VARIABLE FLAVOR NUMBER PARTON DISTRIBUTIONS AT NEXT-TO-NEXT-TO-LEADING ORDER OF QCD*

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Utilizing recent deep inelastic scattering data and our recent NLO analysis, we present a QCD analysis of the proton in order to determine the parton distributions at next-to-next-to-leading order (NNLO) of QCD. We also study the heavy quark contributions to the proton structure function $F_2^i(x, Q^2)$, with $i = c, b$. Our NNLO analysis will be performed within the modified minimal subtraction factorization and renormalization scheme. This analysis is undertake within the framework of the so-called "zero-mass variable flavor number scheme" (ZM-VFNS) parton model predictions at high energy colliders where the heavy quarks (c, b, t) considered as massless partons within the nucleon.

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1. Introduction

Deeply inelastic electron–nucleon scattering at large momentum transfer provides one of the cleanest possibilities to test the predictions of quantum chromodynamics (QCD) and allows to measure the high-precision parton distribution functions (PDFs) of the nucleons together with the strong coupling constant. The structure function $F_2(x, Q^2)$ that describes the scattering cross-section, is well measured in a wide kinematic region, *cf.* Refs. in Table I.

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Heavy quarks $h = c, b, t$ strongly participate in many high energy processes so the study of heavy quark contributions in deep-inelastic electron–proton scattering and proton structure function provides us with important information about heavy-quark production mechanisms. Our motivation to consider the heavy quark contributions stems from the high statistics data for the charm and bottom structure functions provided by the HERA experiments [1–4], where the former, $F_2^c(x, Q^2)$, accounts for a large fraction (up to 30%) of the total structure function F_2. Since having the precise PDFs is important in the analysis of LHC and other high energy colliders data, all calculating in this regards are welcome. The presently available DIS data allow for high precision extractions of PDFs in global fits. The treatment of the heavy quark contributions in these fits is an important issue as it can induce potentially large effects in the PDFs of light quarks and the gluon obtained from these global fits. During the last years, our understanding of PDFs has steadily improved at the NNLO level, and upcoming high-precision data from hadron colliders will continue in this direction [5–7]. In order to perform a consistent QCD analysis of the DIS world data and other hard scattering data, a NNLO analysis is required and this is our motivation to do the NNLO QCD analysis in the present article. Our previous QCD analyses present parameterisations for valence parton distributions up to N^{3}LO, *cf.* Refs. [8, 9].

2. Overview of theoretical framework

In this section, we give a brief overview of the standard theoretical formalism used in the following global analysis. We work within the common modified minimal subtraction ($\overline{\mathrm{MS}}$) factorization and renormalization scheme, where structure functions in DIS, $F_i(x, Q^2)$, can be written as a convolution of coefficient functions, $C_{i,a}$, with PDFs of flavour a in a hadron of type A, $f_{a/A}(x, Q^2)$, *i.e.*

$$F_i\left(x, Q^2\right) = \sum_{a=q,g} C_{i,a} \otimes f_{a/A}\left(x, Q^2\right) . \tag{1}$$

The total structure function can be written as a sum of $F_i(x, Q^2) = F_i^{\mathrm{light}} + F_i^{\mathrm{heavy}}$, where heavy quark contributions are $F_i^{\mathrm{heavy}} = F_i^c + F_i^b$, and the top quark contribution are negligible. In our analysis the contributions of the c and b quarks were calculated in the "zero-mass variable flavor number scheme" (ZM-VFNS), where the heavy quarks evolve according to the splitting functions for massless quarks and it assumes that at high scales, $Q^2 \gg m_h^2$, the massive quarks behave like massless partons and the coefficient functions are simply those in the massless limit. This scheme is accurate in the region, where Q^2 is so much greater than m_h^2 [6,10–12] because much of the DIS data is in an intermediate and higher-Q^2 range. Here we use solely this scheme since we are mainly interested in the large-Q^2 behavior of structure functions. We allow a maximum of five flavors in the evolution and do not include top quarks. Here m stands for the masses of the heavy quark which we fix them at $m_c = 1.41$ GeV and $m_b = 4.50$ GeV in order to include them in the boundary conditions for evolution. The scale dependence of the PDFs is given by the DGLAP evolution equation in terms of the calculable splitting functions, $P_{aa'}$, *i.e.*

$$\frac{\partial f_{a/A}}{\partial \ln Q^2} = \sum_{a'=q,g} P_{aa'} \otimes f_{a'/A} \,. \tag{2}$$

We have performed all Q^2-evolutions in Mellin n-moment space and used the QCD-PEGASUS program [13] for the NNLO evolutions. The strong coupling satisfies the renormalisation group equation, which up to NNLO reads

$$\frac{d}{d\,\ln Q^2}\left(\frac{\alpha_{\rm s}}{4\pi}\right) = -\beta_0\left(\frac{\alpha_{\rm s}}{4\pi}\right)^2 - \beta_1\left(\frac{\alpha_{\rm s}}{4\pi}\right)^3 - \beta_2\left(\frac{\alpha_{\rm s}}{4\pi}\right)^4 - \ldots . \tag{3}$$

It is worth to mention that we include $\alpha_{\rm s}(Q_0^2)$ as a free parameter in our fits, and determine its value together with the parton distributions. The input for the evolution equations in the 3-flavor scheme $f_{a/A}(x, Q_0^2)$ and $\alpha_{\rm s}(Q_0^2)$, at a reference input scale, taken to be $Q_0^2 = 2$ GeV^2, must be determined from a global analysis of data. In the present study, we use the following functions for the valence quark, gluon, and sea-quark distributions at the starting scale $Q_0^2 = 2$ GeV^2 [7,14–16]

$$\begin{aligned}
xu_v\left(x, Q_0^2\right) &= N_u\, x^{\alpha_u}(1-x)^{\beta_u}\left(1 + \gamma_u\sqrt{x} + \eta_u\, x\right)\,, \\
xd_v\left(x, Q_0^2\right) &= N_d\, x^{\alpha_d}(1-x)^{\beta_d}\left(1 + \gamma_d\sqrt{x} + \eta_d\, x\right)\,, \\
x\Delta\left(x, Q_0^2\right) &= N_\Delta\, x^{\alpha_\Delta}(1-x)^{\beta_\Delta}\left(1 + \gamma_\Delta\sqrt{x} + \eta_\Delta\, x\right)\,, \\
x\Sigma\left(x, Q_0^2\right) &= N_\Sigma\, x^{\alpha_\Sigma}(1-x)^{\beta_\Sigma}\left(1 + \gamma_\Sigma\sqrt{x} + \eta_\Sigma\, x\right)\,, \\
xg\left(x, Q_0^2\right) &= N_g\, x^{\alpha_g}(1-x)^{\beta_g}\,,
\end{aligned} \tag{4}$$

where $\Delta \equiv \bar{d} - \bar{u}$ and $\Sigma \equiv \bar{d} + \bar{u}$. The input PDFs listed in Eq. (4) are subject to the constraints from number sum rules $\int_0^1 u_v dx = 2$ and $\int_0^1 d_v dx = 1$, together with the total momentum sum rule $\int_0^1 x(u_v + d_v + 2(\bar{u} + \bar{d} + \bar{s}) + g)dx = 1$. The parameters N_u, N_d and N_g were calculated from the other parameters using above constrains, therefore there are potentially 21 free PDF parameters in the fit, including $\alpha_{\rm s}(Q_0^2)$. In the present analysis which has been done in the 'standard' approach, we choose as usual the strange quark distribution in the symmetric form $s(x,Q_0^2)=\bar{s}(x,Q_0^2)=\frac{\kappa}{2}(\bar{d}(x,Q_0^2)+\bar{u}(x,Q_0^2))$ with a constant $\kappa \approx 0.4$–0.5 and do not parameterise $s+\bar{s}$ as a function of x. Unlike the case in most of our previous analyses [8,9,17–24], the QCD analysis performed in the present paper determines the $\alpha_s(Q_0^2)$ from fit procedure.

3. Global parton analysis

In this section, we present our global PDF analysis performed using the theoretical formalism together with the treatment of heavy flavors given in previous section. In order to perform this global PDF analysis, we use data from HERA (H1 and ZEUS) measurements, the small-x and large-x H1 and ZEUS F_2^p data for $Q^2 \geq 2$ GeV^2 [25–30]. In addition, we have used the fixed target F_2^p data of SLAC [31], BCDMS [32] , E665 [33] and NMC [34] subject to the standard cuts $Q^2 \geq 4$ GeV^2 and $W^2=Q^2(\frac{1}{x}-1)+m_p \geq 10$ GeV^2. The data sets included in the analysis with their fitted overall normalization $\mathcal{N}_n$, extracted from the first minimization, are listed in Table I and ordered according to the type of process. In the present analysis, we have taken into account correlated errors whenever available, this amounts to the total of 1167 data points. Representative comparisons of our results for parton distribution functions, $xf_i(x,Q^2)$, with the results obtained by Dortmund

TABLE I

Data sets fitted in our NNLO QCD analysis. The fitted normalisations $\mathcal{N}_n$ of the data sets included in the global fit, together with the total normalisation uncertainty, $\Delta\mathcal{N}_n$, for each data set n are also shown in the table. The details of corrections to data and the kinematic cuts applied are contained in the text.

Data sets	NNLO	$\Delta\mathcal{N}_n$	$\mathcal{N}_n$
H1 MB 97 e^+p NC	59 [25]	1.5%	1.0056
H1 low Q^2 96–97 e^+p NC	71 [25]	1.7%	1.0048
H1 high Q^2 99–00 e^+p NC	132 [26]	1.5%	0.9995
H1 high Q^2 94–97 e^+p NC	130 [27]	1.5%	0.9988
H1 high Q^2 98–99 e^-p NC	126 [28]	1.8%	0.9995
ZEUS SVX 95 e^+p NC	30 [29]	1.5%	1.0001
ZEUS 96–97 e^+p NC	242 [30]	2%	0.9972
SLAC ep F_2	37 [31]	2%	1.0075
BCDMS μp F_2	164 [32]	3%	0.9924
E665 μp F_2	53 [33]	1.8%	1.0008
NMC μp F_2	123 [34]	2.5%	1.0012
All data sets	**1167**		

group JR09 [7], MSTW08 [6], and valence analysis of KT [17] and BBG [35] are presented in Fig. 1 at $Q^2 = 10$ GeV^2. As can be seen the results of present analysis for parton distribution functions are in good agreement with these theoretical models. It should be emphasized that the perturbatively stable QCD predictions for structure function are in perfect agreement with all recent high statistics measurements of the Q^2-dependence of $F_2(x, Q^2)$ in wide range of x.

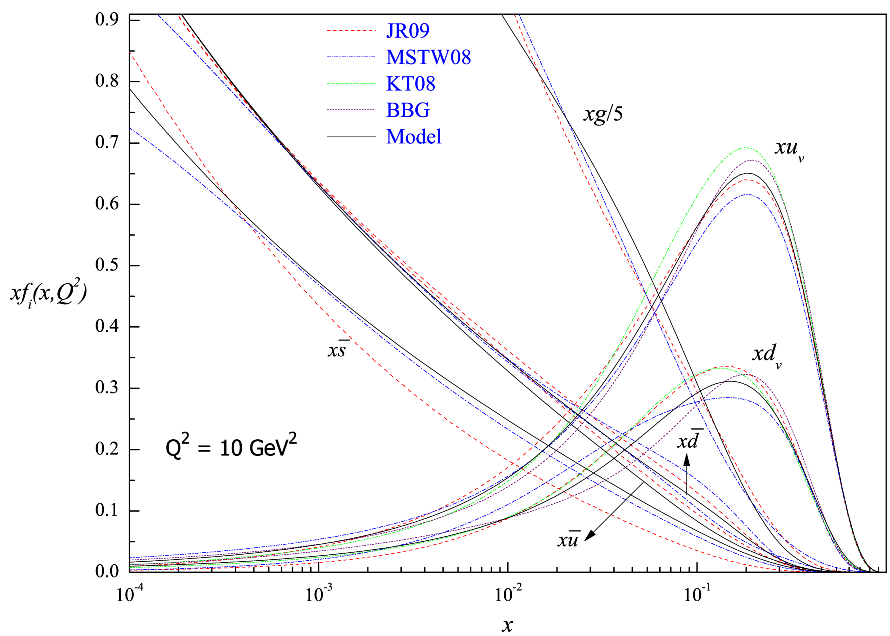

Fig. 1. The parton distribution functions obtained in the NNLO analysis compared with results obtained by MSTW08 [6], JR09 [7], and valence analyses of KT [17] and BBG [35]. The comparison is shown for $Q^2 = 10$ GeV^2.

4. Summary and conclusions

In summary, we have performed a global PDF fit using recent deep inelastic data on proton in the standard parton model approach at next-to-next-to-leading order of perturbative QCD. Obtained proton structure function are fully compatible with all recent high-statistics measurements of the Q^2-dependence of F_2^p in that region. The strong coupling constant obtained from our standard next-to-next-to-leading order (NNLO) QCD analysis $\alpha_s(M_Z^2) = 0.1151 \pm 0.0041$ has been compared with the results obtained by other theoretical models, see Table II. Our present study on parton distribution function and strong coupling constant can be effectively used to obtain the Higgs production cross-section.

TABLE II

Comparison of different measurements of $\alpha_s(M_Z^2)$ at NNLO and higher order.

	$\alpha_s(M_Z^2)$		
N^2LO	Model	0.1151 ± 0.0041	standard approach
	MSTW08 [6]	$0.1171^{+0.0014}_{-0.0014}$	
	JR09 [7]	0.1124 ± 0.0020	dynamical approach
	KT08 [17]	0.1131 ± 0.0019	valence analysis
	BBG [35]	$0.1134^{+0.0019}_{-0.0021}$	valence analysis
N^3LO	KKT09 [8]	0.1139 ± 0.0020	valence analysis
	BBG [35]	$0.1141^{+0.0020}_{-0.0022}$	valence analysis

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