THE SPIN DEPENDENT PARTON DISTRIBUTION FUNCTIONS AND THEIR MOMENTS*

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In the quark parton model (QPM), polarized structure function g_1 is directly related to contributions of the individual quark flavors to the overall spin of the nucleon. Sum rules based on this simple model have provided fertile ground for understanding the origin of the nucleon spin in terms of quark degrees of freedom. Our next-to-leading-order (NLO) analyses of the world g_1 data based on the Jacobi polynomial expansion method have provided indirect information about the components of the nucleon's spin. Moreover, we study the dependence of our results to the number of Jacobi polynomial expansion terms in this paper.

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1. Introduction

The nucleons, the basic building block of nuclear matter, account for more than 99.9% of the mass of visible matter in the Universe. Understanding the nucleon structure is one of the most important issues in modern physics and has challenged us for decades. In the study of nucleon structure,

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spin has played an essential role because it determines their symmetry behavior under space-time transformations. The spin degrees of freedom may be used in high-energy experiments to get informations in the fundamental interactions which are more precise than those obtained with unpolarized beam.

Another aspect of polarization is the question how the spin of non-point like objects, such as the nucleons, is composed of the spins of its constituents, the quarks and gluons. This question can be answered in high-energy experiments because the quarks and gluons behave as (almost) free particles at the energy scale $Q \gg \Lambda_{\rm QCD}$.

This paper is organized as follows. In Section 2 we recall the theoretical background used for the evolution calculation of parton distributions based on the Jacobi polynomial expansion method. Furthermore, contributions from the various quark species in the spin of proton is provided in Section 3. In Section 4, beside the study of the dependence of our results to the number of Jacobi polynomial expansion terms, we try to answer the important question of how much these first moments contribute numerically to the spin of the proton in more details.

2. Theoretical background on the Jacobi polynomial expansion method

We summarize a theoretical background of our method in QCD analysis here. Knowing the fact that world data for g_1 cover a broad range in x and Q^2 with relatively high precision motivates us to extract polarized parton distributions from fits to the data. In our calculation, the polarized parton densities are parametrized at a starting scale $Q_0^2 = 4$ GeV^2 and are evolved to higher factorization scales using a numerical solution of the polarized NLO DGLAP evolution equations.

Two approaches have been widely used for solution of DGLAP equations [1–3]: Solution in x-space or in moment-N space. The first is direct numerical integration of the integro-differential equations in x-space and in the second approach a Mellin transformation is applied to turn the evolution equations into systems of ordinary differential equations (depending on the Mellin variable N) which are more easily accessible to a further analytic treatment. This step is usually performed numerically. Various techniques have been suggested for improving the speed and accuracy of these procedures [4].

The Mellin inversion can be performed using Jacobi polynomials. The x-space behavior of a structure function, or parton distribution, can be reconstructed from a sum over a finite number of Mellin moments weighted by Jacobi polynomials.

It was observed that Jacobi polynomials offer many useful properties to be used [5]: orthogonality for $x \in [0,1]$, a weight function $x^{\beta}(1-x)^{\alpha}$ describing the asymptotic behavior of parton densities and allowing one to factor out an essential part of the x-dependence of the structure function into the weight function [6]. Thus, given the Jacobi moments $a_n(Q^2)$, polarized structure function xg_1 may be reconstructed in form of this series

$$xg_1\left(x,Q^2\right) = x^{\beta}(1-x)^{\alpha}\sum_{n=0}^{N_{\max}} a_n\left(Q^2\right)\Theta_n^{\alpha,\beta}(x)\,, \tag{1}$$

where $N_{\max}$ is the number of polynomials, x is the usual scaling variable, and $\Theta_n^{\alpha,\beta}(x)$ are the Jacobi polynomials of order n,

$$\Theta_n^{\alpha,\beta}(x) = \sum_{j=0}^{n} c_j^{(n)}(\alpha,\beta)x^j\,, \tag{2}$$

where $c_j^{(n)}(\alpha,\beta)$ are the coefficients that expressed through Γ-functions and satisfy the orthogonality relation with the weight $x^{\beta}(1-x)^{\alpha}$ as following

$$\int_0^1 dx\; x^{\beta}(1-x)^{\alpha}\Theta_k^{\alpha,\beta}(x)\Theta_l^{\alpha,\beta}(x) = \delta_{k,l}\,. \tag{3}$$

As $N_{\max}$ becomes arbitrarily large, Eq. (1) is completely general, and in particular the freedom to increase $N_{\max}$ can compensate for injudiciously chosen the constant values of α,β [7]. For the moments, we note that the Q^2 dependence is entirely contained in the Jacobi moments

$$\begin{aligned} a_n\left(Q^2\right) &= \int_0^1 dx\; xg_1\left(x,Q^2\right)\Theta_k^{\alpha,\beta}(x) \\ &= \sum_{j=0}^{n} c_j^{(n)}(\alpha,\beta)\mathbf{M}[xg_1, j+2]\,, \end{aligned} \tag{4}$$

obtained by inverting Eq. (1), using Eqs. (2), (3) and also definition of moments in Mellin-N space,

$$\mathbf{M}[xg_1, N] = \int_0^1 dx\; x^{N-2}xg_1\left(x,Q^2\right)\,. \tag{5}$$

Using Eqs. (1),–(4) now, one can relate the structure function with its moments [8]

$$xg_1^{N_{\max}}\left(x,Q^2\right) \;=\; x^\beta(1-x)^\alpha \sum_{n=0}^{N_{\max}} \Theta_n^{\alpha,\beta}(x) \sum_{j=0}^{n} c_j^{(n)}(\alpha,\beta)\mathbf{M}[xg_1, j+2]\,, \quad (6)$$

where $\mathbf{M}[xg_1, j+2]$ are the moments of the structure function determined by Eq. (5). Given the representation of Eq. (6) for $xg_1^{N_{\max}}(x,Q^2)$, we desire to choose the set $[N_{\max},\alpha,\beta]$ to achieve optimal convergence of this series. In practice, we find that the choice $N_{\max}=9$, $\alpha=3.0$ and $\beta=0.5$ is able to reconstruct $xg_1(x,Q^2)$ with sufficient accuracy through the kinematic region constrained by the data. We did try fits in which α,β were allowed to vary, but they resulted in no noticeable change in χ^2, nor did α,β stray far from their original values. Obviously in Eq. (6), the Q^2-dependence of the polarized structure function is defined by the Q^2-dependence of the moments.

This method was developed and applied to a variety of different QCD analyses for unpolarized [11, 12] and polarized applications [8–10].

We have performed a QCD analysis of the inclusive polarized DIS data at NLO based on the mentioned Jacobi polynomials expansion approach, and extracted the spin structure functions and their moments. The details of the our fitting procedure on of 379 data points are described in Ref. [9,10].

3. Polarized parton distribution functions and their first moments

Considering the fact that we have polarized parton distributions $\delta f(x,Q^2)$ from our performed fitting procedure, we have enough motivation to calculate their first moments,

$$\Delta f_i\left(N,Q^2\right) = \int_0^1 \delta f\left(x,Q^2\right)dx\,, \qquad f=q,\bar{q},g \quad (7)$$

which are the quantities of particular significance since enter the fundamental spin relation in the following sense

$$\tfrac{1}{2} = \tfrac{1}{2}\Delta\Sigma^{\rm p} + \Delta g^{\rm p} + L_z^{\rm p}\,. \quad (8)$$

On the left-hand side we have the spin $+\frac{1}{2}$ of the polarized nucleon state and on the right-hand side a decomposition in terms of the $\Delta\Sigma$, Δg and the relative orbital angular momentum $L_z^{\rm p}$ among all the quarks and gluons [13].

$\Delta(q+\bar{q})$ is the net number of right-handed quarks of flavor q inside a right-handed proton and thus $\frac{1}{2}\Delta\Sigma = \frac{1}{2}(\Delta u + \Delta d + \Delta s + \Delta\bar{u} + \ldots)$ is a measure of how much all quark flavors contribute to the spin of the proton. Similarly, $\Delta g(Q^2)$ represents the total gluonic contribution to the spin of the nucleon.

It should be mentioned that, although $L_z = L_q + L_g$ can be theoretically, formally formulated in a consistent covariant way, there appears to be no direct experimental test on the size as well as of the sign of $L_z(Q^2)$.

4. Results

Since we used an expansion in Jacobi polynomials to facilitate the analysis, it would be helpful to study the dependence of $g_1^{\mathrm{p}}(x,Q^2)$ to the number of Jacobi polynomial expansion terms. In our fitting procedure, we found that the most accurate result would be obtained by $N_{\max} = 9$. In Fig. 1 we present the polarized structure function $g_1^{\mathrm{p}}(x,Q^2)$ in the interval $10^{-4} < x < 1$, and for $Q^2 = 5$ GeV^2 as a function of $N_{\max}$. According to this plot we can study the dependence of polarized proton structure function to the value of $N_{\max}$.

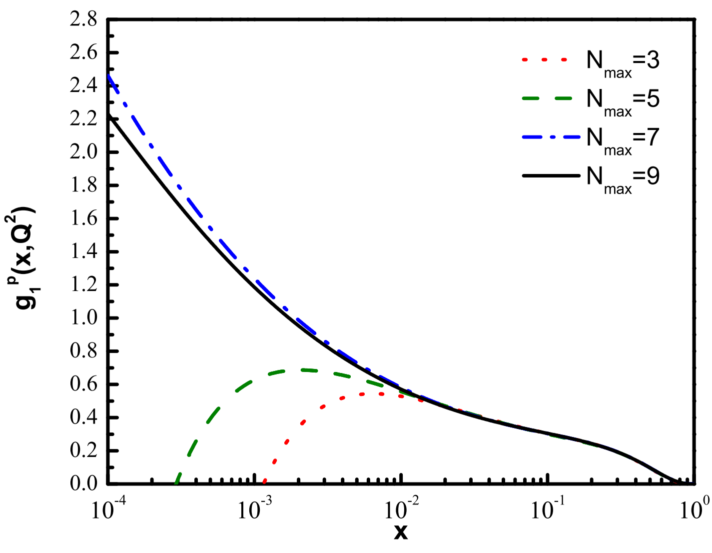

Fig. 1. Our results for $g_1^{\mathrm{p}}(x,Q^2)$ as a function of x in $Q^2 = 5$ GeV^2 and for different values of $N_{\max}$.

In Figs. 2 and 3, the fitted spin dependent sigma and gluon distribution functions in range $x \in [10^{-5}, 1]$ and for different values of Q^2 are shown.

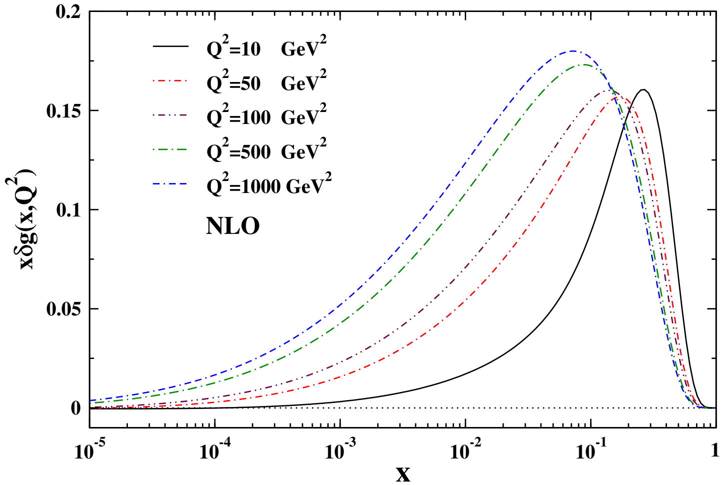

Fig. 2. The polarized gluon distribution as a function of x and for different value of Q^2.

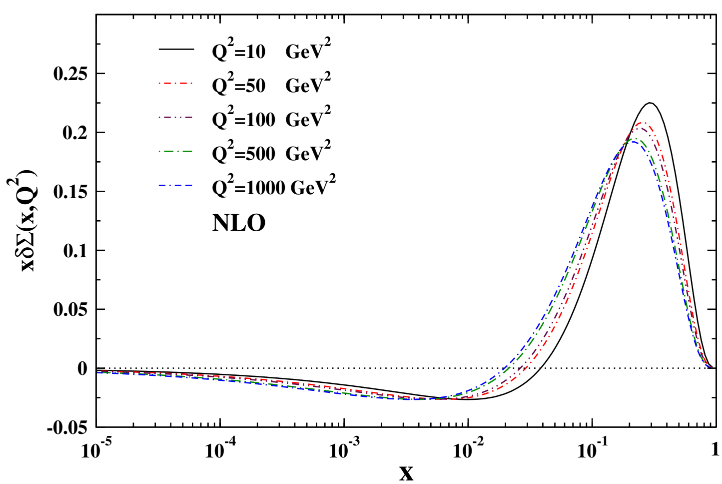

Fig. 3. The polarized sigma distribution as a function of x and for different value of Q^2.

Moments are key ingredients to provide information about the components of the nucleon's spin and unravel some aspects of the quark-gluon structure of nucleon. By having polarized parton distributions, their first moments can be obtained. Table I presents and compares the first moments of our polarized parton densities together with other recent analyses at $Q^2 = 4$ GeV^2 in NLO.

TABLE I

Comparison of the first moments of the polarized parton densities in NLO at $Q^2 = 4$ GeV^2 for different sets of recent parton parametrizations. The second column (Model) contains the first moments which is obtained from our new parametrization based on the Jacobi polynomials expansion method. The BB [14], GRSV [15] and AAC [16] results are also shown.

Distribution	Model	BB [14]	GRSV [15]	AAC [16]
Δu_v	0.928	0.928	0.9206	0.9278
Δd_v	−0.342	−0.342	−0.3409	−0.3416
Δu	0.874	0.866	0.8593	0.8399
Δd	−0.396	−0.404	−0.4043	−0.4295
$\Delta \bar{q}$	−0.054	−0.062	−0.0625	−0.0879
Δg	0.224	0.462	0.6828	0.8076

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