b-QUARK HADRONS — A THEORETICAL LABORATORY FOR COLOR MAGNETIC INTERACTION*

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I discuss several recent highly accurate theoretical predictions for masses of baryons containing the b quark, especially Ω_b (ssb) recently reported by CDF. I also point out an approximate effective supersymmetry between heavy quark baryons and mesons and provide predictions for the magnetic moments of Λ_c and Λ_b. Proper treatment of the color-magnetic hyperfine interaction in QCD is crucial for obtaining these results.

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1. Introduction

QCD describes hadrons as valence quarks in a sea of gluons and $\bar{q}q$ pairs. At distances above ~ 1 GeV^{-1} quarks acquire an effective *constituent mass* due to chiral symmetry breaking. A hadron can then be thought of as a bound state of constituent quarks. In the zeroth-order approximation the hadron mass M is then given by the sum of the masses of its constituent quarks m_i

$$M = \sum_i m_i \,.$$

The binding and kinetic energies are "swallowed" by the constituent quarks masses. The first and most important correction comes from the color Hyper-Fine (HF) chromo-magnetic interaction

$$M \;=\; \sum_i m_i + V_{i<j}^{\mathrm{HF(QCD)}} \,,$$

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$$V_{ij}^{\mathrm{HF(QCD)}} = v_0 \left(\vec{\lambda}_i \cdot \vec{\lambda}_j\right) \frac{\vec{\sigma}_i \cdot \vec{\sigma}_j}{m_i m_j} \langle\psi|\delta\left(r_i - r_j\right)|\psi\rangle\,, \tag{1}$$

where v_0 gives the overall strength of the HF interaction, $\vec{\lambda}_{i,j}$ are the SU(3) color matrices, $\sigma_{i,j}$ are the quark spin operators and $|\psi\rangle$ is the hadron wave function. This is a contact spin–spin interaction, analogous to the EM HF interaction, which is a product of the magnetic moments

$$V_{ij}^{\mathrm{HF(QED)}} \propto \vec{\mu}_i \cdot \vec{\mu}_j = e^2 \frac{\vec{\sigma}_i \cdot \vec{\sigma}_j}{m_i m_j} \tag{2}$$

in QCD, the $\mathrm{SU(3)}_c$ generators take place of the electric charge. From Eq. (1) many very accurate results have been obtained for the masses of the ground-state hadrons. Nevertheless, several caveats are in order. First, this is a low-energy phenomenological model, still awaiting a rigorous derivation from QCD. It is far from providing a complete description of the hadronic spectrum, but it provides excellent predictions for mass splittings and magnetic moments. The crucial assumptions of the model are:

(a) HF interaction is considered as a perturbation which does not change the wave function,

(b) effective masses of quarks are the same inside mesons and baryons,

(c) there are no 3-body effects.

2. Quark masses

As the first example of the application of Eq. (1) we can obtain the $m_c - m_s$ quark mass difference from the $\Lambda_c - \Lambda$ baryon mass difference

$$\begin{aligned} M(\Lambda_c) - M(\Lambda) &= \left(m_u + m_d + m_c + V_{ud}^{\mathrm{HF}} + V_{uc}^{\mathrm{HF}} + V_{dc}^{\mathrm{HF}}\right) \\ &\quad -\left(m_u + m_d + m_s + V_{ud}^{\mathrm{HF}} + V_{us}^{\mathrm{HF}} + V_{ds}^{\mathrm{HF}}\right) \\ &= m_c - m_s\,, \end{aligned} \tag{3}$$

where the light-quark HF interaction terms V_{ud}^{HF} cancel between the two expressions and the HF interaction terms between the heavy and light quarks vanish

$$V_{us}^{\mathrm{HF}} = V_{ds}^{\mathrm{HF}} = V_{uc}^{\mathrm{HF}} = V_{dc}^{\mathrm{HF}} = 0\,,$$

since the u and d light quarks are coupled to a spin-zero diquark and the HF interaction couples to the spin.

Table I below shows the quark mass differences obtained from mesons and baryons [1]. The mass difference between two quarks of different flavors denoted by i and j are seen to have the same value to a good approximation when they are bound to a "spectator" quark of a given flavor.

TABLE I

Quark mass differences from baryons and mesons.

observable	baryons		mesons				Δm_{Bar} MeV	Δm_{Mes} MeV
			$J=1$		$J=0$			
	B_i	B_j	$\mathcal{V}_i$	$\mathcal{V}_j$	$\mathcal{P}_i$	$\mathcal{P}_j$		
$\langle m_s - m_u \rangle_d$	sud	uud	$s\bar{d}$	$u\bar{d}$	$s\bar{d}$	$u\bar{d}$	177	179
	Λ	N	K^*	ρ	K	π		
$\langle m_s - m_u \rangle_c$			$c\bar{s}$	$c\bar{u}$	$c\bar{s}$	$c\bar{u}$		103
			D_s^*	D_s^*	D_s	D_s		
$\langle m_s - m_u \rangle_b$			$b\bar{s}$	$b\bar{u}$	$b\bar{s}$	$b\bar{u}$		91
			B_s^*	B_s^*	B_s	B_s		
$\langle m_c - m_u \rangle_d$	cud	uud	$c\bar{d}$	$u\bar{d}$	$c\bar{d}$	$u\bar{d}$	1346	1360
	Λ_c	N	D^*	ρ	D	π		
$\langle m_c - m_u \rangle_c$			$c\bar{c}$	$u\bar{c}$	$c\bar{c}$	$u\bar{c}$		1095
			ψ	D^*	η_c	D		
$\langle m_c - m_s \rangle_d$	cud	sud	$c\bar{d}$	$s\bar{d}$	$c\bar{d}$	$s\bar{d}$	1169	1180
	Λ_c	Λ	D^*	K^*	D	K		
$\langle m_c - m_s \rangle_c$			$c\bar{c}$	$s\bar{c}$	$c\bar{c}$	$s\bar{c}$		991
			ψ	D_s^*	η_c	D_s		
$\langle m_b - m_u \rangle_d$	bud	uud	$b\bar{d}$	$u\bar{d}$	$b\bar{d}$	$u\bar{d}$	4685	4700
	Λ_b	N	B^*	ρ	B	π		
$\langle m_b - m_u \rangle_s$			$b\bar{s}$	$u\bar{s}$	$b\bar{s}$	$u\bar{s}$		4613
			B_s^*	K^*	B_s	K		
$\langle m_b - m_s \rangle_d$	bud	sud	$b\bar{d}$	$s\bar{d}$	$b\bar{d}$	$s\bar{d}$	4508	4521
	Λ_b	Λ	B^*	K^*	B	K		
$\langle m_b - m_c \rangle_d$	bud	sud	$b\bar{d}$	$c\bar{d}$	$b\bar{d}$	$c\bar{d}$	3339	3341
	Λ_b	Λ_c	B^*	D^*	B	D		
$\langle m_b - m_c \rangle_s$			$b\bar{s}$	$c\bar{s}$	$b\bar{s}$	$c\bar{s}$		3328
			B_s^*	D_s^*	B_s	D_s		

On the other hand, Table I shows clearly that *constituent quark mass differences depend strongly on the flavor of the spectator quark.* For example, $m_s - m_d \approx 180$ MeV when the spectator is a light quark but the same mass difference is only about 90 MeV when the spectator is a b quark.

Since these are *effective masses*, we should not be surprised that their difference is affected by the environment, but the large size of the shift is quite surprising and its quantitative derivation from QCD is an outstanding challenge for theory. A second example shows how we can extract the ratio of the constituent quark masses from the ratio of the the HF splittings in the corresponding mesons. The HF splitting between K^* and K mesons is given by

$$\begin{aligned} M(K^*) - M(K) &= v_0 \frac{\vec{\lambda}_u \cdot \vec{\lambda}_s}{m_u m_s} \Big[(\vec{\sigma}_u \cdot \vec{\sigma}_s)_{K^*} - (\vec{\sigma}_u \cdot \vec{\sigma}_s)_K \Big] \langle \psi | \delta(r) | \psi \rangle \\ &= 4v_0 \frac{\vec{\lambda}_u \cdot \vec{\lambda}_s}{m_u m_s} \langle \psi | \delta(r) | \psi \rangle \end{aligned} \tag{4}$$

and similarly for HF splitting between D^* and D with $s \to c$ everywhere. From (4) and its D analogue we then immediately obtain

$$\frac{M(K^*) - M(K)}{M(D^*) - M(D)} = \frac{4v_0 \dfrac{\vec{\lambda}_u \cdot \vec{\lambda}_s}{m_u m_s} \langle \psi | \delta(r) | \psi \rangle}{4v_0 \dfrac{\vec{\lambda}_u \cdot \vec{\lambda}_c}{m_u m_c} \langle \psi | \delta(r) | \psi \rangle} \approx \frac{m_c}{m_s} \,. \tag{5}$$

2.1. Color hyperfine splitting in baryons

As an example of HF splitting in baryons, let us now discuss the HF splitting in the Σ (uds) baryons. Σ^* has spin $\frac{3}{2}$, so the u and d quarks must be in a state of relative spin 1. The Σ has isospin 1, so the wave function of u and d is symmetric in flavor. It is also symmetric in space, since in the ground state the quarks are in a relative S-wave. On the other hand, the u–d wave function is antisymmetric in color, since the two quarks must couple to a $\mathbf{3}^*$ of color to neutralize the color of the third quark. The u–d wave function must be antisymmetric in flavor $\times$ spin $\times$ space $\times$ color, so it follows it must be symmetric in spin, *i.e.* u and d are coupled to spin one. Since u and d are in spin 1 state in both Σ^* and Σ their HF interaction with each other cancels between the two and thus the u–d pair does not contribute to the $\Sigma^* - \Sigma$ HF splitting

$$M(\Sigma^*) - M(\Sigma) = 6v_0 \frac{\vec{\lambda}_u \cdot \vec{\lambda}_s}{m_u m_s} \langle \psi | \delta(r_{rs}) | \psi \rangle \tag{6}$$

we can then use Eqs. (4) and (6) to compare the quark mass ratio obtained from mesons and baryons

$$\left(\frac{m_c}{m_s}\right)_{\rm bar} = \frac{M_{\Sigma^*} - M_{\Sigma}}{M_{\Sigma_c^*} - M_{\Sigma_c}} = 2.84\,,$$
$$\left(\frac{m_c}{m_s}\right)_{\rm mes} = \frac{M_{K^*} - M_K}{M_{D^*} - M_D} = 2.81\,; \tag{7}$$

$$\left(\frac{m_c}{m_u}\right)_{\rm bar} = \frac{M_{\Delta} - M_p}{M_{\Sigma_c^*} - M_{\Sigma_c}} = 4.36\,,$$
$$\left(\frac{m_c}{m_u}\right)_{\rm mes} = \frac{M_{\rho} - M_{\pi}}{M_{D^*} - M_D} = 4.46\,. \tag{8}$$

We find the same value from mesons and baryons $\pm 2\%$.

The presence of a fourth flavor gives us the possibility of obtaining a new type of mass relation between mesons and baryons. The $\Sigma - \Lambda$ mass difference is believed to be due to the difference between the u–d and u–s HF interactions. Similarly, the $\Sigma_c - \Lambda_c$ mass difference is believed to be due to the difference between the u–d and u–c HF interactions. We therefore obtain the relation

$$\left(\frac{\frac{1}{m_u^2} - \frac{1}{m_u m_c}}{\frac{1}{m_u^2} - \frac{1}{m_u m_s}}\right)_{\rm bar} = \frac{M_{\Sigma_c} - M_{\Lambda_c}}{M_{\Sigma} - M_{\Lambda}} = 2.16\,,$$
$$\left(\frac{\frac{1}{m_u^2} - \frac{1}{m_u m_c}}{\frac{1}{m_u^2} - \frac{1}{m_u m_s}}\right)_{\rm mes} = \frac{(M_{\rho}-M_{\pi})-(M_{D^*}-M_D)}{(M_{\rho}-M_{\pi})-(M_{K^*}-M_K)} = 2.10\,. \tag{9}$$

The meson and baryon relations agree to $\pm 3\%$.

We can write down an analogous relation for hadrons containing the b quark instead of the s quark, obtaining the prediction for splitting between Σ_b and Λ_b

$$\frac{M_{\Sigma_b} - M_{\Lambda_b}}{M_{\Sigma} - M_{\Lambda}} = \frac{(M_{\rho} - M_{\pi}) - (M_{B^*} - M_B)}{(M_{\rho} - M_{\pi}) - (M_{K^*} - M_K)} = 2.51 \tag{10}$$

yielding $M(\Sigma_b) - M(\Lambda_b) = 194$ MeV [1,2].

This splitting was recently measured by CDF [3]. They obtained the masses of the Σ_b^- and Σ_b^+ from the decay $\Sigma_b \to \Lambda_b + \pi$ by measuring the corresponding mass differences in MeV

$$\begin{aligned} M(\Sigma_b^-) - M(\Lambda_b) &= 195.5^{+1.0}_{-1.0}\,(\text{stat.}) \pm 0.1\,(\text{syst.})\,, \\ M(\Sigma_b^+) - M(\Lambda_b) &= 188.0^{+2.0}_{-2.3}\,(\text{stat.}) \pm 0.1\,(\text{syst.}) \end{aligned} \tag{11}$$

with isospin-averaged mass difference $M(\Sigma_b) - M(\Lambda_b) = 192$ MeV, as shown in Fig. 1.

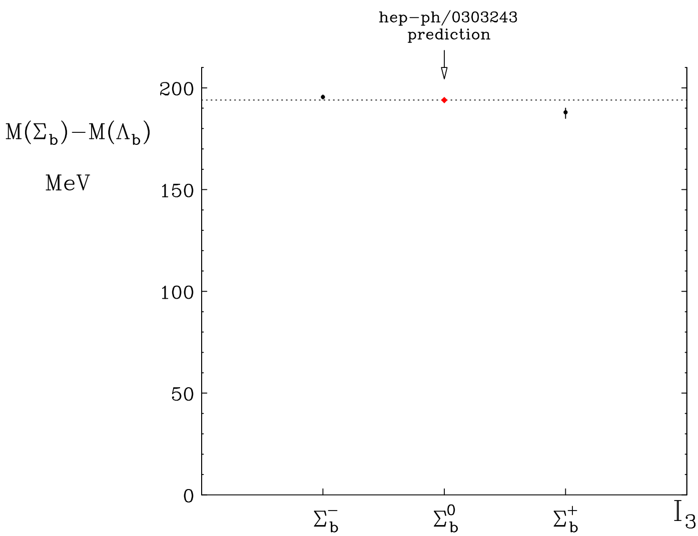

Fig. 1. Experimental results for $\Sigma_b^\pm$ masses [3] *vs* the theoretical prediction in Ref. [1].

There is also the prediction for the spin splittings, good to 5%

$$M(\Sigma_b^*) - M(\Sigma_b) = \frac{M(B^*) - M(B)}{M(K^*) - M(K)}\,[M(\Sigma^*) - M(\Sigma)] = 22 \text{ MeV} \tag{12}$$

to be compared with 21 MeV from the isospin-average of CDF measurements [3].

The relation (10) is based on the assumption that the qq and $q\bar{q}$ interactions have the same flavor dependence. This automatically follows from the assumption that both HF interactions are inversely proportional to the products of the same quark masses. But all that is needed here is the weaker assumption of same flavor dependence

$$\frac{V_{\text{hyp}}(q_i\bar{q}_j)}{V_{\text{hyp}}(q_i\bar{q}_k)} = \frac{V_{\text{hyp}}(q_iq_j)}{V_{\text{hyp}}(q_iq_k)}\,. \tag{13}$$

This yields [1]

$$\begin{aligned} \frac{M_{\Sigma_b} - M_{\Lambda_b}}{(M_\rho - M_\pi) - (M_{B^*} - M_B)} &= 0.32 \\ \approx \frac{M_{\Sigma_c} - M_{\Lambda_c}}{(M_\rho - M_\pi) - (M_{D^*} - M_D)} &= 0.33 \\ \approx \frac{M_{\Sigma} - M_{\Lambda}}{(M_\rho - M_\pi) - (M_{K^*} - M_K)} &= 0.325\,. \end{aligned} \tag{14}$$

The baryon–meson ratios are seen to be independent of the flavor f.

The challenge is to understand how and under what assumptions one can derive from QCD the very simple model of hadronic structure at low energies which leads to such accurate predictions.

3. Effective meson–baryon SUSY

Some of the results described above can be understood [2] by observing that in the hadronic spectrum there is an approximate effective supersymmetry between mesons and baryons related by replacing a light antiquark by a light diquark.

This supersymmetry transformation goes beyond the simple constituent quark model. It assumes only a valence quark of flavor i with a model independent structure bound to "light quark brown muck color antitriplet" of model-independent structure carrying the quantum numbers of a light antiquark or a light diquark, *cf.* Fig. 2. Since it assumes no model for the valence quark, nor the brown muck antitriplet coupled to the valence quark, it holds also for the quark-parton model in which the valence is carried by a current quark and the rest of the hadron is a complicated mixture of quarks and antiquarks.

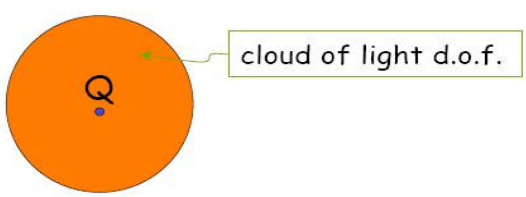

Fig. 2. A heavy quark coupled to "brown muck" color antitriplet.

This light quark supersymmetry transformation, denoted here by T^S_{LS}, connects a meson denoted by $|\mathcal{M}(\bar{q}Q_i)\rangle$ and a baryon denoted by $|\mathcal{B}([qq]_S Q_i)\rangle$ both containing the same valence quark of some fixed flavor Q_i, $i = (u, s, c, b)$

and a light color-antitriplet "brown muck" state with the flavor and baryon quantum numbers respectively of an antiquark $\bar{q}$ (u or d) and two light quarks coupled to a diquark of spin S

$$T^S_{LS}|\mathcal{M}(\bar{q}Q_i)\rangle \quad \equiv \quad \left|\mathcal{B}\left([qq]_S Q_i\right)\right\rangle . \tag{15}$$

The mass difference between the meson and baryon related by this T^S_{LS} transformation has been shown [4] to be independent of the quark flavor i for all four flavors (u, s, c, b) when the contribution of the HF interaction energies is removed. For the two cases of spin-zero [4] $S = 0$ and spin-one $S = 1$ diquarks

$$\begin{aligned} M(N) - \tilde{M}(\rho) &= 323 \text{ MeV} \\ \approx M(\Lambda) - \tilde{M}(K^*) &= 321 \text{ MeV} \\ \approx M(\Lambda_c) - \tilde{M}(D^*) &= 312 \text{ MeV} \\ \approx M(\Lambda_b) - \tilde{M}(B^*) &= 310 \text{ MeV}\,, \end{aligned} \tag{16}$$

$$\begin{aligned} \tilde{M}(\Delta) - \tilde{M}(\rho) &= 517.56 \text{ MeV} \\ \approx \tilde{M}(\Sigma) - \tilde{M}(K^*) &= 526.43 \text{ MeV} \\ \approx \tilde{M}(\Sigma_c) - \tilde{M}(D^*) &= 523.95 \text{ MeV} \\ \approx \tilde{M}(\Sigma_b) - \tilde{M}(B^*) &= 512.45 \text{ MeV}\,, \end{aligned} \tag{17}$$

where

$$\tilde{M}(V_i) \equiv \frac{3M_{\mathcal{V}_i} + M_{\mathcal{P}_i}}{4} \tag{18}$$

are the weighted averages of vector and pseudoscalar meson masses, denoted respectively by $M_{\mathcal{V}_i}$ and $M_{\mathcal{P}_i}$, which cancel their HF contribution, and

$$\tilde{M}(\Sigma_i) \equiv \frac{2M_{\Sigma^*_i} + M_{\Sigma_i}}{3}\,, \qquad \tilde{M}(\Delta) \equiv \frac{2M_\Delta + M_N}{3} \tag{19}$$

are the analogous weighted averages of baryon masses which cancel the HF contribution between the diquark and the additional quark.

4. Magnetic moments of heavy quark baryons

In Λ, Λ_c and Λ_b baryons the light quarks are coupled to spin zero. Therefore, the magnetic moments of these baryons are determined by the magnetic moments of the s, c and b quarks, respectively. The latter are proportional to the chromomagnetic moments which determine the HF splitting in baryon spectra. We can use this fact to predict the Λ_c and Λ_b baryon magnetic moments by relating them to the HF splittings in the same way as given in the original prediction [5] of the Λ magnetic moment,

$$\mu_\Lambda = -\frac{\mu_p}{3}\,\frac{M_{\Sigma^*} - M_\Sigma}{M_\Delta - M_N} = -0.61 \text{ n.m.} \qquad (\text{exp. value} = -0.61 \text{ n.m.})\,. \tag{20}$$

We obtain

$$\begin{aligned}\mu_{\Lambda_c} &= -2\mu_\Lambda\,\frac{M_{\Sigma_c^*} - M_{\Sigma_c}}{M_{\Sigma^*} - M_\Sigma} = 0.43 \text{ n.m.}\,,\\ \mu_{\Lambda_b} &= -\mu_\Lambda\,\frac{M_{\Sigma_b^*} - M_{\Sigma_b}}{M_{\Sigma^*} - M_\Sigma} = -0.067 \text{ n.m.}\,.\end{aligned} \tag{21}$$

We hope these observables can be measured in foreseeable future and view the predictions (21) as a challenge for the experimental community.

5. Testing confining potentials through meson/baryon HF splitting ratio

The ratio of color HF splitting in mesons and baryons is a sensitive probe of the details of the confining potential. This is because this ratio depends only on the value of the wave function at the origin, which in turn is determined by the confining potential and by the ratio of quark masses, as can be readily seen from Eqs. (4) and (6), together with the fact that the color quark–antiquark interaction in mesons is twice as strong as the quark–quark interaction in baryons, $(\vec{\lambda}_u \cdot \vec{\lambda}_s)_{\text{mes}} = 2(\vec{\lambda}_u \cdot \vec{\lambda}_s)_{\text{bar}}$. We then have

$$\frac{M(K^*) - M(K)}{M(\Sigma^*) - M(\Sigma)} = \frac{4}{3}\,\frac{\langle\psi|\delta(\vec{r}_u - \vec{r}_s)|\psi\rangle_{\text{mes}}}{\langle\psi|\delta(\vec{r}_u - \vec{r}_s)|\psi\rangle_{\text{bar}}} \tag{22}$$

and analogous expressions with the s quark replaced by another heavy quark Q. From the experiment we have 3 data points for this ratio, with $Q = s, c, b$. We can then compute the ratio (22) for 5 different representative confining potentials and compare with experiment. The 5 potentials are:

— harmonic oscillator,

— Coulomb interaction,

— linear potential,

— linear + Coulomb, *i.e.* Cornell potential,

— logarithmic.

The results are shown in Fig. 3 and Table II [6].

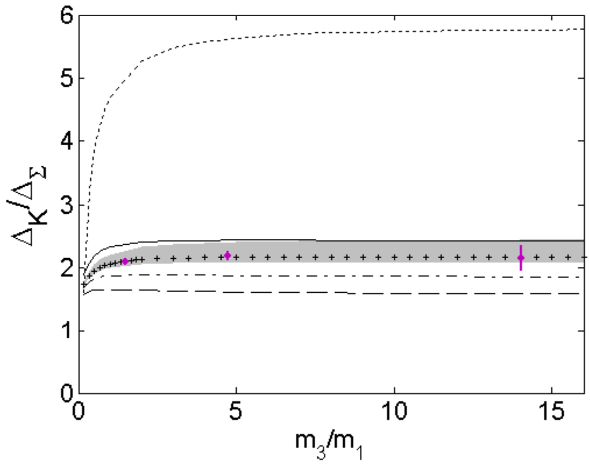

Fig. 3. Ratio of the HF splittings in mesons and baryons, as function of the quark mass ratio. Shaded region: Cornell potential for $0.2 < k < 0.5$; crosses: Cornell, $k = 0.28$; long dashes: harmonic oscillator; short dashes: Coulomb; dot-dashes: linear; continuous: logarithmic; thick dots: experimental data.

TABLE II

Ratio of the HF splittings in mesons and baryons, for different potentials.

	Δ_K/Δ_Σ	$\Delta_D/\Delta_{\Sigma_c}$	$\Delta_B/\Delta_{\Sigma_b}$
m_Q/m_q	1.33	4.75	14
EXP	2.08 ± 0.01	2.18 ± 0.08	2.15 ± 0.20
Harmonic	1.65	1.62	1.59
Coulomb	5.07 ± 0.08	5.62 ± 0.02	5.75 ± 0.01
Linear	1.88 ± 0.06	1.88 ± 0.08	1.86 ± 0.08
Log	2.38 ± 0.02	2.43 ± 0.02	2.43 ± 0.01
Cornell ($k = 0.28$)	2.10 ± 0.05	2.16 ± 0.07	2.17 ± 0.08

For all potentials containing one coupling constant the coupling strength cancels in the meson–baryon ratio. The Cornell potential which is a combination of a Coulomb and linear potential contains two couplings, one of which cancels in the meson–baryon ratio. The remaining coupling is denoted by k. The gray band corresponds to the range of values $0.2 < k < 0.5$ of the Cornell potential. The crosses correspond to $k = 0.28$ which is the value previously used to fit the charmonium data. Clearly the Cornell potential with $k = 0.28$ provides the best fit to the experiment.

6. Predicting the mass of b baryons

On top of the already discussed Σ_b with quark content bqq, there are two additional ground-state b baryons, Ξ_b and Ω_b. We will now discuss the theoretical prediction of their masses and compare it with experiment.

6.1. Ξ_b

The Ξ_Q baryons quark content is Qsd or Qsu. They can be obtained from "ordinary" Ξ (ssd or ssu) by replacing one of the s quarks by a heavier quark $Q = c, b$. There is one important difference, however. In the ordinary Ξ, Fermi statistics dictates that two s quarks must couple to spin-1, while in the ground state of Ξ_Q the (sd) and (su) diquarks have spin zero. Consequently, the Ξ_b mass is given by the expression: $\Xi_q = m_q + m_s + m_u - 3v\langle\delta(r_{us})\rangle/m_u m_s$. The Ξ_b mass can thus be predicted using the known Ξ_c baryon mass as a starting point and adding the corrections due to mass differences and HF interactions

$$\Xi_b = \Xi_c + (m_b - m_c) - 3v\left(\langle\delta(r_{us})\rangle_{\Xi_b} - \langle\delta(r_{us})\rangle_{\Xi_c}\right)/(m_u m_s)\,. \tag{23}$$

Since the Ξ_Q baryon contains a strange quark, and the effective constituent quark masses depend on the spectator quark, the optimal way to estimate the mass difference ($m_b - m_c$) is from mesons which contain both s and Q quarks

$$m_b - m_c = \tfrac{1}{4}(3B_s^* + B_s) - \tfrac{1}{4}(3D_s^* + D_s) = 3324.6 \pm 1.4\,. \tag{24}$$

On the basis of these results we predicted [7] $M(\Xi_b) = 5795 \pm 5$ MeV. Our paper was submitted on June 14, 2007. The next day CDF announced the result [9], $M(\Xi_b) = 5792.9 \pm 2.5 \pm 1.7$ MeV, following up on an earlier D0 measurement, $M(\Xi_b) = 5774 \pm 11 \pm 15$ MeV [8], as shown in Fig. 4 below.

6.2. Mass of the Ω_b

For the spin-averaged Ω_b mass we have

$$\begin{aligned}\tfrac{1}{3}\left(2M(\Omega_b^*) + M(\Omega_b)\right) &= \tfrac{1}{3}\left(2M(\Omega_c^*) + M(\Omega_c)\right) + (m_b - m_c)_{B_s - D_s} \\ &= 6068.9 \pm 2.4 \text{ MeV}\,.\end{aligned} \tag{25}$$

For the HF splitting we obtain

$$M(\Omega_b^*) - M(\Omega_b) = \left(M(\Omega_c^*) - M(\Omega_c)\right)\frac{m_c}{m_b}\frac{\langle\delta(r_{bs})\rangle_{\Omega_b}}{\langle\delta(r_{cs})\rangle_{\Omega_c}} = 30.7 \pm 1.3 \text{ MeV}\,, \tag{26}$$

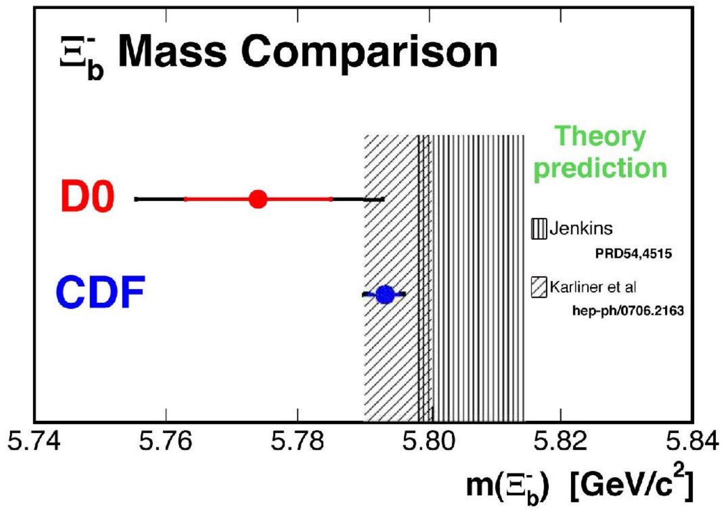

Fig. 4. Ξ_b mass — comparison of theoretical predictions with CDF and X0 data (from a CDF talk by D. Litvinstev).

leading to the following predictions

$$\Omega_b = 6052.1{\pm}5.6 \text{ MeV}\,, \qquad \Omega_b^* = 6082.8{\pm}5.6 \text{ MeV}\,. \tag{27}$$

About four months after our prediction (27) for Ω_b mass was published [10], D0 collaboration published the first measurement of Ω_b mass [11]

$$M(\Omega_b)_{D0} = 6165 \pm 10 \text{ (stat.)} \pm 13 \text{ (syst.) MeV}\,.$$

The deviation from the central value of our prediction was huge, 113 MeV. Understandably, we were very eager to see the CDF result. CDF published their result about nine months later, in May 2009 [12]

$$M(\Omega_b)_{\rm CDF} = 6054 \pm 6.8 \text{ (stat.)} \pm 0.9 \text{ (syst.) MeV}\,.$$

The central values of theoretical prediction and CDF agree to within 2 MeV, or about $1/3$ standard deviation.

Fig. 5 shows a comparison of our predictions for the masses of Σ_b, Ξ_b and Ω_b baryons with the CDF experimental data.

The sign in our prediction

$$M(\Sigma_b^*) - M(\Sigma_b) < M(\Omega_b^*) - M(\Omega_b) \tag{28}$$

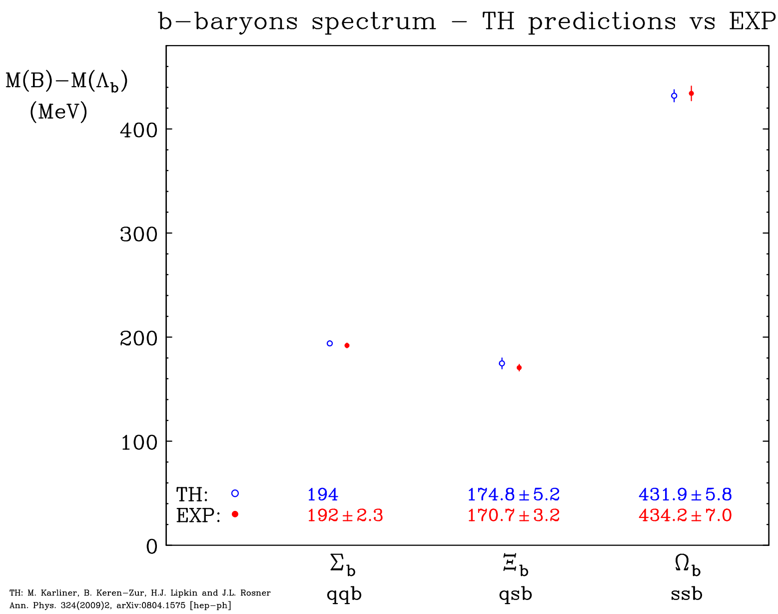

Fig. 5. Masses of b baryons—comparison of theoretical predictions [7, 10] with experiment.

appears to be counterintuitive, since the color HF interaction is inversely proportional to the quark mass. The expectation value of the interaction with the same wave function for Σ_b and Ω_b violates our inequality. When wave function effects are included, the inequality is still violated if the potential is linear, but is satisfied in predictions which use the Cornell potential [6]. This reversed inequality is not predicted by other recent approaches [13–15] which all predict an Ω_b splitting smaller than a Σ_b splitting. However, the reversed inequality is also seen in the corresponding charm experimental data,

$$\begin{array}{ccc} M(\Sigma_c^*) - M(\Sigma_c) & < & M(\Omega_c^*) - M(\Omega_c) \\ 64.3 \pm 0.5 \text{ MeV} & & 70.8 \pm 1.5 \text{ MeV}\,. \end{array} \tag{29}$$

This suggests that the sign of the SU(3) symmetry breaking gives information about the form of the potential. It is of interest to follow this clue theoretically and experimentally.

We have made additional predictions [7, 10] for some excited states of b baryons. Our results are summarized in Table III.

TABLE III

Comparison of predictions for b baryons with those of some other recent approaches [13–15] and with experiment. Masses quoted are isospin averages unless otherwise noted. Our predictions are those based on the Cornell potential.

Quantity	Value in MeV				
	Ref. [13]	Ref. [14]	Ref. [15]	This work	Experiment
$M(\Lambda_b)$	5622	5612	Input	Input	5619.7±1.7
$M(\Sigma_b)$	5805	5833	Input	—	5811.5±2
$M(\Sigma_b^*)$	5834	5858	Input	—	5832.7±2
$M(\Sigma_b^*)-M(\Sigma_b)$	29	25	Input	20.0±0.3	$21.2^{+2.2}_{-2.1}$
$M(\Xi_b)$	5812	5806[a]	Input	5790–5800	5792.9±3.0[b]
$M(\Xi_b')$	5937	5970[a]	5929.7±4.4	5930±5	—
$\Delta M(\ Xi^{\rm b})$[c]	—	—	—	6.4±1.6	—
$M(\Xi_b^*)$	5963	5980[a]	5950.3±4.2	5959±4	—
$M(\Xi_b^*)-M(\ Xi_b')$	26	10[a]	20.6±1.9	29±6	—
$M(\Omega_b)$	6065	6081	6039.1±8.3	6052.1±5.6	6054.4±7[d]
$M(\Omega_b^*)$	6088	6102	6058.9±8.1	6082.8±5.6	—
$M(\Omega_b^*)-M(\Omega_b)$	23	21	19.8±3.1	30.7±1.3	—
$M(\Lambda^*_{b[1/2]})$	5930	5939	—	5929±2	—
$M(\Lambda^*_{b[3/2]})$	5947	5941	—	5940±2	—
$M(\Xi^*_{b[1/2]})$	6119	6090	—	6106±4	—
$M(\Xi^*_{b[3/2]})$	6130	6093	—	6115±4	—

[a] Value with configuration mixing taken into account; slightly higher without mixing.

[b] CDF [9] value of $M(\Xi_b^-)$.

[c] $M(bsd) - M(bsu)$.

[d] CDF [12] value of $M(\Omega_b)$.

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