50 YEARS OF NEUTRINO PHYSICS*

MAREK ZRALEK

Department of Field Theory and Particle Physics
Institute of Physics, University of Silesia
Uniwersytecka 4, 40-007 Katowice, Poland

(Received November 15, 2010)

Some important topics from history of neutrino physics over the last fifty years are discussed. History of neutrinos is older, at 4th December 2010 it will be eightieth anniversary of the *neutrino birth*. In that day W. Pauli wrote the famous letter to participants of the physics conference at Tubingen with the suggestion that "there could exist in the nuclei electrically neutral particle". We will concentrate mostly on the 50 years of neutrino history just to show the long tradition of the Zakopane Theoretical School.

PACS numbers: 13.15.+g, 14.60.Lm, 14.60.St, 14.60.Pq

1. Introduction — neutrinos before 1960

The origins of neutrinos are related to the discovery of β decay of nuclei at the late of 19th century. Observation of the particles after decay pointed to a lack of conservation of energy and momentum in the observed process. Missing energy and momentum could be explained by the existence of some new particle or, as Niels Bohr suggested, having the Quantum Mechanical experience, that perhaps energy and momentum are conserved only statistically. By introducing neutrinos in 1930 [1] Pauli has saved the principle of energy and momentum conservation.

In 1934 the idea of Pauli led Fermi [2] to the formulation of the theory of nuclei β decay, and generally to the theory of weak interaction. Fermi used the analogy of electromagnetic interaction. It was only the effective theory, but now it is considered as the early beginning of the modern gauge theory of weak interaction. Even now the Fermi theory is used to describe four fermion processes for small energy.

* Lecture presented at the L Cracow School of Theoretical Physics "Particle Physics at the Dawn of the LHC", Zakopane, Poland, June 9–19, 2010.

Knowing the neutron lifetime, Fermi was able to predict the value of the so-called *Fermi constant*. Then it was possible to calculate the cross-section for inverse β decay and to predict a probability of neutrino detection. This probability was very small, and Bethe and Peierls claimed [3] that neutrinos might never be observed. Now this strong statement is a warning that in physics such ultimate opinions should not be stated. 22 years later electron neutrinos had been observed.

In 1937 the second charged lepton — a muon was discovered [4]. Muons had very similar properties to almost 200 times lighter electrons. This similarity led in the future to the concept of *lepton universality*.

As we want to be chronological, next we have to mention the Majorana idea of chargeless fermion which are their own antiparticles — and presently are known as *Majorana particles*, even if this idea became popular only in the seventies. In 1937 Majorana [5], using neutrinos, has suggested the existence of such elementary objects. Majorana spinors, as well as the older idea of Weyl spinors [6], was connected with the space and charge parity non-conservation[1]. At that time it was unthinkable to accept breaking of these two symmetries, and both ideas have been rejected.

In 1947, after discovery of muon decay, B. Pontecorvo proposed the *universality* of the Fermi interaction, at that time electron and muon. Then this suggestion was widely discussed in [7] and possibly the origin of the current *generation of leptons or a family* should be linked to this discussion.

In order to explain certain missing decay modes, in 1953 the concept of lepton number (L) was introduced [8]. This is one of well known tested conservation law. In the Fermi theory, in the Standard Model and, what is most important, in all present experiments, L is conserved.

Three years later first neutrino — the *electron neutrino* has been experimentally discovered in the inverse β decay process. After considering several methods, also a possible atomic bomb explosion, Reines and Cowan found antineutrinos from a nuclear reactor [9]. It was the first experiment which we call now *reactor neutrino experiment* where $\overline{\nu}_e$ produced by reactors were used.

In 1956 neutrinos were used by Lee and Yang [10] to put forward the hypothesis, that the symmetries P and C are not satisfied in nature, and a year later [11], to the experimental verification of this fact.

When it became apparent that the parity is broken, the weak Lagrangian has become much more complicated and scalar, vector and tensor terms together with parity violating couplings have to be taken into account. Such complicated situation was simplified in 1958 when the V–A theory of weak

[1] The Weyl spinors do not preserve the C and P symmetries. In the case of Majorana spinors it is not so. As was noted later, using these spinors, it is possible to construct a theory, which satisfies these two discrete symmetries.

β decay has been formulated [12]. Then the Weyl idea of two component spinors, which describe massless fermion, finally found an application [13], but even then it was noted that there is no difference between Weyl and massless Majorana neutrinos [14]. The Weyl two component spinor was simpler and there was no reason to use Majorana bispinors.

In 1958 the polarization of a neutrino has been measured in electron capture reaction $e^- + {}^{152}\mathrm{Eu} \rightarrow {}^{152}\mathrm{Sm}^* + \nu_e$ and subsequent decay ${}^{152}\mathrm{Sm}^* \rightarrow {}^{152}\mathrm{Sm} + \gamma$ [15]. The helicity of neutrinos was negative in full agreement with two component theory of massless neutrino.

At the end of the fifties occurred one more thing that is sure to be noted. The concept of *neutrino oscillation* was proposed by Pontecorvo [16,17]. Motivated by the $K^0 \Leftrightarrow \overline{K}^0$ oscillation phenomena proposed by M. Gell-Mann and A. Pais in 1955, Pontecorvo suggested that similar phenomenon, transition between $\nu \Leftrightarrow \overline{\nu}$ for Majorana neutrinos, can occur. He has interpreted the (wrong) result of the Davis observation of $\overline{\nu} + {}^{37}\mathrm{Cl} \rightarrow e^- + {}^{37}\mathrm{Ar}$ [18] as a result of $\overline{\nu} \Leftrightarrow \nu$ transition and then proper electron production $\nu + {}^{37}\mathrm{Cl} \rightarrow e^- + {}^{37}\mathrm{Ar}$.

One other thing happened in 1958 which have meaning for future discoveries. Feinberg has tried to find muon decay $\mu \rightarrow e + \gamma$ without success [19].

The fifties were indeed very fruitful for neutrino physics. At the end of this period we had information that there are two charged leptons, electron and muon and one neutral — electron neutrino. Lepton number L, which distinguishes leptons (e^-, μ^-, ν_e) from antileptons $(e^+, \mu^+, \overline{\nu}_e)$ was introduced, so all leptons were Dirac particles. The nucleon β decay was described by V–A vector Lagrangian

$$\mathcal{L}_{(\mathrm{V-A})} = \frac{G_\mathrm{F}}{\sqrt{2}} \left(\overline{\nu}_e \gamma^\mu \left(1 - \gamma_5\right) e \right) \left(\overline{n} \gamma_\mu (C_\mathrm{V} - C_\mathrm{A} \gamma_5) p \right) + \mathrm{h.c.} \tag{1}$$

Neutrinos were massless particles described by Weyl two component spinors and took part in the C and P violating, but CP conserving interaction (1). The idea of neutrino oscillation has appeared but not in the correct way, as a neutrino–antineutrino transition.

In the next section we present the origin of the SM, which emerged in the sixties, and the role of neutrinos in that time. In Section 3 neutrino properties in the SM are presented and the first experimental indications, which show that the theory must be extended because the neutrinos do not satisfy the SM predictions and are massive particles. Then, in Section 4, we describe, how the SM has to be minimally extended to predict the massive neutrinos, how the phenomenon of neutrino oscillation is now understood and what kind of experimental information about the neutrino masses and mixing we have today. In Section 5 we present what are the consequences of the observed neutrino properties for physics beyond the SM, and finally in Section 6 some conclusions are given.

2. The road to the Standard Model

The productive period in neutrino physics persisted also in the sixties. Already at the beginning, as in many previous cases, Pontecorvo had a brilliant intuition and suggested [20] that, if neutrino produced in the pion decay $\pi^+ \to \mu^+ + \nu_\mu$ cannot induce e^-, then both neutrinos ν_e and ν_μ are different particles. Such experiment was done in 1962 at Brookhaven National Laboratory by L.M. Lederman, M. Schwartz, J. Steinberger *et al.* [21]. Indeed, neutrinos from pion decay always produced muons but never electrons. Then it was clear that ν_e and ν_μ are different particles and there are at least two different family of leptons, a new neutrino, the *muon neutrino* appeared in particle physics. It is also worth to stress that this Brookhaven experiment was in fact the first, where the beam of neutrinos has been prepared, so it was the first experiment which we now know as *accelerator neutrino experiment*.

In order to explain the smallness of leptonic decay of hyperons and a subtle difference of the Fermi coupling G_νs between μ and β decay, Maki, Nakagawa and Sakata (MNS) introduced neutrino mixing [22]. They assumed that ν_e and ν_μ are not mass eigenstates, but are superposition of two neutrinos with different masses

$$\begin{aligned} \nu_e &= \nu_1 \cos\theta + \nu_2 \sin\theta\,, \\ \nu_\mu &= -\nu_1 \sin\theta + \nu_2 \cos\theta\,. \end{aligned} \tag{2}$$

In the MNS paper there was no discussion about neutrino oscillation. The first intuitive understanding of neutrino mixing and oscillation was presented by Pontecorvo [23] and by Gribov and Pontecorvo [24]. The full theory of neutrino oscillation together with a third generation of leptons was finally developed in 1975–76. The full 3×3 mixing matrix appeared at that time and received the name of the Maki–Nakagawa–Sakata–Pontecorvo (MNSP) mixing matrix. Than it has become possible the violation of CP symmetry in the lepton sector, in the same way as for quarks.

In 1968 the Homestake solar neutrino experiment has started to work [25]. Originally this experiment, by observing neutrinos produced in the sun, had to check the Bethe model [26] for the creation of the solar energy. Nobody predicted that this experiment will have to change its role and start to examine the properties of neutrinos.

In the sixties neutrinos, together with charged leptons and quarks, gave rise to the formulation of the model of electroweak interactions [27]. At that time only three quarks (u,d,s) and four leptons (e,ν_e,μ,ν_μ) were known. All neutrino properties known from the contact Fermi model have been preserved. So, neutrinos remain massless Weyl particles, which interact only through the left-handed currents, and their interaction break up the discrete symmetries C and P maximally. There is also a new property, not known before — family leptons numbers L_e, L_μ, L_τ which now are often used as *flavour lepton numbers.*

Over the next years to date, all the components of this model, with one exception, have been experimentally discovered. Three successive quarks [28], another charged lepton τ [29] and its neutrino ν_τ [30] were found, but the basic particle of the model — the Higgs particle is still missing. Together with strong interactions the full theory, which now describe all elementary particles interactions, is known as the *Standard Model* (SM). In the model there are three families. Four leptons, known before and the new one, form the three families which are called: electron, muon and tau families

$$(\nu_e, e^-)\,, \qquad (\nu_\mu, \mu^-)\,, \qquad (\nu_\tau, \tau^-)\,. \tag{3}$$

In this model neutrinos couple to charged gauge bosons $W^\pm$

$$L_{\rm CC} = \frac{e}{2\sqrt{2}\sin\theta_W} \sum_{\alpha=e,\mu,\tau} \bar{\nu}_\alpha \gamma^\mu (1-\gamma_5) l_\alpha W_\mu^+ + \text{h.c.}\,, \tag{4}$$

and to neutral Z_0 one

$$L_{\rm NC} = \frac{e}{4\sin\theta_W\cos\theta_W} \sum_{\alpha=e,\mu,\tau} \bar{\nu}_\alpha \gamma^\mu (1-\gamma_5) \nu_\alpha Z_\mu\,. \tag{5}$$

Neutrino fields do not appear in any other place of the SM interaction Lagrangian. From (4) and (5) we can easily find that really the family lepton numbers L_e, L_μ and L_τ are separately conserved and, as a consequence, also the total lepton number $L = L_e + L_\mu + L_\tau$ is conserved. In the SM there is no flavour lepton mixing (L_α conservation) and no CP symmetry violation[2] in the lepton sector, neutrinos are stable and have no electromagnetic structure (only the charge radius $\langle r^2 \rangle \neq 0$).

There are three facts that determine that neutrinos are massless in the SM:

1. we do not introduce the right-handed fields $\nu_{\rm R}$,
2. in the model only one Higgs doublet is introduced,
3. we require that the theory is renormalizable.

[2] Both properties, the lack of flavour mixing and the CP symmetry conservation are connected with the assumption that neutrino are massless.

As we see none of these reasons is very basic. Lack of mass of neutrinos is not guaranteed by any fundamental theory so resignation from any of the previous conditions 1–3 results in a mathematically correct theory. So it is very easy to find a theory with massive neutrinos, the problem is, as we will see, why their masses are so remarkably small.

Neutrinos are responsible for the first great success of the SM. In 1973, the measurement of a neutral current reaction in the Gargamelle bubble chamber [31] experiment at CERN has given the first indication that the neutral gauge boson Z_0 exists. Neutrinos from pion decay scattered on the liquid scintillator — freon, and muon has not been produced $\nu_\mu + N \rightarrow \nu_\mu + N$ After this experiment, SM received a good foundation even if successive quarks and leptons have been discovered later. The bases of the model have survived to this day, only the number of generations has grown. So far there are no experimental facts which are incompatible with it, with one exception. From many different experiments it is now obvious that neutrinos are not massless particles.

3. Neutrinos in the SM and the road outside

In the past 40 years MS has achieved a great success and now is firmly established as the model for lepton and quarks interactions. In all experiments, where the particles, including the neutrinos, collide or decay, the energy is much larger than the masses of neutrinos and the correspondence between theory and data is very good or good. Neutrinos also contributed to this success. The measurements in LEP in 1989 of the so-called invisible Z_0 boson decay width, interpreted as the Z_0 decays into unobserved neutrinos, has fixed the number of generations at three [32]. Moreover, this result excluded the existence of other neutrinos, which couple to Z_0 and have mass smaller than half of the Z_0 boson mass.

Since the first experiment of Reines and Cowan [9], where the electron antineutrino $\bar{\nu_e}$ was discovered, neutrinos have been observed continuously in many different processes. The cross-sections for (anti)neutrino + electron, (anti)neutrino + nucleon and (anti)neutrinos + nuclei are measured at different neutrino energies and for different final channels. All measured cross-sections agree with the SM predictions with massless neutrinos, so only upper limit for the neutrino masses can be found. The best upper limit for the neutrino effective mass (m_i are neutrino masses and U_{ei} are the elements of the mixing matrix, see the next section) [33]

$$m_\beta = \sqrt{\sum_{i=1,2,3} |U_{ei}|^2 m_i^2}\,, \qquad m_{\min} < m_\beta < m_{\max}\,, \tag{6}$$

has been found in a tritium β decay [34]

$$m_\beta < 2.2 \text{ eV} . \tag{7}$$

The problems of the SM began with the previously mentioned experiment in Homestake where Davis in 1968 installed the detector in order to catch neutrinos produced inside the Sun [25]. Just from the beginning it was observed that number of neutrinos detected in the Chlorine detector in Homestake was only one third of the one predicted by the so-called Standard Solar Model [35]. This discrepancy between the number of predicted neutrinos and the number measured in this first solar neutrino experiment was known as *The Solar Neutrino Problem*. Many physicists have not believed that the problem can be solved by changing the SM, but rather that the solution to the problem lies with a wrong neutrino flux given by the SSM, or a misinterpretation of the experiment. However, collected over many years the results from Homestake [36] and all subsequent experiments of KAMIOKANDE Collaboration [37], SAGE Collaboration [38] and GALLEX Collaboration [39] confirmed the first results of Davis. It is also worth to stress, that the water detector in Kamioka mine in Japan was not built to study neutrinos, but to look for proton decay. With time the main objective has been changed and Kamiokande and later SuperKamiokande became the most important neutrino detector which looked for the neutrino flavour change.

In the eighties the problem with neutrinos has become wider. Similar alarming phenomenon — the flavour change of neutrinos in the atmospheric neutrino flux — was observed. Several experimental groups have started to observe atmospheric neutrinos deficit [40–42] but not all [43,44] (for a review see [45]). After these experimental facts the situation was still not clear. The problem was resolved at the end of nineties. In 1998 the phenomenon of neutrino oscillation was definitively confirmed [46], showing that neutrinos have mass. This was the first evident indication, that the SM has to be extended. Finally, in 2002 the solar neutrino problem was ultimately resolved [47]. The Sudbury Neutrino Observatory (SNO) Collaboration made a unique measurement in which the total number of neutrinos (having energy above detector threshold) of all types (not only electron neutrinos) was observed. The longest running experiments, the solar neutrino experiments, have finished in 2002 after 34 years and have given spectacular success of the Solar Standard Model. The SNO together with the SuperKamiokande measurements (for recent result see [48] and [49]) show that most of the neutrinos produced in the interior of the Sun as electron neutrinos, are changed into muon and tau neutrinos by the time they reach the Earth. As predicted by B. Pontecorvo neutrinos oscillate. Presently the oscillation phenomena have been confirmed by accelerator neutrino K2K [50] and MINOS [51] as well as reactor neutrino experiments [52]. The last reactor experiment (KamLAND) was very important. Its results combined with all the earlier solar neutrino data established the correct parameters for the solar neutrino deficit. Taking into account all data, especially the SuperKamiokande and KamLAND, we have now very well evidence for neutrino disappearance and reappearance and all non-oscillations models are eliminated.

4. Neutrinos in the SM with small neutrino mass — the νSM

From various neutrino oscillation experiments it results that the SM must be extended at least in such a way, that neutrinos must be massive. There are many beyond the SM (BSM) theories which satisfy such a requirement. The simplest and popular scenario is such that the neutrino mass is the only one visible result of New Physics (NP) at very high scale (*e.g.* unification scale $\sim 10^{16}$ GeV), and all other BSM interaction of quarks and charged leptons are completely negligible at low, experimentally accessible energies. Such model is sometimes called the New SM = νSM. In such model the NP is *visible* by the neutrino mass Lagrangian and neutrino mixing matrix. The mass term and mixing matrix distinguish Dirac from Majorana neutrinos. In the case of Dirac neutrinos the mass term has the form

$$L_{\text{mass}}(D) = \sum_{i=1,2,3} m_i^D \left(\bar{\nu}_{iR}\nu_{iL} + \bar{\nu}_{iL}\nu_{iR}\right) . \tag{8}$$

For Majorana neutrino two kinds of mass Lagrangian are allowed, built using the left-handed chiral fields

$$L^{\text{L}}_{\text{mass}}(M) = \sum_{i=1,2,3} m^M_{Li} \left(\bar{\nu}^c_{iR}\nu_{iL} + \bar{\nu}_{iL}\nu^c_{iR}\right) , \qquad \nu^c_{iR} = i\gamma^2\nu^*_{iL} \tag{9}$$

and the right-handed fields

$$L^{\text{R}}_{\text{mass}}(M) = \sum_{i=1,2,3} m^M_{Ri} \left(\bar{\nu}^c_{iL}\nu_{iR} + \bar{\nu}_{iR}\nu^c_{iL}\right) , \qquad \nu^c_{iL} = i\gamma^2\nu^*_{iR} \, . \tag{10}$$

In the νSM the charged (4) and neutral (5) current Lagrangians have a new form

$$L^{\nu\text{SM}}_{\text{CC}} = \frac{e}{2\sqrt{2}\sin\theta_W} \sum_{\alpha,i} \bar{\nu}_i\gamma^\mu(1-\gamma_5)U^*_{\alpha i} l_\alpha W^+_\mu + \text{h.c.} \tag{11}$$

and for Z_0

$$L^{\nu\text{SM}}_{\text{NC}} = \frac{e}{4\sin\theta_W\cos\theta_W} \sum_{i=1,2,3} \bar{\nu}_i\gamma^\mu(1-\gamma_5)\nu_i Z_\mu \, . \tag{12}$$

As now neutrinos are massive particles the interaction with neutral Higgs particle appears

$$L_H^{\nu\mathrm{SM}} = \frac{e}{2\sin\theta_W} \sum_{i=1,2,3} \left(\frac{m_i}{M_W}\right) \bar{\nu}_i \nu_i H \,, \tag{13}$$

but the ratio $\frac{m_i}{M_W} \ll 1$, and the neutrinos coupling to Higgs particles (Eq. (13)) is negligible small. In such models the lepton flavour numbers are not conserved separately, and neutrinos can oscillate. The total lepton number is (Dirac neutrinos) or is not (Majorana neutrino) conserved. The CP symmetry is broken if the complex phases in the U mixing matrix are different than $\delta_{\mathrm{CP}}, \phi_1, \phi_2 \neq 0, \frac{\pi}{2}, \pi$ (for the present parametrisation of the MNSP mixing matrix see Ref. [53]).

The full theory of neutrino oscillation was worked out in several papers [54], although as we can see from the amount of publications that are constantly emerging, it is still a moot (see *e.g.* [55]). Without going into details[3], we can understand the neutrino oscillation as a typical phenomenon of relativistic quantum mechanics, in which the states of particles with different masses may be added in a coherent way and interfere[4]. This in short can be summarized in the following way:

(i) In each production process, which in the lowest order is described by the CC Lagrangian (11), neutrinos with different masses (m_i) are produced.

(ii) In any realistic production process the mass differences $|m_i - m_k|$ for any two produced neutrinos is much smaller than the neutrino mass uncertainty, determined from measured energies and momenta of all particles in the production/detection process without neutrinos

$$|m_i - m_k| \ll \Delta m \equiv \frac{E\Delta E + p\Delta p}{\sqrt{E^2 - p^2}} \,, \tag{14}$$

where $E, (\Delta E)$ and $p, (\Delta p)$ are energy and momentum of neutrino and their uncertainties.

(iii) From the Lagrangian (11) it follows, that any flavour state $\alpha = e, \mu, \tau$ of produced or detected neutrinos[5] is the linear combination of the mass states

[3] The full description of the oscillation process need the use of the wave packet for all particles which appear in the neutrino production and detection process, see *e.g.* [56].

[4] In the nonrelativistic quantum mechanics, particles of different masses belong to separate Hilbert spaces, and do not interfere.

[5] The neutrino flavour is determined by the charged leptons, which appear in the production or detection processes.

$$|\nu_\alpha, \downarrow\rangle = \sum_i U^*_{\alpha,i} |\nu_i, \downarrow\rangle\,, \qquad |\overline{\nu}_\alpha, \uparrow\rangle = \sum_i U_{\alpha,i} |\overline{\nu}_i, \uparrow\rangle\,, \tag{15}$$

where the arrows ($\downarrow\uparrow$) denote the helicities of neutrino (antineutrino) which all the time in the oscillation process do not change.

(iv) If neutrinos α are produced in the production point, and placed at a distance L a detector is looking for β flavour neutrinos, then the amplitude for flavour $\alpha \to \beta$ change is given by

$$A_{\alpha\to\beta}(E,L) = \left\langle \nu_\beta \left| e^{-iHt} \right| \nu_\alpha \right\rangle\,, \tag{16}$$

where H is the Hamiltonian which describes neutrino propagation in vacuum or in matter[6].

(v) Then the probability of the neutrino flavour change after propagation of distance L in the vacuum, is given by

$$\begin{aligned} P_{\alpha\to\beta}(E,L) &= |A_{\alpha\to\beta}(E,L)|^2 \\ &= \left| \sum_i \sum_k U_{\beta i} U^*_{\alpha k} \left\langle \nu_i \left| e^{-i\sqrt{E_i^2+p_i^2}\frac{L}{v_i}} \right| \nu_k \right\rangle \right|^2 \\ &= \sum_i \sum_k U_{\alpha i} U_{\beta k} U^*_{\alpha k} U^*_{\beta i} e^{i\frac{\delta m^2_{ik} L}{2E}}\,, \end{aligned} \tag{17}$$

where $\delta m^2_{ik} = m_i^2 - m_k^2$ and E is the average energy of neutrinos.

We see why it was so difficult to get any information that neutrinos are massive particles. Independently how small the difference of a neutrino mass square (δm^2_{ik}) is, the other factor L/E depends on our choice and can be large, such that the total phase ($\delta m^2_{ik} L/2E$) is large too, and the effect of neutrino oscillation can be visible. In any laboratory experiment the neutrino detection cross-section *e.g.* on electron $\sigma(\nu_e + e^- \to \nu_e + e^-) \approx 9.5 \times 10^{-49} \left(\frac{E_\nu}{1\ \mathrm{MeV}}\right)$ m^2 or for inverse β decay process $\sigma(\nu_e + n \to e^- + p) \approx 9.3 \times 10^{-48} \left(\frac{E_\nu}{1\ \mathrm{MeV}}\right)$ m^2 is very small and proportional to the neutrino energy in Lab system. So practically detected neutrinos are relativistic, $(m_\nu/E_\nu) \to 0$. Therefore, in any laboratory process where neutrinos are observed, neutrino masses can be neglected. The observed family lepton numbers L_α conservations follow from unitarity of the MNSP mixing matrix.

[6] The presented way of finding the amplitudes works only in frame of the νSM. If in a neutrino production and/or detection processes beyond the SM interactions play a role, a more complicated formalism for neutrino oscillation have to be used (see *e.g.* [57]).

Also the so-called *confusion theorem* was proven [58], which states that differences in all observables for Dirac and Majorana neutrinos due to the different mass Lagrangians (8), (9), (10) smoothly disappear for $m_i \to 0$.

From present experimental data we have information about neutrino masses and about the elements of the MNSP mixing matrix. Direct information about neutrino masses come from the tritium β decay and are given by Eq. (7). Some information also come from a neutrinoless double β decay [59], which can occur only if neutrino are Majorana particles, and if such decay is observed, it is possible to measure the other effective neutrino mass

$$\langle m_{0\nu} \rangle = \left| \sum_{i=1,2,3} U_{ei}^2 m_i \right| , \qquad \langle m_{0\nu} \rangle < m_{\max} . \tag{18}$$

Latest experimental results from the CUORICINO [60] give

$$\langle m_{0\nu} \rangle < 0.19 - 0.68 \text{ eV} \Longrightarrow m_{\max} > 0.68 \text{ eV} , \tag{19}$$

from which we can conclude that the mass of the heaviest neutrino $m_{\max} >$ 0.68 eV. These results have very large systematic error which emerges from large discrepancy between different nuclear matrix element calculations[7]. From neutrino oscillation experiments we also know two differences of neutrino masses squared. The last global fits [61] give

$$\delta m_{21}^2 = m_2^2 - m_1^2 = (7.05 - 8.34) \times 10^{-5} \text{ eV}^2 , \tag{20}$$

$$\left|\delta m_{31}^2\right| = \left|m_3^2 - m_1^2\right| = (2.07{-}2.75)\times 10^{-3} \text{ eV}^2 \Longrightarrow m_{\max} > 0.045 \text{ eV} . \tag{21}$$

From (21) it follows that the mass of the heaviest neutrino must be $m_{\max} >$ 0.045 eV.

The elements of the unitary mixing matrix $U_{\alpha i}$ are parametrized by the three angles $\theta_{12}, \theta_{23}, \theta_{13}$ and one CP violating phase δ. Currently not all these parameters are known. Combined data give [61] (with 3σ interval)

$$\sin^2\theta_{12} \in (0.25 - 0.37) , \quad \sin^2\theta_{23} \in (0.36 - 0.67) , \quad \sin^2\theta_{13} < 0.056 . \tag{22}$$

The result of atmospheric, solar, reactor (KamLAND) and accelerator (K2K and MINOS) neutrino experiments are very well explained by the neutrino oscillations in the framework of the three neutrino mixing. We have a rather precise knowledge of the values of squared-mass difference δm_{21}^2, the absolute value of $|\delta m_{31}^2|$ and the values of two mixing angles θ_{12} and θ_{23}. We expect that the next generation of different experiments

[7] CUORICINO experiment measure the decay timelife of ^{130}Te and they found $T_{1/2}^{0\nu} >$ 3.0×10^{34} y (90% C.L.).

will give us information about: *(i)* the absolute scale of neutrino mass connected with the spectrum of masses (normal hierarchy, inverted hierarchy, degenerate)[8], *(ii)* nature of neutrinos (are they Dirac or Majorana particles), *(iii)* value of θ_{13} mixing angle (is θ_{13} close to zero or rather close to the upper limit)[9] and finally, *(iv)* the CP violating phases ($\delta_{\rm CP}$ — the only one for Dirac neutrinos, or additional two, ϕ_1, ϕ_2 for Majorana neutrinos). It is worth paying attention to the fact that the masses of some neutrinos can be smaller then the experimental error of the charged lepton masses (for electron $(\Delta m_e)_{\rm exp} = 0.013$ eV).

5. Beyond the SM

The very small mass of neutrinos and the completely different leptonic mixing matrix in comparison to quarks require a modification of the SM and some New Physics (NP) beyond the SM must be found. Unfortunately, data are not precise enough to indicate which NP model should be chosen. In the previous section we have considered the NP at the unification scale (10^{15} GeV), now we will concentrate on a NP which appears in the present available energy scale. There are many hints that really NP operates at a 0 (TeV) scale [64]. Such NP is much more interesting, there is a chance to discover it at the LHC, and next high energy machines (*e.g.* ILC) or at the future more precise neutrino experiments. Problem of neutrino mass and mixing can refer to the unification scale as well as to the 0 (TeV) scale. If a NP modifies the neutrino interaction at 0 (TeV) scale then, as we mentioned before, the description of oscillation must be modified too. We shortly discuss ourselves to that.

5.1. Neutrino mass and mixing

Neutrino masses are much smaller than the masses of charged leptons and quarks. For mixing angles it is opposite, there are two large mixing angles for leptons which contrast sharply with the smallness of the quark mixing angles. We would like to know why it is so. On the other hand, the problem of particle masses waits for a solution. Why do we try to solve separately the neutrino mass and the flavour problem? The ratio of the electron mass to neutrino masses $\frac{m_\nu}{m_e} \leq 10^{-6}$ is almost the same as the ratio of the top quark to electron $\frac{m_e}{m_t} \simeq 10^{-5}$. There are several reasons why the smallness of neutrinos masses is interesting. Firstly, the smallness of neutrino mass remains a question even within one family. Quark mass ratio

[8] Up to now we have information that heaviest neutrino mass is in the range 0.05 eV (0.68 eV)$< m_{\rm max} < 2.2$ eV.

[9] Last data including the results from Borexino experiment [62] found non zero value for $\theta_{13}, \sin^2\theta_{13} = 0.0095^{+0.013}_{0.007}$ [63].

in the same family is about 10, while for the same lepton generation the mass ratio is smaller than 10^{-6}. Secondly, the problem of neutrino mass may be connected with their nature. The quarks and charged leptons are Dirac particles. Neutrinos have probably a Majorana nature. And finally, even if the problem of mass is not resolved, the large difference for lepton masses within a single family and completely different structure of the mixing matrix[10] can shed a light on the extension of the SM. This is probably the main reason why the problem of neutrino mass, usually connected with the flavour problem, is so intensively studied in recent years (see *e.g.* [65], for more complete list see [66]).

The simplest way to get massive neutrinos is to add to the SM fields N right-handed chiral neutrino fields ($\nu_{\beta R}, \beta = 1, 2, \ldots N$) and to introduce the neutrino masses in the same way as for the quarks and charged leptons

$$L_Y = -\sum_{\alpha,\beta} f_{\alpha,\beta} \tilde{\psi}_{\alpha L}(-i\sigma_2\varphi^*)\nu_{\beta R} + \text{h.c.}\,, \tag{23}$$

where $\tilde{\psi}_{\alpha L}$ and φ are SU(2) doublets fields of leptons and Higgs particles respectively. There is no fundamental reason why we cannot do that, but we do not like this solution. Neutrino mass matrix is proportional to the Yukawa couplings $f_{\alpha,\beta}$ and there is no good reason why these couplings must be so small. Such a solution does not give any indication how to extend the SM.

The other possibility is to add to the previous model the right-handed mass term

$$L_{\text{RH}} = -\tfrac{1}{2}\sum_{\alpha,\beta} g_{\alpha,\beta} \tilde{\nu}^c_{\alpha L}\nu_{\beta R} + \text{h.c.} \tag{24}$$

Now we have three possibilities. The most popular is the so-called *see–saw mechanism*. The $g_{\alpha,\beta}$ Yukawa constants are very large ($|g_{\alpha,\beta}| \gg |f_{\alpha,\beta}|$), then for $N = 3$ we can get three light and three heavy Majorana particles and B–L symmetry is broken. As usually, if two very different scales exist, we meet with the hierarchy problem. If the large scale has a quantum gravity range, neutrinos obtain too small masses, $m \sim 10^{-5}$ eV. The next possibility is the case where $g_{\alpha,\beta}$ are very small ($|g_{\alpha,\beta}| \ll |f_{\alpha,\beta}|$), then the so-called *pseudo-Dirac neutrino* scenario is realized [67]. The neutrinos are almost Dirac particles with very tiny amount of the Majorana mass. It was found that then the Yukawa coupling must be very small in order to be consistent with current solar neutrino observation [68]. Recently, the third possibility

[10] For the quark mixing, the non-diagonal elements of the CKM mixing matrix are very small, which is presumably due to small ratios of the quark masses $m_c/m_t, m_u/m_c, m_s/m_b, m_d/m_s$, such relations between elements of the MNSP matrix and the ratios of neutrino masses do not exist.

was considered where the Yukawa constants $f_{\alpha,\beta}$ in (23) and $g_{\alpha,\beta}$ are of the same order ($|g_{\alpha,\beta}| \simeq |f_{\alpha,\beta}|$) [69], then some mass states can be Dirac and the others Majorana. The flavour neutrinos which are combination of two Dirac and one Majorana or one Dirac and two Majorana neutrinos was called *schizophrenic neutrinos*.

In the way presented up to now we were able to give mass to neutrinos without a systematic knowledge on how the SM must be extended. There are a lot of various models which in a better or worse way explain small neutrino masses and large two mixing angles. The first option is to continue to maintain symmetry of the SM and, *(i)* modify the fermion sector, *(ii)* enlarge the Higgs sector, and *(iii)* break spontaneously the B–L symmetry (Majoron(s) appears). The first possibility, as we have discussed previously, was not satisfactory. There are three working ways of the Higgs sector enlargement, where (1) additional Higgs triplet Δ, (2) singly charged singlet, h_{-}, or (3) doubly charged gauge singlet k_{++} are introduced. These possibilities are very popular. In models with the Higgs triplet the see–saw mechanism is operating. Models with additional singlets, invented by Zee and Babu [70], are very interesting as NP appears at TeV scale and there is a chance to see some implication at LHC. Neutrino masses are small, as they are generated at either one or two loops. There are also two different realizations of models with Majorons, (1) a gauge singlet and additional right-handed neutrinos are introduced, or (2) the only Higgs sector is extended by adding a Higgs triplet and a singly charged scalar.

The second option is to abandon the symmetry group of the SM and build a model which at low-energy has all features of the SM. Several such models are considered in the literature, *(i)* new gauge group $\mathrm{SU_L}(2) \otimes \mathrm{SU_R}(2) \otimes \mathrm{U}(1)_{\mathrm{B-L}}$ with two Higgs doublets or Higgs doublets and triplets, *(ii)* models of grand unification based on SU(5), SO(10) or E_6 symmetry group, *(iii)* supersymmetric models in several versions, the MSSM, the model with broken R-parity and models based on the supersymmetric Left–Right group.

The next problem is connected with the specific structure of the flavour mixing matrix the MNSP matrix. In order to understand the large values of mixing angles θ_{12}, θ_{23} and much smaller angle θ_{13}, special flavour symmetry are usually imposed in the models.

Despite some successes in understanding of the problem of the small neutrino masses and the large mixing angles, it is difficult to accept that this problem is solved. From the case considered at the beginning of this section we see that so different scenarios of the neutrino mass matrix still agree with experimental data (see–saw mechanism, pseudo-Dirac neutrinos, schizophrenic neutrinos). This situation is probably connected with still too poor experimental knowledge of the neutrino masses and mixing matrix elements and the selection of the best theoretical model is difficult.

5.2. Neutrino oscillation beyond the SM

The original description of the neutrino oscillation phenomena, as we show before, was introduced in the mid seventies of the last century [54]. Such description works well in the case of νSM but does not work if the NP modify neutrino production and detection processes.

Recently a full description has been proposed, which may be used not only for the νSM but can be applied for any model of neutrino interactions, and in which the neutrinos propagate over long distances on mass shell [57]. The neutrino states are obtained from the dynamics of a production/detection process and the entanglement between produced/detected particles is taken into account. This new approach is presented shortly. As a production process the three body decay (*e.g.* muon or nuclear β decay) is considered

$$A \to B + \bar{l}_\alpha + \nu_i(\lambda)\,. \tag{25}$$

The state of produced neutrinos in this process, in the rest frame of the decaying particle A, is described by the density matrix which depends on the dynamics of the process (25). In the base where the neutrino mass (m_i) and helicity λ are specified ($|\nu_i, \lambda\rangle$), the density matrix is given by the well known formula

$$\varrho^\alpha(\lambda, i; \eta, k; E, \theta, \varphi) = \frac{1}{N_\alpha}\sum_{\text{spins}}\int \overline{d\text{Lips}} A_i^\alpha(\lambda_A; \lambda_B, \lambda_l, \lambda; E, \theta, \varphi)\, \varrho_{\lambda_A,\lambda_{A'}} A_k^{\alpha *}\left(\lambda_{A'}; \lambda_B, \lambda_l, \eta; E, \theta, \varphi\right)\,, \tag{26}$$

where the integral $\overline{d\text{Lips}}$ is taken over the part of the phase space, without neutrinos energy (E) and its momentum direction (θ, φ), the $\varrho_{\lambda_A,\lambda_{A'}}$ is the density matrix which describes the polarization of decaying particle (A) and the factor N_α normalizes the density matrix, such that $\text{Tr}\varrho = 1$.

Let us assume that in the detection process the lepton of flavour β is produced in our detector

$$\nu_i + C \to l_\beta + D\,, \tag{27}$$

then the total cross-section for neutrino detection is calculated in the usual way

$$\sigma_{\alpha\to\beta}(E, L) = \frac{1}{64\pi^2 s}\frac{p_f}{p_i}\frac{1}{2s_C+1}\sum_{\text{spins,masses}}\int d\Omega\; f_i^\beta(\lambda)\varrho^\alpha(L; i, \lambda; k, \eta)\; f_k^{\beta *}(\eta)\,, \tag{28}$$

where the $f_i^\beta(\lambda)$ are spin amplitudes for the detection process (27) of neutrino with mass m_i and helicity λ. The $\varrho^\alpha(L; i, \lambda; k, \eta)$ is the density matrix after neutrino propagation, calculated in the following way

$$\varrho^\alpha(E, L) = e^{-iHL} \varrho^\alpha(E, L = 0) e^{iHL} \,. \tag{29}$$

In such proposed approach, depending on the neutrino interaction in the production process, the initial neutrino state can be pure, as in the νSM, or mixed. The final formula for the detection rate does or does not factorize to neutrino oscillation probability times detection cross-section (for details see [57]).

6. Conclusions

Some selected topics from the neutrino history have been described, mainly from the past 50 years. Neutrinos have always given new and unexpected information about elementary interaction. Neutrinos are very special because they only weakly interact. For this reason, in a special way helped in the formulation of the theory of electroweak interactions. A few experiments, which were prepared for other purposes, after some time have begun to explore properties of neutrinos. In such, somewhat accidental way, experiments began, which finally led to the discovery of neutrino mass. The disclosure that neutrinos are massive particle is probably one of the most important discoveries in particle physics in recent years. Although it did not change much in the laboratory experiments, where very small neutrino mass does not play a role, the discovery is of great importance for particle physics, astrophysics and cosmology. In physics of elementary interaction, neutrinos has opened the window into phenomena beyond the SM. In astrophysics, neutrinos allow "glimpses of the interiors of stars" and to verify the theory of the processes taking place inside, *e.g.* verify the Standard Solar Model or different models of supernova explosion. In cosmology, once we manage to develop a detecting method of relic neutrinos, which posses a very low-energy, neutrinos will examine perhaps the evolution of the Universe in the first seconds after the Big Bang. Heavy neutrinos may help to solve the riddle of Dark Matter, and understand the problem of barion asymmetry via leptogenesis. To answer many of these questions better information about neutrino properties are needed. Many of neutrino experiments are still collecting data, new experiments are planned. There are many"working groups" which discuss the new experiments with very intensive beam of neutrinos (beta beams, superbeams, neutrino factories) and larger and better detectors. The field is extremely active. It seems that next years will be very interesting in the physics of neutrinos.

It is a great pleasure to thank the organizers of the L Cracow School of Theoretical Physics for invitation and opportunity to present this material. This work has been supported by the Polish Ministry of Science and Higher Education under grant No. N N202 064936. The author would like to thank the colleagues from the Department of Theoretical Physics and Cosmology at the University of Granada in Spain for the pleasant atmosphere and hospitality during the stay, to F. del Aguila and A. Bueno for many valuable remarks, and the Junta de Andalucía for support (FQM 03048).

REFERENCES

[1] W. Pauli letter of the 4th of December 1930, Pauli Archive at CERN.

[2] E. Fermi, *Nuovo Cim.* **11**, 1 (1934).

[3] H. Bethe, R. Peierls, *Nature* **133**, 532 (1934).

[4] S.H. Neddermeyer, C.D. Anderson, *Phys. Rev.* **51**, 884 (1937).

[5] E. Majorana, *Nuovo Cim.* **14**, 171 (1937).

[6] H. Weyl, *Z. Phys.* **56**, 330 (1929).

[7] G. Puppi, *Nuovo Cim.* **5**, 587 (1948); J. Tiomno, J.A. Wheeler, *Rev. Mod. Phys.* **21**, 144 (1949); T.D. Lee, M. Rosenbluth, C.N. Yang, *Phys. Rev.* **75**, 905 (1949).

[8] E.J. Konopinski, H.M. Mahmoud, *Phys. Rev.* **92**, 1045 (1953).

[9] C.L. Cowan *et al.*, *Science* **124**, 103 (1956).

[10] T.D. Lee, C.-N. Yang, *Phys. Rev.* **104**, 254 (1956).

[11] C.S. Wu *et al.*, *Phys. Rev.* **105**, 1413 (1957).

[12] R.P. Feynman, M. Gell-Mann, *Phys. Rev.* **109**, 193 (1958); E.C.G. Sudarshan, R.E. Marshak, *Phys. Rev.* **109**, 1860 (1958); J.J. Sakurai, *Nuovo Cim.* **7**, 649 (1958).

[13] L.D. Landau, *Nucl. Phys.* **3**, 127 (1957); T.D. Lee, C.-N. Yang, *Phys. Rev.* **105**, 1671 (1957); A. Salam, *Nuovo Cim.* **5**, 299 (1957).

[14] W. Pauli, *Nuovo Cim.* **6**, 204 (1957); I.A. McLennan Jr., *Phys. Rev.* **106**, 821 (1957); K.M. Case, *Phys. Rev.* **107**, 307 (1957); F. Gursey, *Nuovo Cim.* **7**, 411 (1958).

[15] M. Goldhaber, L. Grodzins, A.W. Sunyar, *Phys. Rev.* **109**, 1015 (1958).

[16] B. Pontecorvo, *Sov. Phys.—JETP* **6**, 429 (1957) [*Zh. Eksp. Teor. Fiz.* **33**, 549 (1957)].

[17] B. Pontecorvo, *Sov. Phys.—JETP* **7**, 172 (1958) [*Zh. Eksp. Teor. Fiz.* **34**, 247 (1958)].

[18] R. Davis Jr., *Phys. Rev.* **97**, 766 (1955).

[19] G. Feinberg, *Phys. Rev.* **110**, 1482 (1958).

[20] B. Pontecorvo, *Sov. Phys.—JETP* **10**, 1236 (1960) [*Zh. Eksp. Teor. Fiz.* **37**, 1751 (1959)].

[21] G. Danby *et al.*, *Phys. Rev. Lett.* **9**, 36 (1962).

[22] Z. Maki, M. Nakagawa, S. Sakata, *Prog. Theor. Phys.* **28**, 870 (1962).

[23] B. Pontecorvo, *Sov. Phys.—JETP* **26**, 984 (1968) [*Zh. Eksp. Teor. Fiz.* **53**, 1717 (1967)].

[24] V.N. Gribov, B. Pontecorvo, *Phys. Lett.* **B28**, 493 (1969).

[25] R. Davis Jr., D.S. Harmer, K.C. Hoffman, *Phys. Rev. Lett.* **20**, 1205 (1968).

[26] H.A. Bethe, *Phys. Rev.* **55**, 434 (1939).

[27] S.L. Glashow, *Nucl. Phys.* **22**, 579 (1961); J. Goldston, A. Salam, S. Weinberg, *Phys. Rev.* **127**, 965 (1962); S. Weinberg, *Phys. Rev. Lett.* **19**, 1264 (1967).

[28] J.E. Augustin *et al.*, *Phys. Rev. Lett.* **33**, 1406 (1974); J.J. Aubert, *Phys. Rev. Lett.* **33**, 1404 (1974); S.W. Herb *et al.*, *Phys. Rev. Lett.* **39**, 252 (1977); F. Abe *et al.*, *Phys. Rev. Lett.* **74**, 2626 (1995), S. Abachi *et al.*, *Phys. Rev. Lett.* **74**, 2422 (1995).

[29] M.L. Perl *et al.*, *Phys. Rev. Lett.* **35**, 1489 (1975).

[30] [DONUT Collaboration], *Phys. Lett.* **B504**, 218 (2001).

[31] F.J. Hasert *et al.* [Gargamelle Collaboration], *Phys. Lett.* **B46**, 138 (1973).

[32] [ALEPH, DELPHI, L3, OPAL, and SLD collaborations], *Phys. Rep.* **427**, 257 (2006).

[33] J. Studnik, M. Zralek, `arXiv:hep-ph/0110232v2`.

[34] J. Bonn *et al.*, *Nucl. Phys.* **91**, 273 (2001).

[35] J.N. Bahcall, M.H. Pinsonneault, *Phys. Rev. Lett.* **92**, 121301 (2004).

[36] B.T. Cleveland *et al.*, *Astrophys. J.* **496**, 505 (1998).

[37] [KAMIOKANDE Collaboration], *Phys. Rev. Lett.* **63**, 6 (1989); *Phys. Rev. Lett.* **65**, 1297 (1990); *Phys. Rev. Lett.* **65**, 1301 (1990); *Phys. Rev. Lett.* **66**, 9 (1991); *Phys. Rev.* **D44**, 2241 (1991).

[38] [SAGE Collaboration], *Phys. Rev. Lett.* **67**, 3332 (1991); *Phys. Lett.* **B328**, 234 (1994); *Phys. Rev. Lett.* **83**, 4686 (1990); *Phys. Rev.* **C60**, 055801 (1990).

[39] [GALLEX Collaboration], *Phys. Lett.* **B285**, 376 (1992); *Phys. Lett.* **B285**, 390 (1992); *Phys. Lett.* **B357**, 237 (1995).

[40] K.S. Hirata *et al.*, *Phys. Lett.* **B205**, 416 (1988).

[41] D. Casper *et al.*, *Phys. Rev. Lett.* **66**, 2561 (1991); R. Becker-Szendy *et al.*, *Phys. Rev.* **D46**, 3720 (1992).

[42] W.W.M. Allison *et al.*, *Phys. Lett.* **B391**, 491 (1997).

[43] M. Aglietta *et al.*, *Europhys. Lett.* **8**, 611 (1989).

[44] K. Daum *et al.*, *Z. Phys.* **C66**, 417 (1995).

[45] T. Kajita, Y. Totsuka, *Rev. Mod. Phys.* **73**, 85 (2001).

[46] Y. Fukuda *et al.*, *Phys. Rev. Lett.* **81**, 1562 (1998).

[47] Q.R. Ahmed *et al.*, *Phys. Rev. Lett.* **89**, 011302 (2002).

[48] J. Hosaka *et al.*, *Phys. Rev.* **D73**, 112001 (2006); J.P. Cravens *et al.*, *Phys. Rev.* **D78**, 032002 (2008); K. Abe *et al.*, `arXiv:1010.0118v1 [hep-ex]`.

[49] B. Aharmim *et al.*, *Astrophys. J.* **710**, 540 (2010); B. Aharmim *et al.*, *Phys. Rev.* **C81**, 055504 (2010).

[50] M.H. Ahn *et al.*, *Phys. Rev. Lett.* **90**, 041801 (2003); S. Yamamoto *et al.*, *Phys. Rev. Lett.* **96**, 181801 (2006); M.H. Ahn *et al.*, *Phys. Rev.* **D74**, 072003 (2006).

[51] D.G. Michael *et al.*, *Phys. Rev. Lett.* **97**, 19180 (2006); P. Adamson *et al.*, *Phys. Rev.* **D77**, 072002 (2008); P. Adamson *et al.*, *Phys. Rev.* **D82**, 051102 (2010).

[52] K. Eguchi *et al.*, *Phys. Rev. Lett.* **90**, 021802 (2003); S. Abe *et al.*, *Phys. Rev. Lett.* **100**, 221803 (2008).

[53] K. Nakamura *et al.* [Particle Data Group], *J. Phys. G* **37**, 075021 (2010).

[54] S.M. Bilenky, B. Pontecorvo, *Phys. Lett.* **B61**, 248 (1976); *Lett. Nuovo Cim.* **17**, 569 (1976); *Phys. Rep.* **41**, 225 (1978); S. Eliezer, A. Swift, *Nucl. Phys.* **B105**, 45 (1976); H. Fritzsch, P. Minkowski, *Phys. Lett.* **B62**, 72 (1976).

[55] B. Kayser, *Phys. Rev.* **D24**, 110 (1981); C. Giunti, *J. High Energy Phys.* **11**, 017 (2002); A.G. Cohen, S.L. Glashow, Z. Ligeti, *Phys. Lett.* **B678**, 191 (2009); E.Kh. Akhmedov, J. Kopp, *J. High Energy Phys.* **04**, 008 (2010); E.Kh. Akhmedov, A.Yu. Smirnov, `arXiv:1008.2077v3 [hep-ph]`.

[56] C. Giunti, C.W. Kim, *Fundamentals of Neutrino Physics and Astrophysics*, Oxford University Press, Oxford, UK, 2007.

[57] M. Ochman, R. Szafron, M. Zralek, *J. Phys. G* **35**, 065003 (2008); R. Szafron, M. Zralek, *Prog. Part. Nucl. Phys.* **64**, 210 (2010); R. Szafron, M. Zralek, `arXiv:1010.6034v1 [hep-ph]`.

[58] L.F. Li, F. Wilczek, *Phys. Rev.* **D25**, 143 (1982); B. Kayser, R.E. Shrock, *Phys. Lett.* **B112**, 137 (1982); B. Kayser, *Phys. Rev.* **D26**, 1662 (1982).

[59] W.H. Furry, *Phys. Rev.* **56**, 1184 (1939).

[60] C. Arnaboldi *et al.*, *Phys. Rev.* **C78**, 035502 (2008).

[61] G.L. Fogli *et al.*, *Phys. Rev. Lett.* **101**, 141801 (2008); T. Schwetz, M. Tortola, J.W.F. Valle, *New J. Phys.* **10**, 113011 (2008); see also, M.C. Gonzalez-Garcia, M. Maltoni, *Phys. Rep.* **460** 1 (2008).

[62] G. Bellini *et al.* [BOREXINO Collaboration], *Phys. Rev.* **D82**, 033006 (2010).

[63] M.C. Gonzalez-Garcia, M. Maltoni, J. Salvado, *J. High Energy Phys.* **04**, 056 (2010).

[64] J.D. Lykken, `arXiv:1005.1676v1 [hep-ph]`; J. Ellis, *Int. J. Mod. Phys.* **A25**, 2409 (2010), `arXiv:1004.0648v1 [hep-ph]`; G. Altarelli, `arXiv:1002.4957v1 [hep-ph]`.

[65] A.E. Bernardini, `arXiv:1011.0768v1 [astro-ph.CO]`; Y. Koide, `arXiv:1011.1064v1 [hep-ph]`; S. Ray, *Int. J. Mod. Phys.* **A25**, 4339 (2010); G. Altarelli, F. Feruglio, `arXiv:1002.0211v2 [hep-ph]`; M. Morisi, E. Peinado, *Phys. Rev.* **D81**, 085015 (2010); Y. Lin, *Nucl. Phys.* **B813**, 91 (2009); L. Yin, *Phys. Rev.* **D80**, 076011 (2009) [`arXiv:0903.0831v3 [hep-ph]`]; S.F. King, C. Luhn, *J. High Energy Phys.* **10**, 093 (2009) [`arXiv:0908.1897v2 [hep-ph]`]; I.T. Dyatlov, `arXiv:0910.0153v1 [hep-ph]`; Z.-Q. Guo, B.-Q. Ma, *J. High Energy Phys.* **09**, 091 (2009)

[arXiv:0909.4355v1 [hep-ph]]; A. Albaid, *Phys. Rev.* **D80**, 093002 (2009); D.A. Eby, P.H. Frampton, S. Matsuzaki, *Phys. Rev.* **D80**, 053007 (2009) [arXiv:0907.3425v1 [hep-ph]]; F. Bazzocchi, I. de Medeiros Varzielas, *Phys. Rev.* **D79**, 093001 (2009) [arXiv:0902.3250v1 [hep-ph]]; A. Blum, C. Hagedorn, *Nucl. Phys.* **B821**, 327 (2009); G. Altarelli, D. Meloni, *J. Phys. G* **36**, 085005 (2009); G. Altarelli, F. Feruglio, L. Merlo, *J. High Energy Phys.* **05**, 020 (2009).

[66] M. Zralek, *Acta Phys. Pol. B* **41**, 1477 (2010); C. Giunti, M. Laveder, http://www.nu.to.infn.it/

[67] L. Wolfenstein, *Nucl. Phys.* **B186**, 147 (1981); S.T. Petcov, *Phys. Lett.* **B110**, 245 (1982); M. Doi *et al.*, *Prog. Theor. Phys.* **70**, 1331 (1983).

[68] A. de Gouvea, W.C. Huang, J. Jenkins, *Phys. Rev.* **D80**, 073007 (2009).

[69] R. Allahverdi, B. Dutta, R.N. Mohapatra, arXiv:1008.1232v2 [hep-ph].

[70] A. Zee, *Phys. Lett.* **B93**, 389 (1980) [Erratum-ibid. **B95**, 461 (1980)]; K.S. Babu, *Phys. Lett.* **B203**, 132 (1988).