B - L = -1$.

In our effective theory approach for the study of RH currents [43] the central role is played by a left–right symmetric flavour group $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$, commuting with an underlying $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$ global symmetry and broken only by two Yukawa couplings. The model contains a new unitary matrix $\widetilde{V}$ controlling flavour-mixing in the RH sector and can be considered as the minimally flavour violating generalization to the RH sector. Thus bearing in mind that this model contains non-MFV interactions from the point of view of the standard MFV hypothesis that includes only LH charged currents, we decided to call this model RHMFV.

The new mixing matrix $\widetilde{V}$ can be parametrized in terms of 3 real mixing angles and 6 complex phases. Adopting the standard CKM phase convention, where the 5 relative phases of the quark fields are adjusted to remove 5 complex phases from the CKM matrix, we have no more freedom to remove the 6 complex phases from $\widetilde{V}$. In passing let us remark that the same comments apply to the V_{Hd} and V_{Hl} matrices in the mirror fermion sector of the LHT model.

In the standard CKM basis $\widetilde{V}$ can be parametrized as follows

$$\widetilde{V} = D_U \tilde{V}_0 D_D^\dagger \,, \tag{55}$$

where $\widetilde{V}_0$ is a "CKM-like" mixing matrix, containing only one non-trivial phase and $D_{U,D}$ are diagonal matrices containing the remaining CP-violating phases. It turned out to be useful to choose the following parametrization of $\widetilde{V}_0$ attributing the non-trivial phase of this matrix to the 2–3 mixing, such that

$$\widetilde{V}_0 = \begin{pmatrix} \tilde{c}_{12}\tilde{c}_{13} & \tilde{s}_{12}\tilde{c}_{13} & \tilde{s}_{13} \\ -\tilde{s}_{12}\tilde{c}_{23} - \tilde{c}_{12}\tilde{s}_{23}\tilde{s}_{13}e^{-i\phi} & \tilde{c}_{12}\tilde{c}_{23} - \tilde{s}_{12}\tilde{s}_{23}\tilde{s}_{13}e^{-i\phi} & \tilde{s}_{23}\tilde{c}_{13}e^{-i\phi} \\ -\tilde{c}_{12}\tilde{c}_{23}\tilde{s}_{13} + \tilde{s}_{12}\tilde{s}_{23}e^{i\phi} & -\tilde{s}_{12}\tilde{c}_{23}\tilde{s}_{13} - \tilde{s}_{23}\tilde{c}_{12}e^{i\phi} & \tilde{c}_{23}\tilde{c}_{13} \end{pmatrix} , \tag{56}$$

and

$$D_U = \mathrm{diag}\left(1, e^{i\phi_2^u}, e^{i\phi_3^u}\right) , \qquad D_D = \mathrm{diag}\left(e^{i\phi_1^d}, e^{i\phi_2^d}, e^{i\phi_3^d}\right) . \tag{57}$$

Having this set-up at hand we have performed a detailed phenomenology of RH currents, taking all tree-level constraints into account and solving the V_{ub} problem in this manner. It should be emphasised that this solution chooses the *inclusive* value of $|V_{ub}|$ as the true value of this CKM element and our best value turned out to be in the ballpark of $(4.1 \pm 0.2) \times 10^{-3}$ implying in turn

$$\sin 2\beta = 0.77 \pm 0.05 \,, \tag{58}$$

a value much larger than the measured value of $S_{\psi K_S} = 0.672 \pm 0.023$. This has profound implications as we will see below.

Returning to the RH mixing matrix we find that a good description of the tree-level data is provided by the matrix

$$\widetilde{V}_0 = \begin{pmatrix} \pm\tilde{c}_{12}\frac{\sqrt{2}}{2} & \pm\tilde{s}_{12}\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ -\tilde{s}_{12} & \tilde{c}_{12} & 0 \\ \tilde{c}_{12}\frac{\sqrt{2}}{2} & \tilde{s}_{12}\frac{\sqrt{2}}{2} & \pm\frac{\sqrt{2}}{2} \end{pmatrix}, \tag{59}$$

where $\tilde{s}_{12}$ is free, provided that $\mathrm{sgn}(\tilde{c}_{13}\tilde{s}_{12}) = -1$. Other still possible but less interesting cases are considered in [43].

The novel feature of our analysis as compared with [174] is the determination of the full right-handed matrix, and not only its selected elements, making use of its unitarity. We also find that while RH currents are very welcome to solve the "$|V_{ub}|$ problem" they do not have a significant impact on the determination of $|V_{cb}|$ (as also pointed out in [176]).

The matrix $\widetilde{V}_0$ in (59) has then been used in our analysis of FCNCs. In fact the particular structure of this matrix is a key ingredient to generate a sizable non-standard contribution to $S_{\psi\phi}$. As a result, after we require a large $S_{\psi\phi}$, most of our conclusions listed below do not depend on the choice of this ansatz. It should also be stressed that, contrary to the CKM case, having a zero in $\widetilde{V}_0$ does not prevent non-vanishing CP-violating effects thanks to the extra phases in (57).

The mixing structures relevant to the three down-type $\Delta F = 2$ and FCNC amplitudes in the SM (LH sector) and in the RH sector are shown in Table I. We observe that the $\tilde{c}_{12}$ and $\tilde{s}_{12}$ dependencies in the three systems considered are non-universal with the observables in the K mixing, B_d mixing and B_s mixing dominated by $\tilde{c}_{12}\tilde{s}_{12}$, $\tilde{c}_{12}$ and $\tilde{s}_{12}$, respectively. Since both $\Delta S = 2$ and B_d mixing are strongly constrained, and the data from CDF and D0 give some hints for sizable NP contributions in the B_s mixing, it is natural to assume that $\tilde{c}_{12} \ll 1$. The phenomenological analysis is then rather constrained.

TABLE I

Mixing structures relevant to the three down-type $\Delta F = 2$ and FCNC amplitudes in the SM (LH sector) and in the RH sector. In the SM case approximate expressions of the CKM factors expanded in powers of $\lambda = |V_{us}|$ are also shown. In the RH case the parametrization for the RH matrix is the one in (59).

Mixing term	$s \to d$	$b \to d$	$b \to s$
$V_{ti}^* V_{tj}$	$V_{ts}^* V_{td} \approx -\lambda^5 e^{-i\beta}$	$V_{tb}^* V_{td} \approx \lambda^3 e^{-i\beta}$	$V_{tb}^* V_{ts} \approx -\lambda^2 e^{-i\beta_s}$
$\widetilde{V}_{ti}^* \widetilde{V}_{tj}$	$\frac{1}{2}\tilde{c}_{12}\tilde{s}_{12} e^{i\left(\phi_2^d - \phi_1^d\right)}$	$\pm\frac{1}{2}\tilde{c}_{12} e^{i\left(\phi_3^d - \phi_1^d\right)}$	$\pm\frac{1}{2}\tilde{s}_{12} e^{i\left(\phi_3^d - \phi_2^d\right)}$

Before presenting the main results of our analysis let us emphasise that the non-standard contributions to $\Delta S = 2$ amplitudes are exceedingly large compared to the SM term (and compared with data) unless the Wilson coefficients of the relevant operators

$$Q_1^{\rm VRR} = (\bar{s}^\alpha \gamma_\mu P_R d^\alpha)\left(\bar{s}^\beta \gamma^\mu P_R d^\beta\right), \quad Q_1^{\rm LR} = (\bar{s}^\alpha \gamma_\mu P_L d^\alpha)\left(\bar{s}^\beta \gamma^\mu P_R d^\beta\right), \tag{60}$$

are very small. This can be for instance achieved if one of the two mixing terms $\tilde{c}_{12}$ or $\tilde{s}_{12}$ is very small. The most problematic is the second operator for which the P_i parameter in (25) takes the value $P_1^{\rm LR}(K) \approx -52$ to be compared with $P_1^{\rm VRR} \approx 0.5$. Due to the $(V-A) \times (V+A)$ structure of $Q_1^{\rm LR}$, its contributions are known to be strongly enhanced at low energies through renormalization group effects and in the case of ε_K and ΔM_K through its chirally enhanced hadronic matrix elements. Consequently these observables put severe constraints on the model parameters as known from various studies in explicit left–right symmetric models [167] and also RS models.

Having determined the size and the flavour structure of RH currents that is consistent with the present data on tree-level processes and which removes the "$|V_{ub}|$-problem", we have investigated how this NP would manifest itself in neutral current processes, including particle–antiparticle mixing, $Z \to b\bar{b}$, $B_{s,d} \to \mu^+\mu^-$, $B \to \{X_s, K, K^*\}\nu\bar{\nu}$ and $K \to \pi\nu\bar{\nu}$ decays. Most importantly, we have also addressed the possibility to explain a non-standard CP-violating phase in B_s mixing in this context and the issue of the anomalies in the UT-triangle.

The main messages from our analysis of these processes are as follows:

1. The desire to generate large CP-violating effects in B_s mixing, hinted for by the enhanced value of $S_{\psi\phi}$ observed by the CDF and D0 collaborations, in conjunction with the ε_K-constraint, implies additional constraints on the shape of $\tilde{V}$. In particular $\tilde{c}_{12} \ll 1$ and consequently $\tilde{s}_{12} \approx 1$. The pattern of deviations from the SM in this model is then as follows.

2. The $S_{\psi\phi}$ and ε_K anomalies can be understood.

3. As a consequence of the large value of $\tilde{s}_{12}$, it should be possible to resolve the presence of RH currents also in $s \to u$ charged-current transitions. Here RH currents imply a $\mathcal{O}(10^{-3})$ deviation in the determination of $|V_{us}|$ from $K \to \pi\ell\nu$ and $K \to \ell\nu$ decays.

4. The "true value" of $\sin 2\beta$ determined in our framework, namely the determination of the CKM phase β on the basis of the tree-level processes only, and in particular of $|V_{ub}|$, is $\sin 2\beta = 0.77 \pm 0.05$. This result is

roughly 2σ larger than the measured value $S_{\psi K_S} = 0.672 \pm 0.023$. This is a property of any explanation of the "$|V_{ub}|$-problem" by means of RH currents, unless the value of $|V_{ub}|$ from inclusive decays will turn out to be much lower than determined presently.

5. In general, such a discrepancy could be solved by a negative new CP-violating phase φ_{B_d} in $B_d^0 - \bar{B}_d^0$ mixing. However, we have demonstrated that this is not possible in the present framework once the ε_K constraint is imposed and large $S_{\psi\phi}$ is required. Indeed as seen in Table I for $\tilde{c}_{12} \ll 1$ the NP effects in $S_{\psi K_S}$ are tiny. Thus we pointed out that simultaneous explanation of the "$|V_{ub}|$-problem", of $S_{\psi K_S} = 0.672 \pm 0.023$ and large $S_{\psi\phi}$ is problematic through RH currents alone.
6. The present constraints from $B_{s,d} \to \mu^+\mu^-$ eliminate the possibility of removing the known anomaly in the $Z \to b\bar{b}$ decay with the help of RH currents. On top of it, the constraint from $B \to X_s l^+ l^-$ precludes $B_s \to \mu^+\mu^-$ to be close to its present experimental bound. However, still values as high as 1×10^{-8} are possible. Moreover, NP effects in $B_d \to \ell^+\ell^-$ are found generally smaller than in $B_s \to \ell^+\ell^-$.
7. Contributions from RH currents to $B \to \{X_s, K, K^*\}\nu\bar{\nu}$ and $K \to \pi\nu\bar{\nu}$ decays can still be significant. Most important, the deviations from the SM in these decays would exhibit a well-defined pattern of correlations.

Thus our analysis casts a shadow on the explanation of the "$|V_{ub}|$-problem" with the help of RH currents alone unless the $S_{\psi\phi}$ anomaly goes away and $\tilde{c}_{12}$ can be large solving the problem with $S_{\psi K_S}$ naturally.

Particularly interesting is the comparison with the $2\mathrm{HDM}_{\overline{\mathrm{MFV}}}$ model, where the $S_{\psi\phi}$ and ε_K anomalies can also be accommodated [41] as seen in our presentation of this model. What clearly distinguishes these two models at low-energies is how they face the "$|V_{ub}|$-problem" (which can be solved only in the RHMFV case) and the "$\sin 2\beta$–$S_{\psi K}$ tension" (which can be softened only in the 2HDM case). But also the future results on rare B and K decays listed above could in principle help to distinguish these two general NP frameworks.

Restricting the discussion to these two NP frameworks, it appears that a model with an extended scalar sector and RH currents could provide solutions to all the existing tensions in flavour physics simultaneously. This possibility can certainly be realized in explicit left–right symmetric models, where an extended Higgs sector is also required to break the extended gauge symmetry. However, these extensions contain many free parameters and clear cut conclusions on the pattern of flavour violation cannot be as easily reached as it was possible in the simple frameworks like RHMFV [43] and $2\mathrm{HDM}_{\overline{\mathrm{MFV}}}$ [41].

5.11. A Randall–Sundrum Model with custodial protection

When the number of space-time dimensions is increased, new solutions to the hierarchy problems are possible. Most ambitious proposals are models with a warped extra dimension first proposed by Randall and Sandrum (RS) [177] which provide a geometrical explanation of the hierarchy between the Planck scale and the EW scale. Moreover, when the SM fields, except for the Higgs field, are allowed to propagate in the bulk [178–180], these models naturally generate the hierarchies in the fermion masses and mixing angles [179, 180] through different localisations of the fermions in the bulk. Yet, this way of explaining the hierarchies in masses and mixings necessarily implies FCNC transitions at the tree-level [181–184]. Most problematic is the parameter ε_K which receives tree-level KK-gluon contributions and some fine tuning of parameters in the flavour sector is necessary in order to achieve consistency with the data for KK scales in the reach of the LHC [183, 186].

Once this fine tuning is made, the RS-GIM mechanism [181, 182], combined with an additional custodial protection of flavour violating Z couplings [185–187], allows yet to achieve the agreement with existing data for other observables without an additional fine tuning of parameters[1]. New theoretical ideas addressing the issue of large FCNC transitions in the RS framework and proposing new protection mechanisms occasionally leading to MFV can be found in [188–193].

Entering some details it should be emphasised that to avoid problems with electroweak precision tests (EWPT) and FCNC processes, the gauge group is generally larger than the SM gauge group [173, 194, 195]

$$G_{\rm RSc} = {\rm SU}(3)_c \times {\rm SU}(2)_L \times {\rm SU}(2)_R \times {\rm U}(1)_X\,, \tag{61}$$

and similarly to the LHT model new heavy gauge bosons are present. The increased symmetry provides a custodial protection. We will denote such framework by RSc.

The lightest new gauge bosons are the KK-gluons, the KK-photon and the electroweak KK gauge bosons $W_H^\pm$, $W'^\pm$, Z_H and Z', all with masses $M_{\rm KK}$ around 2–3 TeV as required by the consistency with the EWPT [173, 194, 195]. The fermion sector is enriched through heavy KK-fermions (some of them with exotic electric charges) that could in principle be discovered at the LHC. The fermion content of this model is explicitly given in [196], where also a complete set of Feynman rules has been worked out. Detailed analyses of electroweak precision tests and of the parameter ε_K in a RS model without and with custodial protection can also be found in [197, 198]. These authors analysed also rare and non-leptonic decays in [199]. Possible flavour protections in warped Higgsless models have been presented in [192].

[1] See, however, comments at the end of this subsection.

Here we summarise the main results obtained in Munich:

1. The CP asymmetry $S_{\psi\phi}$ can reach values as high as 0.8 to be compared with its SM value 0.04.

2. The branching ratios for $K^+ \to \pi^+\nu\bar{\nu}$, $K_{\rm L} \to \pi^0\nu\bar{\nu}$, $K_{\rm L} \to \pi^0 l^+ l^-$ can be enhanced relative to the SM expectations up to factors of 1.6, 2.5 and 1.4, respectively, when only moderate fine tuning in ε_K is required. Otherwise the enhancements can be larger. $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$ and $\mathrm{Br}(K_{\rm L} \to \pi^0\nu\bar{\nu})$ can be simultaneously enhanced but this is not necessary as the correlation between these two branching ratios is not evident in this model. On the other hand, $\mathrm{Br}(K_{\rm L} \to \pi^0\nu\bar{\nu})$ and $\mathrm{Br}(K_{\rm L} \to \pi^0 l^+ l^-)$ $(l = e, \mu)$ are strongly correlated and the enhancement of one of these three branching ratios implies the enhancement of the remaining two.

3. A large enhancement of the short distance part of $\mathrm{Br}(K_{\rm L} \to \mu^+\mu^-)$ is possible, up to a factor of 2–3, but not simultaneously with $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$.

4. More importantly simultaneous large NP effects in $S_{\psi\phi}$ and $K \to \pi\nu\bar{\nu}$ channels are very unlikely and this feature is even more pronounced than in the LHT model.

5. The branching ratios for $B_{s,d} \to \mu^+\mu^-$ and $B \to X_{s,d}\nu\bar{\nu}$ remain SM-like: the maximal enhancements of these branching ratios amount to 15%.

6. The relations [14, 200] between various observables present in models with CMFV can be strongly violated.

In particular this pattern of flavour violation implies that in the case of the confirmation of large values of $S_{\psi\phi}$ by future experiments significant deviations of $\mathrm{Br}(K_{\rm L} \to \pi^0\nu\bar{\nu})$ and $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$ from their SM values in this framework similarly to the LHT model are very unlikely. On the other hand, SM-like value of $S_{\psi\phi}$ will open the road for large enhancements of these branching ratios that could be tested by KOTO at J-Parc and NA62 at CERN, respectively.

Next, let me just mention that large NP contributions in the RS framework that require some tunings of parameters in order to be in agreement with the experimental data have been found in $\mathrm{Br}(B \to X_s\gamma)$ [201], $\mathrm{Br}(\mu \to e\gamma)$ [202–204] and EDM's [182,205], that are all dominated by dipole operators. Also the new contributions to ε'/ε can be large [206]. Moreover, it appears that the fine tunings in this ratio are not consistent necessarily with the ones required in the case of ε_K.

Finally a personal comment. My excursion into the fifth dimension was very interesting and I learned a lot about the structure of weak interactions in this NP scenario. Yet after this study I am sceptical that the nature is so violent to provide this physics already at the LHC scales. Therefore, I am delighted to be back in $D = 4$.

5.12. "DNA's" of flavour models

The "DNA's" of flavour physics effects for the most interesting observables constructed in [66] and extended by the recent results obtained in the $2\mathrm{HDM}_{\overline{\mathrm{MFV}}}$, SM4 and RHMFV are presented in Tables II and III. These tables only indicate whether large, moderate or small NP effects in a given observable are still allowed in a given model but do not exhibit correlations between various observables characteristic for a given model. Still they could turn out to be useful in eliminating models in which large NP effects for certain observables, hopefully seen soon in the data, are not possible.

TABLE II

"DNA" of flavour physics effects for the most interesting observables in a selection of SUSY models. ★★★ signals large NP effects, ★★ moderate to small NP effects and ★ implies that the given model does not predict visible NP effects in that observable. From [66].

	AC	RVV2	AKM	δLL	FBMSSM
D^0–$\bar{D}^0$	★★★	★	★	★	★
ϵ_K	★	★★★	★★★	★	★
$S_{\psi\phi}$	★★★	★★★	★★★	★	★
$S_{\phi K_S}$	★★★	★★	★	★★★	★★★
$A_{\mathrm{CP}}(B \to X_s\gamma)$	★	★	★	★★★	★★★
$A_{7,8}(K^*\mu^+\mu^-)$	★	★	★	★★★	★★★
$B_s \to \mu^+\mu^-$	★★★	★★★	★★★	★★★	★★★
$K^+ \to \pi^+\nu\bar{\nu}$	★	★	★	★	★
$K_{\mathrm{L}} \to \pi^0\nu\bar{\nu}$	★	★	★	★	★
$\mu \to e\gamma$	★★★	★★★	★★★	★★★	★★★
$\tau \to \mu\gamma$	★★★	★★★	★	★★★	★★★
$\mu + N \to e + N$	★★★	★★★	★★★	★★★	★★★
d_n	★★★	★★★	★★★	★★	★★★
d_e	★★★	★★★	★★	★	★★★
$(g-2)_\mu$	★★★	★★★	★★	★★★	★★★

Concerning correlations we have discussed them above and they will also enter the discussion below. In Table IV the references to papers from my group that analysed various correlations in all models discussed above have been collected for convenience.

TABLE III

"DNA" of flavour physics effects for the most interesting observables in a selection of non-SUSY models. ★★★ signals large NP effects, ★★ moderate to small NP effects and ★ implies that the given model does not predict visible NP effects in that observable. Empty spaces reflect my present ignorance about the given entry.

	LHT	RSc	4G	2HDM	RHMFV
D^0–$\bar{D}^0$ (CPV)	★★★	★★★	★★	★★	
ϵ_K	★★	★★★	★★	★★	★★
$S_{\psi\phi}$	★★★	★★★	★★★	★★★	★★★
$S_{\phi K_S}$	★	★	★★		
$A_{CP}(B \to X_s\gamma)$	★		★		
$A_{7,8}(K^*\mu^+\mu^-)$	★★	★	★★		
$B_s \to \mu^+\mu^-$	★	★	★★★	★★★	★★
$K^+ \to \pi^+\nu\bar{\nu}$	★★★	★★★	★★★		★★
$K_L \to \pi^0\nu\bar{\nu}$	★★★	★★★	★★★		★★
$\mu \to e\gamma$	★★★	★★★	★★★		
$\tau \to \mu\gamma$	★★★	★★★	★★★		
$\mu + N \to e + N$	★★★	★★★	★★★		
d_n	★	★★★	★	★★★	
d_e	★	★★★	★	★★★	
$(g-2)_\mu$	★	★★	★		

TABLE IV

References to correlations in various models.

Model	Reference
CMFV	[13–15, 49]
$2HDM_{\overline{MFV}}$	[41, 42]
ACD Model	[207, 208]
LH	[87, 88]
LHT	[95, 107]
SM4	[137–139]
AC, RVV2, AMK, δLL	[66, 160]
FBMSSM	[62]
RHMFV	[43]
RSc	[185, 186]

6. Facing anomalies and distinguishing between various BSM models through correlations

6.1. Preliminaries

Armed with the information on patterns of flavour violation in a large number of concrete models and NP scenarios, we will now look from a different angle at various anomalies observed in the data. The readers, who followed the previous sections will notice certain repetitions of statements made before but now these statements are in a different context and could be useful anyway. On the other hand, the readers who skipped all previous sections should be able, because of these repetitions, to follow this section, only occasionally looking up the summaries of specific models presented before.

6.2. The ε_K–$S_{\psi K_S}$ anomaly

It has been pointed out in [155, 156] that the SM prediction for ε_K implied by the measured value of $S_{\psi K_S} = \sin 2\beta$, the ratio $\Delta M_d/\Delta M_s$ and the value of $|V_{cb}|$ turns out to be too small to agree well with experiment. This tension between ε_K and $S_{\psi K_S}$ has been pointed out from a different perspective in [154, 157, 158]. These findings have been confirmed by a UTfitters analysis [209]. The CKMfitters having a different treatment of uncertainties find less significant effects [210].

The main reasons for this tension are on the one hand a decreased value of the relevant non-perturbative parameter $\hat{B}_K = 0.724 \pm 0.008 \pm 0.028$ [44] resulting from unquenched lattice calculations and on the other hand, the decreased value of ε_K in the SM arising from a multiplicative factor, estimated first to be $\kappa_\varepsilon = 0.92 \pm 0.02$ [155]. This factor took into account the departure of ϕ_ε from $\pi/4$ and the long distance (LD) effects in $\mathrm{Im}\Gamma_{12}$ in the K^0–$\bar{K}^0$ mixing. The recent inclusion of LD effects in $\mathrm{Im}M_{12}$ modified this estimate to $\kappa_\varepsilon = 0.94 \pm 0.02$ [211]. Very recently also NNLO–QCD corrections to the QCD factor η_{ct} in ε_K [34] have been calculated enhancing the value of ε_K by 3%. Thus while in [155] the value $|\varepsilon_K|_{\mathrm{SM}} = (1.78 \pm 0.25) \times 10^{-3}$ has been quoted and with the new estimate of LD effects and new input one finds $|\varepsilon_K|_{\mathrm{SM}} = (1.85 \pm 0.22) \times 10^{-3}$, including NNLO corrections gives the new value [34]

$$|\varepsilon_K|_{\mathrm{SM}} = (1.90 \pm 0.26) \times 10^{-3}\,, \tag{62}$$

significantly closer to the experimental value $|\varepsilon_K|_{\mathrm{exp}} = (2.23 \pm 0.01) \times 10^{-3}$.

Consequently, the ε_K-anomaly softened considerably but it is still alive. Indeed, the $\sin 2\beta = 0.74 \pm 0.02$ from UT fits is visibly larger than the experimental value $S_{\psi K_S} = 0.672 \pm 0.023$. The difference is even larger if one wants to fit ε_K exactly: $\sin 2\beta \approx 0.85$ [154–159].

One should also recall the tension between inclusive and exclusive determinations of $|V_{ub}|$ with the exclusive ones in the ballpark of 3.5×10^{-3} and the inclusive ones typically above 4.0×10^{-3}. As discussed in detail in the previous section, an interesting solution to this problem is the presence of RH charged currents, which selects the inclusive value as the true value, implying again $\sin 2\beta \approx 0.80$ [43].

As discussed in [154,155] and subsequent papers of these authors a small negative NP phase φ_{B_d} in $B_d^0 - \bar{B}_d^0$ mixing would solve both problems, provided such a phase is allowed by other constraints. Indeed we have then

$$S_{\psi K_S}(B_d) = \sin(2\beta + 2\varphi_{B_d}) \,, \qquad S_{\psi\phi}(B_s) = \sin(2|\beta_s| - 2\varphi_{B_s}) \,, \quad (63)$$

where the corresponding formula for $S_{\psi\phi}$ in the presence of a NP phase φ_{B_s} in B_s^0–$\bar{B}_s^0$ mixing has also been given. With a negative φ_{B_d} the true $\sin 2\beta$ is larger than $S_{\psi K_S}$, implying a higher value on $|\varepsilon_K|$, in reasonable agreement with data and a better UT-fit. This solution would favour the inclusive value of $|V_{ub}|$ as chosen *e.g.* by RH currents but as pointed out in [43] this particular solution of the "$|V_{ub}|$-problem" does not allow for a good fit to $S_{\psi K_S}$ if large $S_{\psi\phi}$ is required.

Now making a universality hypothesis of $\varphi_{B_s} = \varphi_{B_d}$ [155,212], a negative φ_{B_d} would automatically imply an enhanced value of $S_{\psi\phi}$ which in view of $|\beta_s| \approx 1^\circ$ amounts to roughly 0.04 in the SM. However, in order to be in agreement with the experimental value of $S_{\psi K_S}$ this type of NP would imply $S_{\psi\phi} \leq 0.25$. All this shows that correlations between various observables are very important in this game.

The universality hypothesis of $\varphi_{B_s} = \varphi_{B_d}$ in [155, 212] is clearly *ad hoc*. Recently, in view of the enhanced value of $S_{\psi\phi}$ at CDF and D0 a more dynamical origin of this relation has been discussed by other authors and different relations between these two phases corresponding still to a different dynamics have been discussed in the literature. Let us elaborate on this topic in more detail.

6.3. Facing an enhanced CPV in the B_s mixing

Possibly the most important highlight in flavour physics in 2008, 2009 [213] and even more in 2010 was the enhanced value of $S_{\psi\phi}$ measured by the CDF and D0 Collaborations, seen either directly or indirectly through the correlations with various semi-leptonic asymmetries. While in 2009 and in the spring of 2010 [214], the messages from Fermilab indicated good prospects for $S_{\psi\phi}$ above 0.5, the recent messages from ICHEP 2010 in Paris, softened such hopes significantly [215]. Both CDF and D0 find the enhancement by only one σ. Yet, this does not yet preclude $S_{\psi\phi}$ above 0.5, which would really be a fantastic signal of NP. But $S_{\psi\phi}$ below 0.5 appears more likely at present. Still even a value of 0.2 would be exciting. Let us hope that the future data from Tevatron and in particular from the LHCb, will measure this asymmetry with sufficient precision so that we will know to which extent NP is at work here. One should also hope that the large CPV in dimuon CP asymmetry from D0, that triggered new activities, will be better understood. I have nothing to add here at present and can only refer to numerous papers [67, 68, 210, 216, 217].

Leaving the possibility of $S_{\psi\phi} \geq 0.5$ still open but keeping in mind that also $S_{\psi\phi} \leq 0.25$ could turn out to be the final value, let us investigate how different models described in Section 5 would face these two different results and what kind of dynamics would be behind these two scenarios.

6.3.1. $S_{\psi\phi} \geq 0.5$

Such large values can be obtained in the RSc model due to KK-gluon exchanges and also heavy neutral KK electroweak gauge boson exchanges. In the supersymmetric flavour model with the dominance of RH currents like the AC model, double Higgs penguins constitute the dominant NP contributions responsible for $S_{\psi\phi} \geq 0.5$, while in the RVV2 model where NP LH current contributions are equally important, also gluino boxes are relevant. On the operator level, it is $Q_2^{\rm LR}$ operator in (18) with properly changed quark flavours, which is primarily responsible for this enhancement.

Interestingly the SM4 having only $(V-A)\times(V-A)$ operator $Q_1^{\rm VLL}$ is also capable in obtaining high values of $S_{\psi\phi}$ [130, 133, 137] but not as easily as the RSc, AC and RVV2 models. The lower scales of NP in the SM4 relative to the latter models and the non-decoupling effects of t' compensate to some extent the absence of LR scalar operators. In the LHT model where only $(V-A)\times(V-A)$ operators are present and the NP enters at higher scales than in the SM4, $S_{\psi\phi}$ above 0.5 is out of reach [95]. Similar comment applies to the AKM model.

All these models contain new sources of flavour and CP violation and it is not surprising that in view of many parameters involved large values of $S_{\psi\phi}$ can be obtained. The question then arises whether strongly enhanced values of this asymmetry would uniquely imply new sources of flavour violation beyond the MFV hypothesis. The answer to this question is as follows:

- In models with MFV and FBPs set to zero, $S_{\psi\phi}$ remains indeed SM-like.
- In supersymmetric models with MFV even in the presence of non-vanishing FBPs, at both small and large $\tan\beta$, the supersymmetry constraints do not allow values of $S_{\psi\phi}$ visibly different from the SM value [62, 66, 67].
- In the $2\mathrm{HDM}_{\overline{\mathrm{MFV}}}$ in which at one-loop both Higgs doublets couple to up- and down-quarks, the interplay of FBPs with the CKM matrix allows to obtain $S_{\psi\phi} \geq 0.5$ while satisfying all existing constraints [41].

In the presence of a large $S_{\psi\phi}$ the latter model allows also for a simple and unique softening of the ε_K-anomaly and of the tensions in the UT analysis if the FBPs in the Yukawa interactions are the dominant source of new CPV.

In this case the NP phases φ_{B_s} and φ_{B_d} are related through

$$\varphi_{B_d} \approx \frac{m_d}{m_s}\varphi_{B_s} \approx \tfrac{1}{17}\varphi_{B_s}\,, \tag{64}$$

in visible contrast to the hypothesis $\varphi_{B_s} = \varphi_{B_d}$ of [155, 212]. Thus in this scenario large φ_{B_s} required to obtain values of $S_{\psi\phi}$ above 0.5 imply a unique small shift in $S_{\psi K_S}$ that allows to lower $S_{\psi K_S}$ from 0.74 down to 0.70, that is closer to the experimental value 0.672 ± 0.023. This in turn implies that it is $\sin 2\beta = 0.74$ and not $S_{\psi K_S} = 0.67$ that should be used in calculating ε_K resulting in a value of $\varepsilon_K \approx 2.0 \times 10^{-3}$ within one σ from the experimental value. The direct Higgs contribution to ε_K is negligible because of small masses $m_{d,s}$. We should emphasize that once φ_{B_s} is determined from the data on $S_{\psi\phi}$ by means of (63), the implications for ε_K and $S_{\psi K_S}$ are unique. All these correlations are explicitly seen in (44) and (45).

It is remarkable that such a simple set-up allows basically to solve all these tensions provided $S_{\psi\phi}$ is sufficiently above 0.5. The plots of ε_K and $S_{\psi K_S}$ *versus* $S_{\psi\phi}$ in [41] show this very transparently. On the other hand, this scenario does not provide any clue for the difference between inclusive and exclusive determinations of $|V_{ub}|$.

6.3.2. $S_{\psi\phi} \approx 0.25$

Now, as signalled recently by CDF and D0 data [215], $S_{\psi\phi}$ could be smaller. In this case all non-MFV models listed above can reproduce such values and in particular this time also the LHT model [95] and another supersymmetric flavour model (AKM) analysed by us stay alive [66].

Again MSSM–MFV cannot reproduce such values. On the other hand, the $2\mathrm{HDM}_{\overline{\mathrm{MFV}}}$ can still provide interesting results. Yet as evident from the plots in [41] the FBPs in Yukawa interactions cannot now solve the UT tensions. Indeed the relation in (64) precludes now any interesting effects in ε_K and $S_{\psi K_S}$: $S_{\psi\phi}$ and the NP phase φ_{B_s} are simply too small. Evidently, this time the relation

$$\varphi_{B_d} = \varphi_{B_s} \tag{65}$$

would be more appropriate.

Now, the analyses in [67, 216] indicate how such a relation could be obtained within the $2\mathrm{HDM}_{\overline{\mathrm{MFV}}}$. This time the FBPs in the Higgs potential are at work, the relation in (65) follows and the plots of ε_K and $S_{\psi K_S}$ *versus* $S_{\psi\phi}$ are strikingly modified: the dependence is much stronger and even moderate values of $S_{\psi\phi}$ can solve all tensions. This time not Q_2^{LR} but Q_1^{SLL} in (18) is responsible for this behaviour.

Presently it is not clear which relation between φ_{B_s} and φ_{B_d} fits best the data but the model independent analysis of [216] indicates that φ_{B_s} should be significantly larger than φ_{B_d}, but this hierarchy appears to be smaller than in (64). Therefore as pointed out in [42] in the $2\mathrm{HDM}_{\overline{\mathrm{MFV}}}$ the best agreement with the data is obtained by having these phases both in Yukawa interactions and the Higgs potential, which is to be expected in any case. Which of the two flavour-blind CPV mechanisms dominates depends on the value of $S_{\psi\phi}$, which is still affected by a sizable experimental error, and also by the precise amount of NP allowed in $S_{\psi K_S}$.

Let us summarise the dynamical picture behind an enhanced value of $S_{\psi\phi}$ within $2\mathrm{HDM}_{\overline{\mathrm{MFV}}}$. For $S_{\phi\phi} \geq 0.7$ the FBPs in Yukawa interactions are expected to dominate. On the other hand, for $S_{\phi\phi} \leq 0.25$ the FBPs in the Higgs potential are expected to dominate the scene. If $S_{\psi\phi}$ will eventually be found somewhere between 0.3 and 0.6, a hybrid scenario analysed in [42] would be most efficient although not as predictive as the cases in which only one of these two mechanism is at work.

6.4. Implications of an enhanced $S_{\psi\phi}$

6.4.1. Preliminaries

Let us then assume that indeed $S_{\psi\phi}$ will be found to be significantly enhanced over the SM value. The studies of different observables in different models summarised in the previous section allow then immediately to make some concrete predictions for a number of observables which makes it possible to distinguish different models. This is important as $S_{\psi\phi}$ alone is insufficient for this purpose.

In view of space limitations I will discuss here only the implications for $B_{s,d} \to \mu^+\mu^-$ and $K \to \pi\nu\bar{\nu}$ decays, which we declared to be the superstars of the coming years. Subsequently I will make brief comments on a number of other superstars: EDMs, $(g-2)_\mu$, lepton flavour violation and ε'/ε.

6.4.2. $S_{\psi\phi} \geq 0.5$ scenario

The detailed studies of several models in which such high values of $S_{\psi\phi}$ can be attained imply the following pattern:

- In the AC model and the $2\mathrm{HDM}_{\overline{\mathrm{MFV}}}$, $\mathrm{Br}(B_{s,d} \to \mu^+\mu^-)$ will be automatically enhanced up to the present upper limit of roughly 3×10^{-8} from CDF and D0. The double Higgs penguins are responsible for this correlation [41, 42, 66].

- In the SM4 this enhancement will be more moderate: up to $(6–9) \times 10^{-9}$, that is a factor of 2–3 above the SM value [133, 137].

- In the non-Abelian supersymmetric flavour model RVV2, $\mathrm{Br}(B_{s,d} \to \mu^+\mu^-)$ can be enhanced up to a few 10^{-8} but it is not uniquely implied due to the pollution of double Higgs penguin contributions to B_s^0–$\bar{B}_s^0$ mixing through gluino boxes, that disturbs the correlation between $S_{\psi\phi}$ and $\mathrm{Br}(B_{s,d} \to \mu^+\mu^-)$ present in the AC model [66] and $2\mathrm{HDM}_{\overline{\mathrm{MFV}}}$.

- In the RSc, $\mathrm{Br}(B_{s,d} \to \mu^+\mu^-)$ is SM-like independently of the value of $S_{\psi\phi}$ [185]. If the custodial protection for Z^0 flavour violating couplings is removed values of 10^{-8} are possible [185, 199].

The question then arises what kind of implications does one have for $\mathrm{Br}(B_d \to \mu^+\mu^-)$. Our studies show that:

- The $2\mathrm{HDM}_{\overline{\mathrm{MFV}}}$ implies automatically an enhancement of $\mathrm{Br}(B_d \to \mu^+\mu^-)$ with the ratio of these two branching ratios governed solely by $|V_{td}/V_{ts}|^2$ and weak decay constants.

- This familiar MFV relation between the two branching ratios $\mathrm{Br}(B_{s,d} \to \mu^+\mu^-)$ is strongly violated in non-MFV scenarios like AC and RVV2 models and as seen in Fig. 5 of [1] taken from [66] for a given $\mathrm{Br}(B_s \to \mu^+\mu^-)$ the range for $\mathrm{Br}(B_d \to \mu^+\mu^-)$ can be large with the values of the latter branching ratios being as high as 5×10^{-10}.

- Interestingly, in the SM4, large $S_{\psi\phi}$ accompanied by large $\mathrm{Br}(B_s \to \mu^+\mu^-)$ precludes a large departure of $\mathrm{Br}(B_d \to \mu^+\mu^-)$ from the SM value 1×10^{-10} [137].

We observe that simultaneous consideration of $S_{\psi\phi}$ and $\mathrm{Br}(B_{s,d} \to \mu^+\mu^-)$ can already help us in eliminating some NP scenarios. Even more insight will be gained when $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$ and $\mathrm{Br}(K_\mathrm{L} \to \pi^0\nu\bar{\nu})$ will be measured:

- First of all the supersymmetric flavour models mentioned above predict by construction tiny NP contributions to $K \to \pi\nu\bar{\nu}$ decays. However, it does not mean that in supersymmetric models large effects in these decays are not possible. Examples of large enhancements of the rates for $K \to \pi\nu\bar{\nu}$ decays in supersymmetric theories can be found in [143, 218–222] and are reviewed in [223].

- In the RSc model significant enhancements of both branching ratios are generally possible [185, 199] but not if $S_{\psi\phi}$ is large. Similar comments would apply to the LHT model where the NP effects in $K \to \pi\nu\bar{\nu}$ can be larger than in the RSc [95]. However, the LHT model has difficulties to reproduce a very large $S_{\psi\phi}$ and does not belong to this scenario.

- Interestingly, in the SM4 large $S_{\psi\phi}$, $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$ and $\mathrm{Br}(K_\mathrm{L} \to \pi^0\nu\bar{\nu})$ can coexist with each other [137].

6.4.3. $S_{\psi\phi} \approx 0.25$ scenario

In this scenario many effects found in the large $S_{\psi\phi}$ scenario are significantly weakened. Prominent exceptions are:

- In the SM4, $\mathrm{Br}(B_s \to \mu^+\mu^-)$ is not longer enhanced and can even be suppressed, while $\mathrm{Br}(B_d \to \mu^+\mu^-)$ can be significantly enhanced [137].

- The branching ratios $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$ and $\mathrm{Br}(K_{\mathrm{L}} \to \pi^0\nu\bar{\nu})$ can now be strongly enhanced in the LHT model [95] and the RSc model [185, 199] with respect to the SM but this is not guaranteed.

These patterns of flavour violations demonstrate very clearly the power of flavour physics in distinguishing different NP scenarios.

6.5. EDMs, $(g-2)_\mu$ and Br $(\mu \to e\gamma)$

These observables are governed by dipole operators but describe different physics as far as CP violation and flavour violation is concerned. EDMs are flavour conserving but CP-violating, $\mu \to e\gamma$ is CP-conserving but lepton flavour violating and finally $(g-2)_\mu$ is lepton flavour conserving and CP-conserving. A nice paper discussing all these observables simultaneously is [224].

In concrete models there exist correlations between these three observables of which EDMs and $\mu \to e\gamma$ are very strongly suppressed within the SM and have not been seen to date. $(g-2)_\mu$, on the other hand, has been very precisely measured and exhibits a 3.2σ departure from the very precise SM value (see [225] and references therein). Examples of these correlations can be found in [62, 66]. In certain supersymmetric flavour models with non-MFV interactions the solution of the $(g-2)_\mu$ anomaly implies simultaneously d_e and $\mathrm{Br}(\mu \to e\gamma)$ in the reach of experiments in this decade. In these two papers several correlations of this type have been presented. We have listed them in the previous section.

The significant FBPs required to reproduce the enhanced value of $S_{\psi\phi}$ in the $2\mathrm{HDM}_{\overline{\mathrm{MFV}}}$ model, necessarily imply large EDMs of the neutron, thallium and mercury atoms. Yet, as a detailed analysis in [42] shows the present upper bounds on the EDMs do not forbid sizable non-standard CPV effects in B_s mixing. However, if a large CPV phase in B_s mixing will be confirmed, this will imply hadronic EDMs very close to their present experimental bounds, within the reach of the next generation of experiments. For a recent model independent analysis of EDMs see [226].

6.6. *Waiting for precise predictions of* ε'/ε

The flavour studies of the last decade have shown that provided the hadronic matrix elements of QCD-penguin and electroweak penguin operators will be known with sufficient precision, ε'/ε will play a very important role in constraining NP models. We have witnessed recently an impressive progress in the lattice evaluation of $\hat{B}_K$ that elevated ε_K to the group of observables relevant for precision studies of flavour physics. Hopefully this could also be the case of ε'/ε already in this decade.

6.7. $B \to K^* l^+ l^-$

Let us next mention briefly these decays that will be superstars at the LHCb. While the branching ratios for $B \to X_s l^+ l^-$ and $B \to K^* l^+ l^-$ put already significant constraints on NP, the angular observables, CP-conserving ones like the well known forward–backward asymmetry and CP-violating ones will definitely be very useful for distinguishing various extensions of the SM. A number of detailed analyses of various CP averaged symmetries and CP asymmetries provided by the angular distributions in the exclusive decay $B \to K^*(\to K\pi) l^+ l^-$ have been performed in [227–229]. In particular the zeroes of some of these observables can be accurately predicted. Belle and BaBar provided already interesting results for the best known forward–backward asymmetry but the data have to be improved in order to see whether some sign of NP is seen in this asymmetry. Future studies by the LHCb, Belle II and SFF in Rome will be able to contribute here in a significant manner.

6.8. $B^+ \to \tau^+ \nu$ *and* $B^+ \to D^0 \tau^+ \nu$

Another prominent anomaly in the data not discussed by us so far is found in the tree-level decay $B^+ \to \tau^+ \nu$. The relevant branching ratio is given by

$$\mathrm{Br}(B^+ \to \tau^+ \nu)_{\mathrm{SM}} = \frac{G_{\mathrm{F}}^2 m_{B^+} m_\tau^2}{8\pi} \left(1 - \frac{m_\tau^2}{m_{B^+}^2}\right)^2 F_{B^+}^2 |V_{ub}|^2 \tau_{B^+} \,. \tag{66}$$

In view of the parametric uncertainties induced in (66) by F_{B^+} and V_{ub}, in order to find the SM prediction for this branching ratio one can rewrite it as follows [66]

$$\mathrm{Br}(B^+ \to \tau^+ \nu)_{\mathrm{SM}} = \frac{3\pi \Delta M_d}{4 \eta_B S_0(x_t) \hat{B}_{B_d}} \frac{m_\tau^2}{M_W^2} \left(1 - \frac{m_\tau^2}{m_{B^+}^2}\right)^2 \left|\frac{V_{ub}}{V_{td}}\right|^2 \tau_{B^+} \,. \tag{67}$$

Here ΔM_d is supposed to be taken from experiment and

$$\left|\frac{V_{ub}}{V_{td}}\right|^2 = \left(\frac{1}{1-\lambda^2/2}\right)^2 \frac{1+R_{\rm t}^2 - 2R_{\rm t}\cos\beta}{R_{\rm t}^2}\,, \tag{68}$$

with $R_{\rm t}$ and β determined by means of $\Delta M_d/\Delta M_s$ and $S_{\psi K_{\rm S}}$, respectively. In writing (67), we used $F_B \simeq F_{B^+}$ and $m_{B_d} \simeq m_{B^+}$. We then find [66]

$$\mathrm{Br}(B^+ \to \tau^+\nu)_{\rm SM} = (0.80 \pm 0.12) \times 10^{-4}\,. \tag{69}$$

This result agrees well with the result presented by the UTfit Collaboration [230].

On the other hand, the present experimental world average based on results by BaBar and Belle reads [230]

$$\mathrm{Br}(B^+ \to \tau^+\nu)_{\rm exp} = (1.73 \pm 0.35) \times 10^{-4}\,, \tag{70}$$

which is roughly by a factor of 2 higher than the SM value. We can talk about a tension at the 2.5σ level.

With a higher value of $|V_{ub}|$ as obtained through inclusive determination this discrepancy can be decreased significantly. For instance with a value of 4.4×10^{-3}, the central value predicted for this branching ratio would be more like 1.25×10^{-4}. Yet, this would then require NP phases in B_d^0–$\bar{B}_d^0$ mixing to agree with the data on $S_{\psi K_{\rm S}}$. In any case values of $\mathrm{Br}(B^+ \to \tau^+\nu)_{\rm exp}$ significantly above 1×10^{-4} will signal NP contributions either in this decay or somewhere else.

While the final data from BaBar and Belle will lower the experimental error on $\mathrm{Br}(B^+ \to \tau^+\nu)$, the full clarification of a possible discrepancy between the SM and the data will have to wait for the data from Belle II and SFF in Rome. Also improved values for F_B from lattice and $|V_{ub}|$ from tree-level decays will be important if some NP like charged Higgs is at work here. The decay $B^+ \to D^0\tau^+\nu$ being sensitive to different couplings of $H^\pm$ can contribute significantly to this discussion but formfactor uncertainties make this decay less theoretically clean. A thorough analysis of this decay is presented in [231] where further references can be found.

Interestingly, the tension between theory and experiment in the case of $\mathrm{Br}(B^+ \to \tau^+\nu)$ increases in the presence of a tree-level $H^\pm$ exchange which interferes destructively with the $W^\pm$ contribution. As addressed long time ago by Hou [232] and in modern times calculated first by Akeroyd and Recksiegel [233], and later by Isidori and Paradisi [234], one has in the MSSM with MFV and large $\tan\beta$

$$\frac{\mathrm{Br}(B^+ \to \tau^+\nu)_{\rm MSSM}}{\mathrm{Br}(B^+ \to \tau^+\nu)_{\rm SM}} = \left[1 - \frac{m_B^2}{m_{H^\pm}^2}\,\frac{\tan^2\beta}{1+\epsilon\tan\beta}\right]^2\,, \tag{71}$$

with ϵ collecting the dependence on supersymmetric parameters. This means that in the MSSM this decay can be strongly suppressed unless the choice of model parameters is such that the second term in the parenthesis is larger than 2. Such a possibility that would necessarily imply a light charged Higgs and large $\tan\beta$ values seems to be very unlikely in view of the constraints from other observables [235]. Recent summaries of $H^\pm$ physics can be found in [236, 237].

6.9. Rare B decays $B \to X_s\nu\bar{\nu}$, $B \to K^*\nu\bar{\nu}$ and $B \to K\nu\bar{\nu}$

Finally, we discuss these three superstars that provide a very good test of modified Z penguin contributions [238, 239], but their measurements appear to be even harder than those of the rare $K \to \pi\nu\bar{\nu}$ decays discussed previously. Recent analyses of these decays within the SM and several NP scenarios can be found in [240, 241].

The inclusive decay $B \to X_s\nu\bar{\nu}$ is theoretically as clean as $K \to \pi\nu\bar{\nu}$ decays but the parametric uncertainties are a bit larger. The two exclusive channels are affected by formfactor uncertainties but last year in the case of $B \to K^*\nu\bar{\nu}$ [240] and $B \to K\nu\bar{\nu}$ [241] significant progress has been made. In the latter paper this has been achieved by considering simultaneously also $B \to Kl^+l^-$. Last year also non-perturbative tree-level contributions from $B^+ \to \tau^+\nu$ to $B^+ \to K^+\nu\bar{\nu}$ and $B^+ \to K^{*+}\nu\bar{\nu}$ at the level of roughly 10% have been pointed out [242].

The interesting feature of these three $b \to s\nu\bar{\nu}$ transitions, in particular when taken together, is their sensitivity to right-handed currents [240]. Belle II and SFF in Rome machines should be able to measure them at a satisfactory level and various ideas put forward in the latter paper will be tested.

7. The 3×3 flavour code matrix

7.1. Basic idea

In Section 5 we have reviewed a large number of BSM models with rather different patterns of flavour and CP violation. There are other models in the literature that I did not discuss here but it appears to me that already on the basis of the models considered by us a rough picture is emerging. The question then is, how to draw a grand picture of all these NP effects and to summarise them in a transparent manner. The problem with this goal is the multitude of free parameters present basically in all extensions of the SM, making any transparent classification a real challenge.

In this context let me give one example. At first sight it is evident that the presence of the $Q_2^{\rm LR}$ operator in a NP scenario promises interesting and often dangerous effects due to large RG effects accompanying this operator (large anomalous dimension) and the chiral enhancements of its matrix elements in the K system. Such effects are typically much smaller in models dominated by LH currents and the study of the LHT model confirms this picture: the CP asymmetry $S_{\psi\phi}$ in this model cannot be as large as in models containing RH currents which in collaboration with LH currents produce the $Q_2^{\rm LR}$ operator. Yet, this clear picture is polluted to some extent by the results obtained in the SM4. This model has only LH currents but still is capable of obtaining values for $S_{\psi\phi}$ above 0.5, even if this is not as easy as in the case of models with RH currents.

Thus the question is, what are the right "degrees of freedom" or "coordinates" for such a grand picture. We have made the first steps in this direction in [1], where a 2×2 Flavour Matrix has been proposed. This matrix distinguishes between models with SM and non-SM operators on the one hand and between MFV models and models with non-MFV sources on the other hand. The previous sections demonstrate clearly that this matrix is too small to transparently uncover all possibilities so that a proper distinction between models belonging to a given element of this matrix cannot be made.

There are three aspects which are missing in the 2×2 Flavour Matrix:

- The distinction between models having significant FBPs and those not having them.
- The distinction between BSM models with dominant LH currents, RH currents and scalar (SH) currents.
- Moreover, there are models in which LH, RH and SH currents play comparable role.

This situation indicates that it is probably a better idea to invent a new classification of various NP effects by means of a coding system in a form of a 3×3 *flavour code matrix* (FCM) which instead of attaching a given model to a specific entry of a flavour matrix describes each model separately. Thus each NP model is characterised by a special code in which only some entries of the matrix in question are occupied. MFV, non-MFV sources and FBPs on the one hand and LH currents, RH currents and SH currents on the other hand, are the fundamental coordinates in this code. They allow to distinguish compactly the models discussed in Section 5. The basic structure of a FCM is shown schematically in Table V.

In other words, the main goal is to identify the main fundamental "ingredient" of NP models that lead to specific/distinct features in flavour observables. Once these ingredients are identified one has a classification of a NP model simply based on the ingredients that are present in that model. The FCM presented here should be a good starting point for such a classification scheme but more elaborate schemes could also be constructed. We will return to them in the future.

TABLE V

The Flavour Code Matrix for a given model. FBPs denotes important flavour-blind phases. BMFV denotes new flavour violating interactions. LH denotes the left-handed currents, RH denotes right-handed currents and SH denotes scalar currents.

Model	LH	RH	SH
MFV	F_{11}	F_{12}	F_{13}
BMFV	F_{21}	F_{22}	F_{23}
FBPs	F_{31}	F_{32}	F_{33}

Ultimately a given *flavour code* should correspond to a unique flavour DNA pattern representing a given model and just having this code should be sufficient to determine the interesting flavour observables that can probe this model.

7.2. Arguments for the choice of coordinates of FCM

Let us still elaborate on our choice of the coordinates.

First, MFV restricts both flavour and CP violation to the CKM matrix. Next, FBPs only provide additional sources of CP violation. Finally, BMFV allows for non-trivial complex flavour structures, *i.e.* both new sources of flavour and CP violation are present. This choice of *flavour* coordinates MFV, non-MFV and FBPs is then rather convincing even if the non-MFV interactions can vary from model to model by a significant amount. However, some differences between models with non-MFV sources arise precisely from the different structure of contributing operators and this difference is taken care of at least partially by the remaining three coordinates LH, RH and SH.

The classification of Lorentz structures in only LH, RH and SH over-simplifies of course the general situation but we think that it is sufficient as the leading order approximation. There would be no point in choosing as coordinates all different operators involved as one would end in very large code matrices.

Let us note that for $\Delta F = 1$ semi-leptonic processes the division in LH, RH and SH is rather transparent as one can convince oneself by looking at the operators involved that we listed in Section 4.

The case of $\Delta F = 2$ processes is still simple if one only considers diagrams with exclusively LH currents and separately diagrams with only RH currents[2]. Then operators Q_1^{VLL} and Q_1^{VRR} result, respectively. This is also

[2] I would like to thank Wolfgang Altmannshofer and Paride Paradisi for interesting and inlighting discussions related to the points presented here and other parts of this section.

the case of box diagrams with internal squarks and gluinos in SUSY models. $d_{\rm LL}$ and $d_{\rm RR}$ mass insertions when considered separately generate $Q_1^{\rm VLL}$ and $Q_1^{\rm VRR}$, respectively. The necessary γ_μ is provided by the gluino propagator. These two cases are also easily distinguishable from the third coordinate related to scalar interactions.

The situation becomes more complicated when LH and RH currents or $d_{\rm LL}$ and $d_{\rm RR}$ mass insertions are considered simultaneously. Then the operators $Q_1^{\rm LR}$ and $Q_2^{\rm LR}$ enter and have to be considered simultaneously as they mix under QCD renormalization. Moreover, up to the colour indices these two operators are related by Fierz transformation. But this is not really a problem for our classification as simply the presence of LH and RH currents just generates such operators with related phenomenological implications. Yet, one should stress that the operator $Q_2^{\rm LR}$ can also be generated by scalar exchanges and the renormalization under QCD generates then $Q_1^{\rm LR}$ as well. For large $\tan\beta$ the scalar currents can, however, dominate and as our studies of RHMFV and $2{\rm HDM}_{\overline{\rm MFV}}$ indicate the phenomenology of RH currents and LH currents and of SH currents is quite different in each case.

In summary, the coordinates chosen here appear as a good starting point towards some optimal classification of flavour dynamics at very short distance scales.

7.3. Examples of flavour code matrices

We will now present FCMs for models discussed in the text. While the patterns of flavour violation in a given model are in details specific for this model, models having similar codes will have similar predictions for various observables. We distinguish MFV, BMFV and FBPs by different shapes (LH, RH and SH by different colours. The latter colour coding is evident: red for LH, black for RH and Bavarian blue for SH.).

It should be emphasized that although in each model considered by us almost all entries of the related FCM could be filled, we indicate only those which play a significant role in the phenomenology.

In Tables VI–XI we show the FMCs for models discussed in Section 5.

In Table VI we summarise CMFV and $2{\rm HDM}_{\overline{\rm MFV}}$. The NP effects in CMFV are generally small and CPV is SM-like. In $2{\rm HDM}_{\overline{\rm MFV}}$ the presence of scalar currents accompanied by FBPs allows to obtain, in spite of MFV, interesting CPV effects in FCNC processes and EDMs. Also significantly enhanced ${\rm Br}(B_{s,d}\to\mu^+\mu^-)$ are possible in the latter case.

In Table VII we show FCMs for the LHT model and the SM4. These FCMs look identical and, in fact, the patterns of flavour violation in these two models show certain similarities although NP effects in the SM4 can be

TABLE VI

The FCM for the CMFV models (left) and $2\mathrm{HDM}_{\overline{\mathrm{MFV}}}$ (right).

CMFV	LH	RH	SH
MFV	$\textcolor{red}{\bigstar}$		
BMFV			
FBPs			

$2\mathrm{HDM}_{\overline{\mathrm{MFV}}}$	LH	RH	SH
MFV	$\textcolor{red}{\bigstar}$		$\textcolor{blue}{\bigstar}$
BMFV			
FBPs	$\textcolor{red}{\blacktriangle}$		$\textcolor{blue}{\blacktriangle}$

larger. A prominent difference is noticed in the case of $\mathrm{Br}(B_{s,d} \to \mu^+\mu^-)$. They can be enhanced up to factors of 2–3 in the SM4, while this is not possibly in the LHT.

TABLE VII

The FCM for the LHT model (left) and SM4 (right).

LHT	LH	RH	SH
MFV	$\textcolor{red}{\bigstar}$		
BMFV	$\textcolor{red}{\blacksquare}$		
FBPs			

SM4	LH	RH	SH
MFV	$\textcolor{red}{\bigstar}$		
BMFV	$\textcolor{red}{\blacksquare}$		
FBPs			

In Table VIII we show the FBMSSM and the FS model with the dominance of LH currents (δLL). The pattern of flavour violations in these two models is similar but the presence of BMFV sources makes the δLL model less constrained. In particular in the FBMSSM the ratio of $\mathrm{Br}(B_d \to \mu^+\mu^-)$ to $\mathrm{Br}(B_s \to \mu^+\mu^-)$ is MFV-like as in (29), while in the (δLL) model due to BMFV sources this relation can be strongly violated. This different pattern could be used to distinguish these two models. Characteristic for these two models are sizable effects in $S_{\phi K_{\mathrm{S}}}$ and SM-like $S_{\psi\phi}$. Finally, let us note that within (δLL) model and all other SF models discussed by us, one assumes that CP is a symmetry of the theory (hence no FBPs) that is broken only by flavour effects after the breaking of the flavour symmetry.

TABLE VIII

The FCM for the FBMSSM model (left) and δLL (right).

FBMSSM	LH	RH	SH
MFV	$\textcolor{red}{\bigstar}$		$\textcolor{blue}{\bigstar}$
BMFV			
FBPs	$\textcolor{red}{\blacktriangle}$		

δLL	LH	RH	SH
MFV	$\textcolor{red}{\bigstar}$		$\textcolor{blue}{\bigstar}$
BMFV	$\textcolor{red}{\blacksquare}$		$\textcolor{blue}{\blacksquare}$
FBPs			

In Table IX we show FCMs for RHMFV and RSc. The presence of sizable contributions from BMFV sources in LH and RH currents in RSc makes the effects in this model generally larger than in the RHMFV and some fine tuning of parameters is required in order to be in accordance with the data. In turn some observables, in particular $\mathrm{Br}(B_{s,d} \to \mu^+\mu^-)$, turn out to be SM-like in the RSc. The RHMFV has a much simpler structure than RSc and the pattern of flavour violation is more transparent.

TABLE IX

The FCM for the RHMFV model (left) and RSc (right).

RHMFV	LH	RH	SH
MFV	★		
BMFV		■	
FBPs			

RSc	LH	RH	SH
MFV	★		
BMFV	■	■	
FBPs			

In Table X we show the FCM's of AMK and AC models. It should be emphasized that the AMK model is based on the SU(3) flavour symmetry, while the AC model is an Abelian model. Moreover, whereas the RH currents in the AMK model are CKM-like, they are $\mathcal{O}(1)$ in the case of the AC model. The small (red) box in the FCM of the AMK model indicates that the LH currents in this model as analysed by us are much weaker than the RH currents. Still many implications of these two models are similar although the effects in the AC model are generally larger. The smoking gun of both models is the strong correlation between $S_{\psi\phi}$ and the lower bounds on $\mathrm{Br}(B_{s,d} \to \mu^+\mu^-)$ that can be for large $S_{\psi\phi}$ by an order of magnitude larger than the SM values. The neutral Higgs exchanges with large $\tan\beta$ are responsible for this strong correlation.

TABLE X

The FCM for the AMK model (left) and AC model (right).

AMK	LH	RH	SH
MFV	★		★
BMFV	▪	■	■
FBPs			

AC	LH	RH	SH
MFV	★		★
BMFV		■	■
FBPs			

Finally, in Table XI we show FCM of the RVV2 model that having SU(3) flavour symmetry has some similarities with the AMK model but the presence of larger LH currents in the RVV2 model leads to differences as summarised in Section 5. We have left in several models the entries related to FBPs empty as this issue requires further study. Moreover, in certain models, flavour diagonal but not flavour-blind phases like in the RSc model are present.

TABLE XI

The FCM for the RVV2 model.

RVV2	LH	RH	SH
MFV	★		★
BMFV	■	■	■
FBPs			

This discussion shows that the idea of FCM's, although more powerful than the 2×2 matrix discussed by us previously, cannot yet fully depict all properties of a given model and the differences between various models. However, in conjunction with the correlations between various observables it could turn out to be a useful step towards a grand picture of various patterns of flavour and CP violation. The references to such correlations have been collected in Table IV.

Let us also note that the entry (MFV,RH) is always empty as the MFV flavour structure implies automatically the absence of RH currents or their suppression by mass ratios m_s/m_b, m_d/m_b and m_d/m_s.

8. Grand summary

Our presentation of various BSM models is approaching the end. I hope I convinced the readers that flavour physics is a very rich field which necessarily will be a prominent part of a future theory of fundamental interactions both at large and short distance scales. While MFV could work to the first approximation, various studies show that models attempting the explanation of the hierarchies of fermion masses and of its hierarchical flavour violating and CP-violating interactions in most cases imply non-MFV interactions. This is evident from the study of supersymmetric flavour models [66] and more general recent studies [243, 244].

What role will be played by flavour-blind phases in future phenomenology depends on the future experimental data on EDMs. Similar comment applies to LFV. A discovery of $\mu \to e\gamma$ rate at the level of 10^{-13} would be a true mile stone in flavour physics. Also the discovery of $S_{\psi\phi}$ at the level of 0.3 or higher would have a very important impact on quark flavour physics. The measurements of $\mathrm{Br}(B_{s,d} \to \mu^+\mu^-)$ in conjunction with $S_{\psi\phi}$, $K^+ \to \pi^+\nu\bar{\nu}$ and at later stage $K_{\mathrm{L}} \to \pi^0\nu\bar{\nu}$ will allow to distinguish between various models as explicitly shown in Section 6. Here the correlations between various observables will be crucial. It is clearly important to clarify the origin of the tensions between ε_K, $S_{\psi K_{\mathrm{S}}}$, $|V_{ub}|$ and $\mathrm{Br}(B^+ \to \tau^+\nu_\tau)$ but this possibly has to wait until Belle II and later SFF will enter their operation.

At the end of our presentation we made a new attempt to classify various extensions of the SM with the help of a 3×3 flavour code matrix. Whether this classification and earlier proposed DNA tests of flavour physics will turn out to be a step forward should be clear in the next five years when new measurements will be available hopefully showing clear patterns of deviations from the SM. In any case I have no doubts that we will have a lot of fun with flavour physics in this decade and that this field will offer very important insights into the short distance dynamics.

I would like to thank the organizers of the Cracow School 2010, in particular Michał Praszałowicz, for inviting me to such a pleasant school. I wish my Cracow colleagues next fruitful 50 years in Zakopane. I also thank all my collaborators for exciting time we spent together exploring the short distance scales with the help of flavour violating processes and in particular Wolfgang Altmannshofer and Paride Paradisi for comments on the manuscript. I would also like to thank Walter Grimus and Helmut Neufeld for the great hospitality extended to me during my stay at the University of Vienna as Schrödinger guest professor where the final part of these lectures has been written. This research was partially supported by the Cluster of Excellence "Origin and Structure of the Universe" and by the German "Bundesministerium für Bildung und Forschung" under contract 05H09WOE.

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