

LIMITING SOFT PARTICLE PRODUCTION AND QCD*

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We present some basic elements of the treatment of particle multiplicities in jets from high energy collisions within perturbative QCD. Then we discuss the universal features of the inclusive particle spectrum for the limiting case of momentum $p \rightarrow 0$ (or $p_T \rightarrow 0$) as expected from soft QCD gluon bremsstrahlung. The energy independence of the invariant particle density in this limit $I_0 = E \frac{dN}{d^3p} |_{p \rightarrow 0}$ is predicted as well as the dependence of this quantity on the colour factors characteristic of the underlying partonic processes. These properties are first recalled from e^+e^- collisions and then extended to pp and nuclear collisions according to W. Ochs, V.A. Khoze, M.G. Ryskin, *Eur. Phys. J. C* **68**, 141 (2010). Present data support these predictions. It will be interesting to see whether new incoherent contributions show up in the new energy regime of LHC.

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1. Introduction

The production of multi-hadron final states at high energies is described within QCD as a two-phase process: in the first phase there is some hard scattering (strong or electroweak) of the incoming elementary objects like quarks, leptons or gauge bosons. The produced quarks and gluons (“partons”) will form jets of partons by gluon bremsstrahlung and quark pair production according to the rules of QCD perturbation theory for a characteristic cut-off scale Q_0 . In a second phase, the partons reinteract and hadrons are formed which ultimately decay into stable particles. These processes are not accessible in perturbation theory and particular models are applied for their description.

The simplest and best understood high energy process is e^+e^- annihilation into hadrons. It is initiated by the process $e^+e^- \rightarrow q\bar{q}$ which evolves into two hadronic jets dominantly. In pp collisions the protons in the primary

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hard collision act as a collection of partons and in an event triggered for large transverse energy the partons scattered into large angles form sidewise jets while the spectator jets follow the direction of the incoming protons. The soft particles in this case form the “underlying event” and this phenomenon is under intense investigation today. In addition, there is the possibility of multiple independent parton–parton interactions considered important at the highest available energies. There are also the untriggered “minimum bias” events which may result from small angle parton–parton scatterings. Finally, in nucleus–nucleus collisions there may occur hard parton–parton interactions as in pp collisions, in addition there are parton reinteractions in the large nucleus, multiple nucleon–nucleon interactions and, with special interest, the new collective phenomena like quark gluon plasma formation.

Although very different phenomena appear in the various processes there are some remarkable simplifications for very soft particles with momentum in the limit $p \rightarrow 0$ in all processes. We consider for inclusive particle distributions the limit for the particle density

$$I_0 = E \frac{dN}{d^3p} \Big|_{p \rightarrow 0}. \quad (1)$$

In this limit the Born term in the perturbative expansion dominates and this leads to some universal features for all processes

1. inclusive spectra become energy independent,
2. the relative normalisation of spectra in different processes is given by the colour factors relevant for the minimal partonic process.

This holds for QCD partons, but we assume the same is true also for hadrons.

These properties can be understood qualitatively as follows. A soft gluon is coherently emitted from all final partons. Having a large wavelength it cannot resolve any detailed intrinsic jet structure. It “sees” only the total colour charge which is carried by the primary partons, and these are represented by the Born term for the minimal partonic process in the perturbative expansion.

2. Inclusive properties of QCD jets

2.1. QCD evolution equations

We begin by recalling the main tools to derive the inclusive observables for parton jets. They are obtained analytically in QCD using the concept of evolution equations (see, for example, Refs. [2–4], some more recent results will be added). Let us consider the partons in a jet emerging from

a primary parton of energy E within the opening angle Θ . First, we consider the global observables like mean multiplicity $\langle n \rangle$, factorial moments $f_q = \langle n(n-1) \dots (n-q+1) \rangle$ of the multiplicity distribution which can be derived from a generating function

$$Z(Q, u) = \sum_{n=1}^{\infty} P_n(Q) u^n \quad (2)$$

for the jet scale $Q = E\Theta$ at small angles Θ and the probability P_n for production of n particles as

$$\bar{n} = \left. \frac{\partial Z(Q, u)}{\partial u} \right|_{u=1}, \quad f_q = \left. \frac{\partial^q Z(Q, u)}{\partial u^q} \right|_{u=1}. \quad (3)$$

In a corresponding way we obtain inclusive distributions $D(k) \equiv \frac{dn}{d^3k}$, *i.e.* the number of particles in the interval d^3k , and, more generally, inclusive correlation functions $D^{(n)}(k_1 \dots k_n)$ from a generating functional which depends on a probing function $u(k)$

$$Z(Q, u) = \sum_n \int d^3k_1 \dots d^3k_n P_n(k_1 \dots k_n) u(k_1) \dots u(k_n) \quad (4)$$

$$D^{(n)}(k_1 \dots k_n) = \left. \frac{\delta^n Z(\{u\})}{\delta(u(k_1) \dots \delta(u(k_n))} \right|_{u=1}, \quad (5)$$

where P_n is the probability distribution of momenta k_i .

For this generating function or functional an evolution equation is derived in the scale $Q = E\Theta$ which yields, by appropriate differentiation, the equations for the observables like multiplicities and inclusive spectra. In differential form one finds the coupled equation for quark and gluon jets ($a = q, g$) [2, 5]¹. They can be considered as extensions of the well known DGLAP evolution equations towards low particle energies taking into account soft gluon coherence as realized in a probabilistic way by angular ordering [8, 9]

$$\frac{d}{dY} Z_a(Y, u) = \sum_{b,c} \int_{z_c}^{1-z_c} dz \frac{\alpha_s(\tilde{k}_T)}{2\pi} P_{bc}(z) \times \{Z_b(Y + \ln z, u) Z_c(Y + \ln(1-z), u) - Z_a(Y, u)\}, \quad (6)$$

$$Z_a(0, u) = u. \quad (7)$$

¹ Simplified forms have been obtained before [6, 7].

The evolution variable is taken as $Y = \ln(E\Theta/Q_0)$ with the non-perturbative k_T cut-off Q_0 ; the argument of the running coupling is $\tilde{k}_T = \min(z, 1-z)E\Theta$. The evolution equation (6) describes the decay of a parton jet a at scale E into two parton jets b, c at scale zE and $(1-z)E$ with probability $P_{bc}(z)$, the so-called DGLAP splitting functions. The second equation (7) represents the initial conditions at threshold ($E\Theta = Q_0$) and means that the parton jets start just with the initial parton a .

Asymptotic solutions can be obtained in the Double Logarithmic Approximation (DLA) which includes only the dominant contributions from the singularities at small angles and energies in the emission probability. In this approximation the splitting function $P_{gg}(z) \sim 1/z$ in (6); the next to leading single logarithmic terms are included in the Modified Leading Logarithmic Approximation (MLLA). Up to this order the results from Eq. (6) are complete; further logarithmic contributions beyond MLLA can be calculated, but they are not complete and neglect in particular process dependent large angle emissions. Nevertheless, they improve the results considerably as they take into account energy conservation with increasing accuracy. The full solution of Eq. (6), corresponding to the summation of all logarithmic orders can be obtained numerically. Alternatively, one may calculate results of the QCD cascade from a Monte Carlo generator, such as ARIADNE [10], which applies the same k_T cut-off procedure as Eq. (6).

2.2. Parton hadron duality approaches

So far, we have discussed the properties of a jet of partons obtained from perturbation theory using an artificial cut-off at low scales Q_0 . The application to multiparticle observables needs an additional assumption about the hadronization process at large distances which is governed by the colour-confinement forces not accessible by perturbation theory.

The simplest idea is to treat hadronization as long-distance process, involving only small momentum transfers, and to compare directly the perturbative predictions at the partonic level with the corresponding measurements at the hadronic level. This can be applied at first to the total cross-sections, where at the low energies the resonance structures are represented in an average sense. The perturbative approach also describes jet production for a given resolution; here, the collection of partons is compared to hadronic jets at the same resolution and kinematics. This approach has led to spectacular successes and has built up our present confidence in the correctness of QCD as the theory of strong interactions.

In a next step, one may carry on such a dual correspondence further to the level of partons and final hadrons. This procedure turns out successful for “infrared and collinear safe” observables which do not change if a soft

particle is added or one particle splits into two collinear particles. Such observables become insensitive to the cut-off Q_0 for small Q_0 . Quantities of this type are energy-flows and -correlations as well as global event shapes like thrust *etc.*

Further on, one may compare partons and hadrons for observables which count individual particles, for example, particle multiplicities, inclusive spectra and multiparton correlations. Such observables depend explicitly on the cut-off Q_0 (the smaller the cut-off, the larger the particle multiplicity).

According to the hypothesis of Local Parton Hadron Duality (LPHD) [11] the hadron spectra are proportional to the parton spectra, where the conversion of partons into hadrons occurs at a low virtuality scale, of the order of hadronic masses, *i.e.* $Q_0 \sim$ few hundred MeV, independent of the scale of the primary hard process.

While this hypothesis has been suggested originally for single inclusive spectra it can be generalised to more complex situations of the form [3, 4]

$$O(x_1, x_2, \dots)|_{\text{hadrons}} = K O(x_1, x_2, \dots, Q_0, A)|_{\text{partons}}, \quad (8)$$

where the non-perturbative cut-off Q_0 and the “conversion coefficient” K have to be determined by experiment. The conversion coefficient should be a true constant independent of the hardness of the underlying process.

In a more recent analysis mean multiplicities and higher multiplicity moments have been calculated both for sub-jets of variable cut-off scale Q_c (“jet virtuality”) and for hadrons with cut-off Q_0 in e^+e^- annihilation [12, 13] with the smooth transition from jets to hadrons for $Q_c \rightarrow Q_0^2$. For jets at fixed cut-off Q_c the normalisation is $K = 1$ in (8). It turns out that a unified description of jets and hadrons was possible with the common normalisation

$$K \approx 1. \quad (9)$$

In this case, the hadronic cascade has been represented by the partonic cascade in the average with the same multiplicity of partons and hadrons. So the resonance bumps in the multi-particle spectra are just represented by the corresponding smooth perturbative spectra in the average. This *parton-hadron-jet correspondence* implies that *a hadron corresponds to a parton jet of resolution Q_0 .*

When comparing differential parton and hadron distributions there can be a mismatch near the soft limit caused by the mass effects (partons are taken as massless in general). This mismatch can be avoided by a proper choice of energy and momentum variables. In a simple model [15, 16] partons

² For jets and sub-jets the so-called Durham-algorithm [14] which corresponds to a cut-off $k_T > Q_c$ has been applied.

and hadrons are compared at the same energy (or transverse mass) using an effective mass Q_0 for the hadrons, *i.e.*

$$E_{T,\text{parton}} = k_{T,\text{parton}} \Leftrightarrow E_{T,\text{hadron}} = \sqrt{k_{T,\text{hadron}}^2 + Q_0^2}, \quad (10)$$

then, the corresponding lower limits are $k_{T,\text{parton}} \rightarrow Q_0$ and $k_{T,\text{hadron}} \rightarrow 0$.

We should remark that these duality approaches are justified primarily by their phenomenological success and their intrinsic simplicity and not yet by a convincing theoretical derivation from QCD. In particular, they allow compact analytical solutions for the observable quantities in the available approximations to QCD (DLA, MLLA...), which is not possible for the phenomenological hadronization models of high complexity.

3. Quark and gluon jets: global observables versus soft limit

3.1. Global observable: particle multiplicities

Before we derive the soft properties of particle spectra we discuss the mean particle multiplicity in a jet as the most simple example of a global event characteristic. Here, the higher orders in the QCD perturbation theory are very important. By differentiation (3) of the evolution equations of the generating functions (6) one obtains the evolution equations for the parton multiplicities $\mathcal{N}_a$ in quark and gluon jets ($a = q, g$) [12]

$$\begin{aligned} \frac{d\mathcal{N}_a(Y)}{dY} &= \frac{1}{2} \sum_{b,c} \int_0^1 dz \frac{\alpha_s(\tilde{k}_T)}{\pi} P_a^{bc}(z) \\ &\times [\mathcal{N}_b(Y + \ln z) + \mathcal{N}_c(Y + \ln(1-z)) - \mathcal{N}_a(Y)] \end{aligned} \quad (11)$$

with initial conditions

$$\mathcal{N}_a(Y)|_{Y=0} = 1, \quad (12)$$

which imply there is only one particle in a jet at threshold. Starting with this initial condition one can obtain by iteration of the evolution equation the perturbative expansion.

The asymptotic behaviour can be derived from (11) using the ansatz

$$\mathcal{N}_g(Y) \sim \exp \left(\int^Y \gamma(y) dy \right), \quad (13)$$

where the anomalous dimension γ has an expansion in $\gamma_0 = \sqrt{2N_C\alpha_s/\pi}$

$$\gamma = \gamma_0 (1 - a_1\gamma_0 - a_2\gamma_0^2 - a_3\gamma_0^3 \dots), \quad (14)$$

and for the ratio of gluon and quark jet multiplicities

$$r \equiv \frac{\mathcal{N}_g}{\mathcal{N}_q} = \frac{C_A}{C_F} (1 - r_1 \gamma_0 - r_2 \gamma_0^2 - r_3 \gamma_0^3 \dots) \quad (15)$$

with QCD colour factors

$$C_A = N_C = 3, \quad C_F = \frac{4}{3}. \quad (16)$$

The coefficients a_i and r_i can be derived from the evolution equations. At high energies the leading behaviour in MLLA for both quark and gluon jet multiplicities is given by

$$\ln \mathcal{N}(Y) \sim c_1 / \sqrt{\alpha_s(Y)} + c_2 \ln \alpha_s(Y) + c_3, \quad (17)$$

$$c_1 = \sqrt{96\pi}/b, \quad c_2 = \frac{1}{4} + \frac{10}{27} n_f/b, \quad b = \frac{11}{3} C_A - \frac{2}{3} n_f, \quad (18)$$

with the arbitrary constant c_3 , and this behaviour describes well the data in e^+e^- annihilation at LEP-1 and LEP-2, for review, see [18], more recent results have been presented by DELPHI [19–21] and OPAL [22, 23].

The important role of higher logarithmic orders can be studied in the behaviour of the multiplicity ratio r in (15). The asymptotic limit $r = C_A/C_F$ acquires large finite energy corrections in NLL0 [24, 25] and 2NLL0 order [26, 27]

$$r_1 = 2 \left(h_1 + \frac{N_f}{12N_C^3} \right) - \frac{3}{4}, \quad (19)$$

$$r_2 = \frac{r_1}{6} \left(\frac{25}{8} - \frac{3N_f}{4N_C} - \frac{C_F N_f}{N_C^2} - \frac{7}{8} - h_2 - \frac{C_F}{N_C} h_3 + \frac{N_f}{12N_C} h_4 \right) \quad (20)$$

with $h_1 = \frac{11}{24}$, $h_2 = \frac{67-6\pi^2}{36}$, $h_3 = \frac{4\pi^2-15}{24}$ and $h_4 = \frac{13}{3}$, also 3NLL0 results have been derived [28]. Results from these approximations [18] are shown in Fig. 1 together with the numerical solution of the MLLA evolution equations (11) obtained in 1998 [12], which takes into account all higher order corrections from this equation and fulfils the (non-perturbative) boundary condition (12). All curves are absolute predictions, as the parameter Λ (and Q_0 in case of the numerical calculation) is adjusted from the growth of the total particle multiplicity in the e^+e^- jets. The slow convergence of this $\sqrt{\alpha_s}$ expansion can be seen and there are still considerable effects beyond 3NLL0. The numerical solution is also in close agreement with the MC result at the parton level obtained [23] from the HERWIG MC above the jet energy $E_{\text{jet}} > 15$ GeV ($E_{\text{jet}} = Q/2$ in e^+e^- annihilation) and $\sim 20\%$

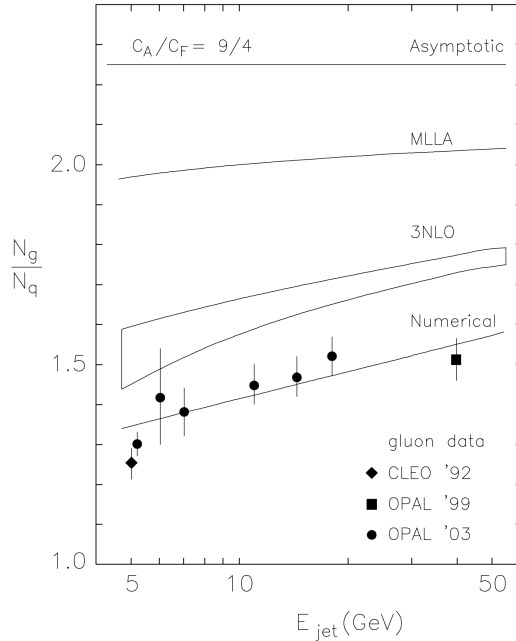


Fig. 1. The ratio of the mean multiplicities in gluon jets and quark jets N_g and N_q obtained from e^+e^- experiments; results from perturbative QCD show the large higher order corrections for a global observable (from [17]).

larger at $E_{\text{jet}} \sim 5$ GeV. This overall agreement suggests that the effects not included in the MLLA evolution equation, such as large angle emission, are small.

These numerical results are also compared in Fig. 1 with data from OPAL [23] where the data on gluon jets are derived from 3-jet events in e^+e^- -annihilation. Note also that a proportionality constant K relating partons and hadrons according to LPHD drops in the ratio r . The results obtained from DELPHI [20] fall slightly below the curve by about 20% at the lowest energies but converge for the higher ones; the CDF Collaboration comparing quark and gluon jets at high p_T in pp collisions [29] finds the ratio r in the range $5 < E_{\text{jet}} < 15$ GeV a bit larger, closer to the 3NLO prediction, but with larger errors and therefore still consistent with the LEP results.

3.2. Inclusive energy spectrum: soft limit

Next, we consider the inclusive distribution $D(\xi, Y)$ of partons in the momentum fraction $x = k/E$ or $\xi = \ln(1/x)$ within a jet with primary parton energy E and opening angle Θ . The evolution equation for D can be

obtained by functional differentiation of (6) (for a review, see [4]). At small x (large ξ) the angular ordering [8] of the cascade evolution which takes into account the soft gluon interferences in a probabilistic way plays an important role. For large x the equations approach the well known DGLAP evolution equations.

For the present discussion we restrict ourselves to the simplest approximation, the DLA with fixed coupling, where only the most singular terms in the splitting functions for $g \rightarrow gg$ and $q \rightarrow qg$ are kept. Then the evolution equation for parton a reads

$$D_a^g(\xi, Y) = \delta_a^g \delta(\xi) + \int_0^\xi d\xi' \int_0^{Y-\xi} dy \frac{C_a}{N_C} \gamma_0^2(y) D_g^g(\xi', y). \quad (21)$$

This equation can be solved by iteration. For fixed $\gamma_0 \sim \sqrt{\alpha_s}$ one obtains the perturbative expansion

$$D_a^g(\xi, Y) = \delta_a^g \delta(\xi) + \frac{C_a}{N_C} \gamma_0^2(Y - \xi) + \frac{1}{2} \frac{C_a}{N_C} \gamma_0^4 \xi(Y - \xi)^2 + \dots \quad (22)$$

$$= \delta_a^g \delta(\xi) + \frac{C_a}{N_C} \gamma_0 \sqrt{\frac{Y - \xi}{\xi}} I_1 \left(2\gamma_0 \sqrt{\xi(Y - \xi)} \right) \quad (23)$$

with the modified Bessel function I_1 . The coherent soft gluon emission leads to a depletion of the spectrum at large ξ , also called “the hump-backed plateau”. From the inclusive distribution (22) one can obtain the double differential distribution in energy and angle by the differentiation over Y which yields $\frac{dN_a}{d\xi dY} = \frac{C_a}{N_C} \gamma_0^2 + \frac{C_a}{N_C} \gamma_0^4 \xi(Y - \xi) + \dots$, or, in the original variables

$$\frac{dN_a}{dk d\Theta} = \frac{2}{\pi} \frac{C_a}{k\Theta} \alpha_s + \frac{4N_C}{\pi^2} \frac{C_a}{k\Theta} \alpha_s^2 \ln \frac{E}{k} \ln \frac{k_T}{Q_0} + \dots \quad (24)$$

Here we recognise in the leading term of $O(\alpha_s)$ the well known Born term for soft gluon bremsstrahlung as in QED, but with the appropriate QCD colour factors. One observes that only the Born term survives in the soft limit where $k_T \rightarrow Q_0$; in this limit we find the simple universal properties emphasised in the introduction: the particle density becomes independent of energy E and is proportional to the relevant colour factor for the minimal process, that is here the gluon emission from the quark or gluon jet with $C_a = C_F$ or $C_a = C_A$, respectively.

This result can be generalised to the accuracy of DLA with running coupling which is obtained by iterating (21) accordingly up to $O(\alpha_s^2)$ which is appropriate for the low momentum region, furthermore results within

MLLA have been derived as well [16]. The above properties in the soft limit remain unaltered. A comparison of these calculations is shown in Fig. 2 where a model dependent kinematic relation as in (10) is used. One can see that the data are rather well described by the model which predicts an energy independent particle density in the soft limit $p \rightarrow 0$.

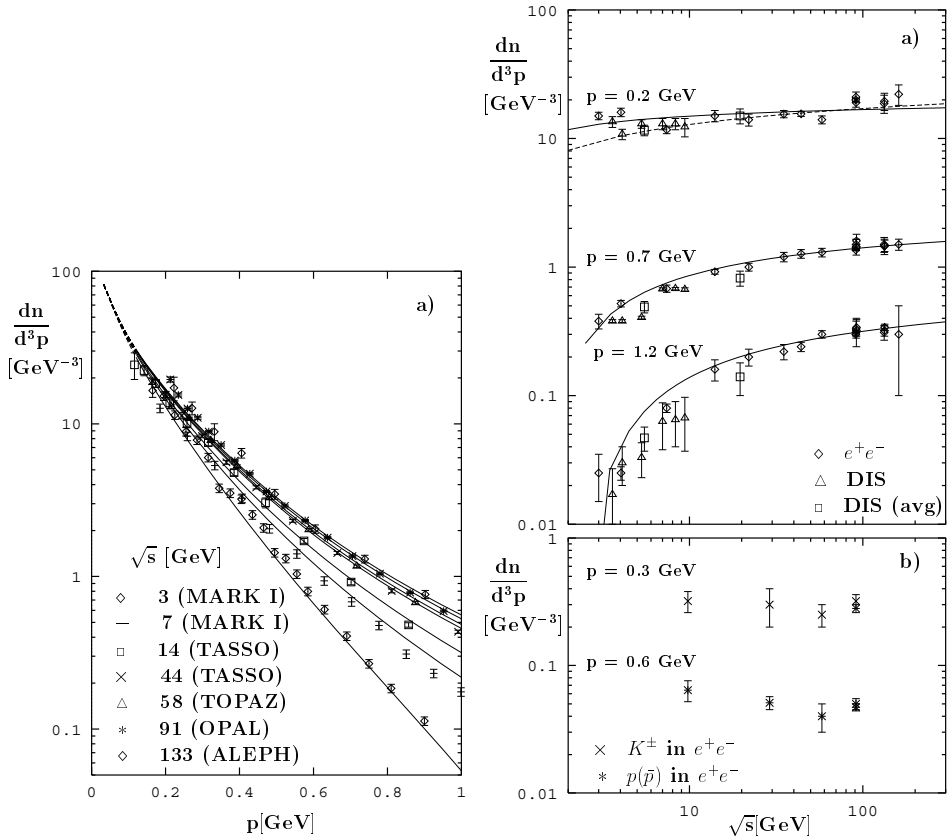


Fig. 2. Inclusive particle densities $\frac{dn}{d^3p}$ of charged particles in e^+e^- annihilations; left panel: momentum spectra for different energies, right panel energy $\sqrt{s}$ -dependence for fixed momenta p , lower right panel: kaons and protons; calculations within MLLA accuracy.

3.3. Colour factors in quark and gluon jets

In order to check the sensitivity to the colour factors in the Born term, we should study the dependence of the soft limit of momentum spectra in quark and gluon jets. Separating quark and gluon jets results in uncertainties

increasing with smaller momenta. Therefore, an alternative procedure has been suggested [16] which studies the radiation into a cone perpendicular to the production plane of 3-jet events in e^+e^- annihilation.

One can consider two extreme limits with two jets aligned:

- (a) a quark and a gluon jet are parallel and recoil against a quark jet, in this configuration the soft perpendicular radiation cannot separate the two parallel jets and the intensity is as in a $q\bar{q}$ dipole, proportional to the colour factor C_F ;
- (b) the quark and antiquark are parallel and recoil against the gluon, in this case the primary configuration acts like a gg dipole with colour factor C_A .

Of course, in a realistic experiment one cannot go to such extreme limits because of the finite width of the jets, but one can measure a certain range of the angles in between the jets which interpolates between the limits.

The soft gluon bremsstrahlung from a “colour dipole antenna” ($q\bar{q}$ or gg) is given by [2]

$$\frac{dN_{A,F}}{d\Omega dk} = \frac{\alpha_s}{(2\pi)^2} \frac{1}{k} W_{A,F}(\vec{n}_g), \quad W_{A,F}(\vec{n}_g) = 2C_{A,F} \left(\widehat{i, j} \right), \quad (25)$$

$$\left(\widehat{i, j} \right) = \frac{1 - \cos \Theta_{ij}}{(1 - \cos \Theta_{is})(1 - \cos \Theta_{js})}, \quad (26)$$

where Θ_{ij} is the angle between partons i and j and s denotes the soft gluon. Then, for the aligned $q\bar{q}$ antenna one obtains $W_F = 4C_F / \sin^2 \Theta_{qs}$.

In a 3-jet event $e^+e^- \rightarrow q\bar{q}g$ one finds in lowest order

$$W_{q\bar{q}g}(\vec{n}_g) = C_A \left[(\widehat{q, g}) + (\widehat{\bar{q}, g}) - \frac{1}{N_C^2} (\widehat{q, \bar{q}}) \right], \quad (27)$$

i.e. there are two dipoles between each of the quarks and the gluon of strength C_A and a colour suppressed dipole between the q and the $\bar{q}$. For $q\|g$ one finds $W = 4C_F / \sin^2 \Theta_{qs}$ like a $q\bar{q}$ dipole, and for $q\|\bar{q}$ one obtains $W = 4C_A / \sin^2 \Theta_{gs}$ like a $g\bar{g}$ dipole, as anticipated above.

The radiation perpendicular to the 3-jet plane ($\cos \Theta_{is} = 0$) normalised to the same radiation in 2-jet events is then given by the simple formula

$$\frac{N_{\perp}^{q\bar{q}g}}{N_{\perp}^{q\bar{q}}} = \frac{W_{\perp}^{q\bar{q}g}}{W_{\perp}^{q\bar{q}}} \equiv \frac{C_A}{C_F} r(\Theta_{ij}), \quad (28)$$

$$r(\Theta_{ij}) = \frac{1}{4} \left[(1 - \cos \Theta_{qg}) + (1 - \cos \Theta_{\bar{q}g}) - \frac{1}{N_C^2} (1 - \cos \Theta_{q\bar{q}}) \right]. \quad (29)$$

Such a measurement has been carried out by the DELPHI Collaboration [30, 31]. The above formulae should apply in the soft limit where the Born term of $O(\alpha_s)$ dominates. The p_T spectra in the cone perpendicular to the 3-jet plane are found all very similar for $p_T \lesssim 1$ GeV, therefore one can study instead the integrated multiplicity in the respective cones. One observes first that the multiplicities in the cone are well described by the above formula (29), the data are accurate enough to even notice the $1/N_C^2$ term in (29). Furthermore, the data for the ratio on the l.h.s. of (28) are found to depend linearly on the function $r(\Theta_{ij})$ and one obtains from the slope

$$\frac{C_A}{C_F} = 2.211 \pm 0.014 \text{ (stat.)} \pm 0.053 \text{ (syst.)} \quad (30)$$

which is well consistent with the expected $C_A/C_F = 9/4$ in QCD.

From these studies of jets in e^+e^- annihilation we can conclude that the soft particle density indeed follows the prediction of the soft gluon Born terms emphasised in the introduction:

- (a) the spectra become independent of energy for $p \rightarrow 0$;
- (b) the soft particle density varies with the orientation of the colour antenna as predicted, this implies that the soft particle density in quark and gluon jets becomes proportional to the colour factors C_A and C_F . This is in strong contrast to the behaviour of global observables like the mean multiplicity, which obtains large higher order corrections from the integral over the perturbative expansion such as (24) and the ratio $r = N_g/N_q$ is as low as 1.5 at LEP energies instead of 2.25 (see Fig. 1). In the soft limit there is no phase space for subsequent emissions, nor for energy momentum conservation effects. With this experience we now investigate the hadronic collisions.

4. High energy pp collisions

We discuss here the “minimum bias” events, which we consider as non-diffractive events. In order to estimate the very soft particle production we look for the minimal partonic process which could be responsible for the soft gluon bremsstrahlung. We assume that the relevant process is a semihard $2 \rightarrow 2 + g_s$ scattering of lowest perturbative order where any two partons inside the proton can scatter with one-gluon exchange at small angles. The exchange of a colour octet gluon at small angle transfers the colour from the colour singlet protons to the two outgoing partonic clusters which are the colour octet sources of soft gluon bremsstrahlung. In the minimal configuration each proton splits into a quark–diquark pair which scatter by gluon

exchange. In large N_C approximation the process corresponds to two radiating colour triplet antennae responsible for bremsstrahlung from initial and final partons. It should be added that also more complex partonic processes will end up in the production of two colour octet systems as discussed in [1].

Therefore, according to our general rules, we expect the energy independent limiting soft radiation density in pp collisions I_0^{pp} for $p \rightarrow 0$ and, furthermore, we expect this density to occur in a fixed ratio to the corresponding density in e^+e^- collisions as ratio of colour octet and colour triplet dipole sources

$$p \rightarrow 0 : \quad I_0^{pp}/I_0^{e^+e^-} \approx C_A/C_F, \quad (31)$$

just like the ratio of the spectra in gluon and quark jets discussed in the last section.

This kind of relation (31) has been suggested by Brodsky and Gunion already in 1976 [32], but relating the integrated multiplicities in the central rapidity region to these colour factors. From our QCD analysis we find Eq. (31) to be valid only in the soft limit while the ratio of integrated multiplicities is found closer to unity (see below). Similarly, relations of the kind (31) appear in some early phenomenological models, but again for the integrated densities only, as outlined in [1].

Our expectation of an energy independent I_0^{pp} is based on a coherent process. It would be violated if there were multiple parton-parton interactions (processes like $4 \rightarrow 4 + g_s$) added incoherently. Such processes appear in some models at high energies (see, for example, PYTHIA [33]) and so the measurements at LHC can shed some light onto the contributions from such processes.

We have studied the energy dependence of I_0^{pp} using the results of fits to the invariant cross-sections $E \frac{d\sigma}{d^3p}$ measured in the energy range $\sqrt{s} = 20 \dots 1800$ GeV obtained from the colliders at CERN, Fermilab and Brookhaven and measurements of inelastic cross-sections σ_{in} . The p_T spectra look qualitatively similar to those in Fig. 2 for e^+e^- annihilations converging towards small p_T but falling more steeply at high momentum. The p_T spectra have been fitted to distributions which at small p_T behave like

$$E \frac{d\sigma}{d^3p} = A \exp(Bp_T + \dots). \quad (32)$$

These fits are good down to the smallest measured $p_T \sim 0.1$ GeV. Then one finds $I_0^{pp} = A/\sigma_{\text{in}}$ from extrapolation $p_T \rightarrow 0$. The functional form (32) is not analytic at $p_T = 0$ and is, therefore, theoretically not satisfactory. This problem is avoided using the “thermal” parametrisation in terms of m_T instead of p_T

$$E \frac{d\sigma}{d^3p} = \frac{A}{(\exp(m_T/T) - 1)}, \quad m_T = \sqrt{m^2 + p_T^2} \quad (33)$$

as applied by PHOBOS [34] in nuclear collisions and a good fit down to the smaller $p_T \sim 0.03$ GeV has been obtained. The extrapolated values I_0^{pp} are smaller by about 25% as compared to the fit (32).

The results from the available published exponential extrapolations are shown in Fig. 3. One observes a rather flat energy dependence over the two decades in energy with an average $I_0^{pp} \approx (7 \pm 1)$ GeV $^{-2}$. Note, that over this energy range the rapidity density $\frac{dN}{dy}$ would rise by about a factor 2. The different extrapolations (32) or (33) should not affect the trend of the energy dependence. For comparison with e^+e^- annihilation it is better to normalise by the non-diffractive cross-section which we take as 15% lower than the inelastic one, which yields $I_{0,\text{nd}}^{pp} \approx (8 \pm 1)$ GeV $^{-2}$. Then for the thermal fit and non-diffractive (minimum bias) events we obtain

$$I_0^{pp} \approx (6 \pm 1) \text{ GeV}^{-2}. \quad (34)$$

So far, there is not yet a fit result from LHC to be used for comparison.

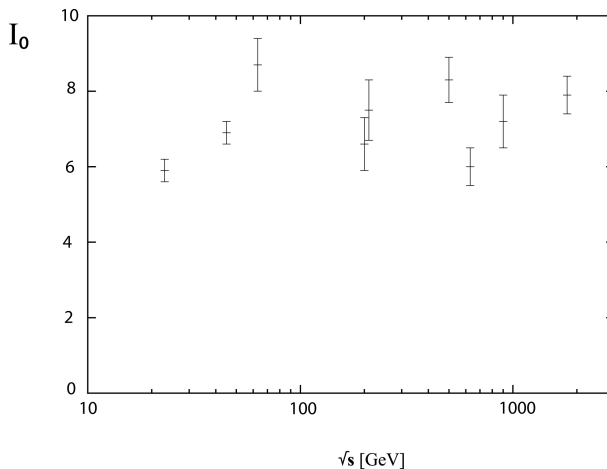


Fig. 3. Soft limit I_0^{pp} of the invariant density $E \frac{dn}{d^3p}$ of charged particles $[(h^+ + h^-)/2]$ in pp collisions as a function of c.m.s. energy $\sqrt{s}$ (from exponential extrapolation).

Next, we also compare this result with the soft limit in e^+e^- annihilation to test our prediction (31). We present two results.

1. There is a dedicated comparison by the TPC Collaboration [35] who compared their own data on e^+e^- annihilation with pp data from the British-Scandinavian Collaboration [36] on the p_T spectra of the invariant density. The TPC data are presented as function of p_T as determined from the sphericity jet axis. In the average over p_T both data sets for pions, kaons and protons are similar. A closer look,

however, reveals, that the spectra fall more steeply with p_T in the pp collisions and there is a crossover of the spectra at low p_T . The appropriate extrapolation down to p_T near zero yields a larger density for pp collisions by a factor 2.0–2.7 depending on the kind of fit.

2. Most other experiments present fits to the spectra in particle energy E (not p_T). Using the fit results from various experiments in the range $\sqrt{s} = 10 \dots 29$ GeV yields $I_0^{e^+e^-} \approx (3.3 \pm 0.5) \text{ GeV}^{-2}$ or the ratio

$$I_0^{pp}/I_0^{e^+e^-} \approx (1.8 \pm 0.4) \div (2.4 \pm 0.5), \quad (35)$$

where the first (preferred) number refers to the thermal and the second to the exponential extrapolation. This result is consistent with our QCD based expectation for this ratio $C_A/C_F = 2.25$.

5. Nucleus–nucleus scattering

For the nucleus–nucleus (AA) cross-sections we may consider two limiting cases in the relation to the pp cross-section.

1. In the case of a point like interaction the particle densities in nuclear collisions are obtained by rescaling the densities in pp collisions by N_{coll} , the number of nucleon–nucleon collisions, or, “the nuclear modification factor”

$$R_{AA}^{N_{\text{coll}}} = \frac{1}{N_{\text{coll}}} \frac{dN_{AA}/dp_T}{dN_{pp}/dp_T}, \quad (36)$$

is unity. The number N_{coll} can be obtained from the Glauber model.

2. In the case of soft particle production we expect that such particles with a large wave length $1/p_T \gtrsim r$ are coherently emitted over a range r (from a nucleon $r_N \sim 1/m_\pi$ or a nucleus $r_A \sim 1/(30 \text{ MeV})$, which results in a reduced rate. Indeed, the inspection of the RHIC data [34, 37–39] shows the ratio $R_{AA}^{N_{\text{coll}}}$ falling below unity for small p_T . An alternative way presenting data in the soft region is the normalisation to the number of “participating nucleons”

$$R_{AA}^{N_{\text{part}}} = \frac{1}{(N_{\text{part}}/2)} \frac{dN_{AA}/dp_T}{dN_{pp}/dp_T}. \quad (37)$$

This concept has been introduced already in 1976 by Bialas, Bleszynski and Czyz [40], who found the number of participating nucleons, called there “wounded nucleons” as relevant scaling factor for soft production ($R_{AA}^{N_{\text{part}}} \approx 1$), *i.e.* each interacting nucleon should be counted only once and the rescatterings of the same nucleon be disregarded.

Again we consider the energy dependence and the normalisation of the particle production at central rapidity in the limit $p_T \rightarrow 0$. A detailed study for various centralities (from peripheral to central AA collisions) by PHOBOS [34] shows that the inclusive p_T spectra in normalisation (36) or (37) approach about the same densities at 200 and at 62.4 GeV. This implies that the energy dependence is the same for nuclei and protons, *i.e.* there is no sizable energy dependence.

Concerning the normalisation, the same data [34] show that the quantity (37) approaches unity for all centralities (within about 30%) at the lower limit of $p_T \approx 200$ MeV. The STAR Collaboration [41] has measured this quantity with high precision down to 500 MeV and an extrapolation to the “participant scaling” for $p_T = 0$ is indicated. We combined fits of PHOBOS low p_T and STAR AuAu data at 200 GeV using the thermal parametrisation and also the STAR pp data which yields the result

$$I_0^{AA}/I_0^{pp} \approx 160 \pm 17. \quad (38)$$

This agrees with the calculated $N_{\text{part}}/2 = 172$ ($\pm 15\%$), and is therefore consistent with

$$p_T \rightarrow 0 : \quad R_{AA}^{N_{\text{part}}} \rightarrow 1 \quad \text{and} \quad I_0^{\text{AuAu}} \approx \frac{N_{\text{part}}}{2} I_0^{pp}. \quad (39)$$

This density for AuAu collisions is about six times smaller than expected for an incoherent superposition of collisions with $N_{\text{coll}} = 1040$, where this number is obtained from Glauber model calculations. It is remarkable, that the “wounded nucleon” model works to the precision of about 10%, the accuracy of measurements and theoretical calculations. Note that this agreement is obtained only in the limit $p_T \rightarrow 0$ as can be seen from the STAR data [41]; already for $p_T = 0.5$ GeV the deviation from “participant scaling” amounts to about 50%.

How can this scaling result be understood? An incoming nucleon scatters successively at a number of nucleons in the nucleus (see Fig. 4 (b), (c) for representative diagrams of pA scattering). The successive gluon exchanges yield again an outgoing colour octet state as in pp scattering, such that the rescatterings of the nucleon are not causing any production of additional particles. This happens if in the low p_T interaction only a quark and the diquark appear as active partons, so that also a multi-gluon exchange cannot produce a higher colour multiplet than an octet³. Alternatively, one can think of a larger number of exchanged gluons but taking into account that the colour octet exchange gives the dominant Leading Logarithmic contribution both in DGLAP and BFKL kinematics. Then, from both viewpoints, each scattered nucleon produces dominantly a colour octet flow as in pp interactions, in agreement with the phenomenological result (39).

³ A model based on wounded quarks and diquarks has been developed in Ref. [42], but for the description of p_T -integrated rapidity distributions.

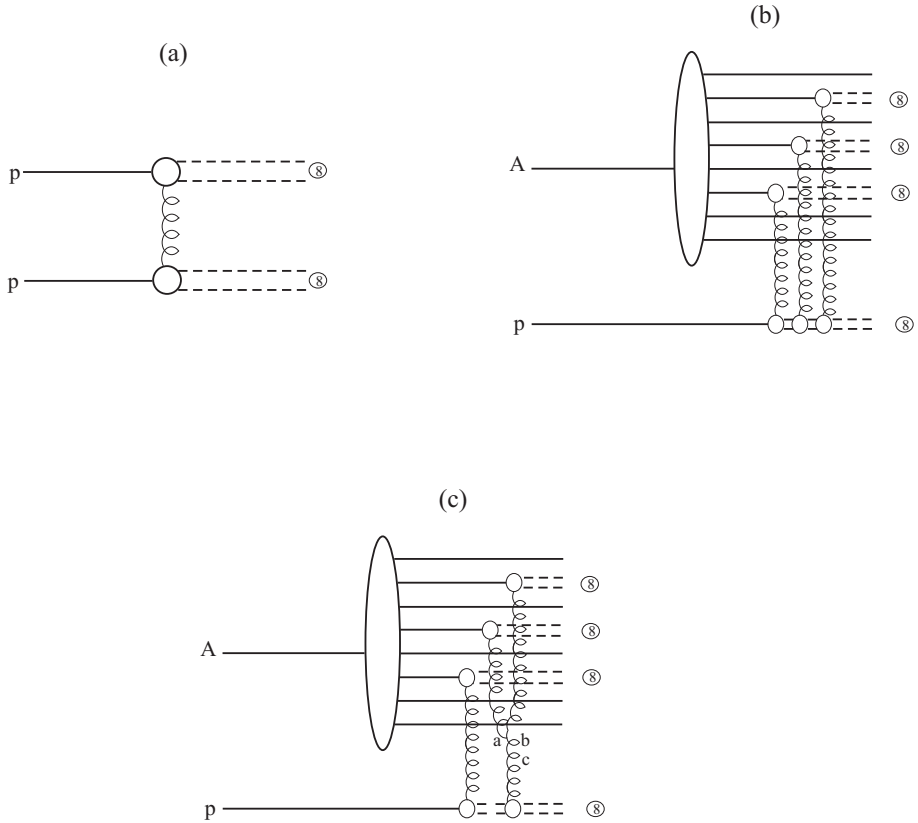


Fig. 4. Diagrams, contributing to the pp and pA collisions in the minimal model for soft particle production: (a) In the pp collisions the exchanged gluon interacts with the colour triplet constituents q or qq in the proton to form an outgoing colour octet system; (b) (c) In the pA collisions the proton can rescatter inside the nucleus and then forms a colour octet system again. This implies that the multiple gluon exchange acts as a single gluon exchange in the particle production. In this example: $N_{\text{coll}} = 3$, $N_{\text{part}}/2 = 2$.

6. Universal composition of softly produced particles

Finally, we may ask, whether the universal production of the soft particles from gluon bremsstrahlung also reflects in their composition as detected by particle ratios. Such a universality can be expected if the source of the bremsstrahlung are the colour triplet dipoles generated in pp and AA collisions by gluon exchange. In that case not only hadronic collisions but also e^+e^- annihilations have universal dipole sources.

A similarity of particle ratios K/π and $\bar{p}/\pi$ in the e^+e^- and pp reactions at $p_T < 0.5$ GeV has been indeed noted already some time ago by the TPC Collaboration [35]. In this measurement the transverse momentum p_T for the particle collisions was defined with respect to the sphericity axis.

The p_T dependence of the particle ratios for several hadronic collisions have been compared by PHENIX [43]. While at the large $p_T > 2$ GeV the ratios p/π and K/π tend to approach large values ~ 1 in the central AuAu collisions, these ratios are reduced for non-central and minimum bias pp collisions. Remarkably, these ratios converge for all the different processes towards lower $p_T < 1$ GeV. In Fig. 5 we collect data in the low p_T region on the ratio K^-/π^- from the e^+e^- , pp and AA interactions. As one can see, these particle ratios, indeed, approach each other towards low $p_T < 0.4$ GeV pointing towards a dominance of multiple $q\bar{q}$ dipole radiation in all processes.

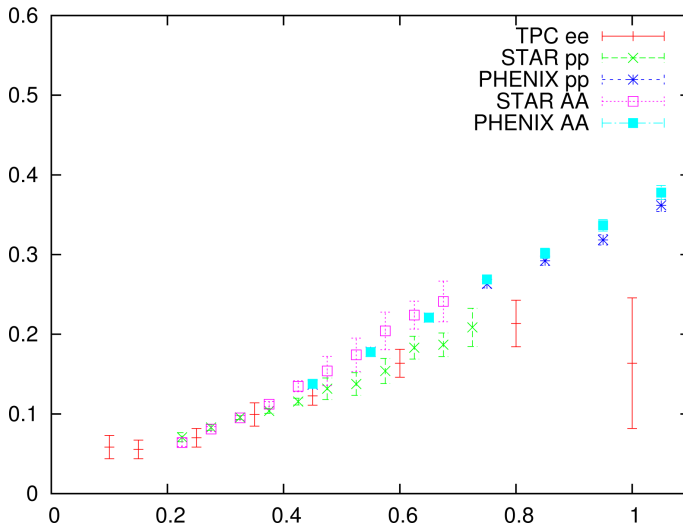


Fig. 5. Convergence of particle ratios K^-/π^- towards small p_T for various processes: e^+e^- annihilation (TPC [35] data, p_T with respect to sphericity axis), pp (minimum bias) and central (0–5%) AuAu collisions (PHENIX [43] and STAR [44] collaborations).

7. Summary

We note some universal features of the particle production in the limit $p, p_T \rightarrow 0$ which we derive from the dominance of coherent QCD gluon bremsstrahlung in this limit. We consider the particle density I_0 in this limit for which we predict

1. the energy independence, and

2. the dependence on colour factors according to the minimal partonic process (Born-term):

(a) e^+e^- annihilation: $I_0^{g-\text{jet}}/I_0^{q-\text{jet}} = C_A/C_F$;

(b) pp scattering: $I_0^{pp}/I_0^{e^+e^-} = C_A/C_F$;

(c) AA scattering: $I_0^{AA}/I_0^{pp} = (N_{\text{part}}/2) C_A/C_A$.

These expectations are well met by the data. In consequence, the soft particles do not follow a universal thermal behaviour independent of the initial state.

There is also some universality in the particle ratios which tend to converge to those from $q\bar{q}$ dipoles. Soft hadrons in the central region are produced first. In AA collisions these slow particles stay behind and do not participate in the equilibration.

It will be interesting to study the soft limit at LHC energies. If there are new incoherent sources, as expected in some models with multiple interactions, the soft density I_0 could start rising with energy.

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