SUBTHRESHOLD $\bar{N}N$ AMPLITUDES FROM THE LIGHTEST ANTIPROTONIC ATOMS*

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The data from the simplest antiprotonic-hydrogen and -helium atoms can be used to extract the elastic antiproton–nucleon scattering amplitudes below the antinucleon–nucleon threshold. These are interesting since two quasi-bound S and P antiproton–nucleon states are likely to exist in this region. Such states, indicated by the phenomenological Paris potential, find support in the atomic level widths. Additional evidence for the broad S-wave state comes from J/ψ physics. The other narrow P-wave state requires further experimental checks.

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1. Introduction

The lightest hadronic atoms involve hadron–nucleon interactions on bound nucleons. This offers a chance to extract the hadron–nucleon scattering amplitudes just below the thresholds. This energy region is of special interest in cases of bound or quasi-bound states in the hadron–nucleon systems. The two cases of current concern, the $\bar{p}N$ and $\bar{K}N$ states, are similar in this respect. The point of interest is the existence of exotics in the $\bar{N}N$

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and $\bar{K}N$ systems: quasi-bound, virtual or resonant states. One way to detect such states is to measure the scattering lengths for a given spin and isospin: a large length signals a bound state. Scattering experiments are the way to achieve this, but a unique separation of the quantum states is not easy. A complementary measurement of X-ray transitions in exotic atoms may be profitable, especially when the spin structure of the atomic levels is known. Such resolution of fine structure has been so far obtained only in antiprotonic hydrogen, [1,2]. In deuterium [2] and helium [3] the states are not resolved yet. The energies of atomic states are determined essentially by the Coulomb potential, but, because of the nuclear interactions, these energies are shifted. Atomic states become also unstable as a result of the nucleon–antinucleon annihilation. In X-ray studies the level shifts and widths may be measured and from these one can extract the $\bar{N}N$ amplitudes in the subthreshold energy region. In this note we argue that the existing atomic data indicate the existence of two $\bar{N}N$ exotic states. One is a broad S-wave state, the other is a narrow P-wave state. There is no spin and isospin selectivity without a fine structure resolution, so we cannot specify quantum numbers of these states. However, there are other experiments which, to some extent, allow this.

One can reach selected states in formation experiments. In this way BES Collaboration observes a $p\bar{p}$ threshold enhancement in the decay $J/\psi \to \gamma p\bar{p}$ [4]. This enhancement may be attributed to a quasi-bound or a virtual state in the $^{2t+1,2s+1}L_J = {}^{1,1}S_0$ state. Another measurement, $J/\psi \to \gamma\eta'\pi^+\pi^-$, finds a peak in the meson invariant mass spectrum. This phenomenon named $X(1835)$ is tentatively attributed to an intermediate $p\bar{p}$ system in the same $^{1,1}S_0$ state [5].

On can correlate these findings in terms of the recent Paris potential for the $N\bar{N}$ interactions. This meson-exchange semi-phenomenological potential incorporates about 4000 scattering and atomic data [6] and predicts one S- and one P-wave bound state close to the $p\bar{p}$ threshold. On the other hand, the $N\bar{N}$ potential of Juelich group generates no S-wave bound state [7]. We argue here that the PAX-Collaboration at CERN [8] might provide an additional test of these models and check the coupling of the $X(1835)$ to the $N\bar{N}$ channel.

2. Atomic experiments

Experimental atomic level widths are measured in ^{1}H$(1S, 2P)$, ^{2}H$(1S, 2P)$, ^{3}He$(2P, 3D)$ and ^{4}He$(2P, 3D)$. The shifts are known in the lower of these states. In the first step of our presentation we recall that these shifts are equivalent to the zero energy scattering lengths (S waves) and volumes (P waves) *etc.* for the antiproton scattering on each of these light nuclei.

2.1. The $\bar{N}$-nucleus scattering parameters extracted from atomic data

The atomic level shifts and widths are expected to be proportional to the overlap of atomic and nuclear densities. If the shift is small, compared to the level spacing, it has to be proportional to the zero energy scattering parameter A_L, where L is the atomic angular momentum. The proper relation is known as Trueman formula [9]. It follows the expansion of level shift into A_L/B^L. The latter is a small parameter as the Bohr radius B is usually much larger than the characteristic lengths of nuclear interactions. For S waves, this relation is

$$\Delta E_{nS} - i\Gamma_{nS}/2 = E_{nS} - \varepsilon_{nS} = -\frac{2\pi}{\mu}|\psi_n(0)|^2 A_0(1+\lambda A_0/B)\,, \qquad (1)$$

where ψ is the atomic Coulomb wave function. The electromagnetic energy ε_{1S} in Eq. (1) is composed of the Bohr's atom energy $\varepsilon^o_{1S} = -\mu(\alpha)^2/2$ corrected for relativity and electric polarization. The Bohr radius is given by $= 1/(\alpha\mu)$ where μ is the reduced mass. In the $1S$ states one has $\lambda = 3.154$, and with $A_0 \approx 1$ fm the second order term in Eq. (1) constitutes a few percent correction. For higher angular momentum states a simpler relation, linear in A_L/B^L, [10], is sufficient

$$\Delta E_{nL} - i\Gamma_{nS}/2 = \varepsilon^o_{nL}\frac{4}{n}\Pi^L_{i=1}\left(\frac{1}{i^2}-\frac{1}{n^2}\right)A_L/B^{2L+1} \equiv \Omega_{nL}A_L\,. \qquad (2)$$

TABLE I

Some properties of the simplest antiprotonic atoms. The second column gives the Bohr radii and the three other columns give overlaps Ω relating the level shifts to the nuclear scattering parameters.

Atom	B [fm]	Ω_{1S} [keV/fm]	Ω_{2P} [meV/fm^3]	Ω_{3D}[μeV/fm^5]
$\bar{p}\ ^1$H	57.639	0.8668	24.46	0.3591
$\bar{p}\ ^2$H	43.247	1.5398	77.19	2.0129
$\bar{p}\ ^3$He	19.22	15.59	3958	522.6
$\bar{p}\ ^4$He	18.02	17.77	5125	770.1

The scattering parameters, lengths, volumes *etc.* are commonly used to parameterize the strong interaction part of the scattering amplitudes at low energies. The standard partial wave expansion is

$$f\left(\boldsymbol{p},E,\boldsymbol{p}'\right) = \Sigma_L(2L+1)P_L\left(\hat{p}\cdot\hat{p'}\right)\left(pp'\right)^L A_L\,, \qquad (3)$$

where $\boldsymbol{p}$ and $\boldsymbol{p}'$, of modulus p and p', respectively, are the initial and final momenta in the c.m. system. Here $\hat{p}\cdot\hat{p}' = \boldsymbol{p}\cdot\boldsymbol{p}'/(pp')$. The scattering lengths are defined in the baryon–nucleon convention, $A = A_r + i|A_i|$. The amplitudes f and scattering parameters A_L involve "the inner" effects of Coulomb interactions at the nuclear distances. Thus, on a phenomenological level, the measurements of atomic level shifts are equivalent to the measurements of low energy scattering amplitudes.

2.2. Relation between zero energy parameters A_L and $\bar{N}N$ scattering amplitudes

The $\bar{p}N$ interactions in atoms can be described by two scattering amplitudes: the length $a(E)$-for S waves and the volume $b(E)$-for P waves. For a zero energy antiproton scattering on a bound nucleon, the energies, involved in the $\bar{p}N$ center of mass, span certain region below the $\bar{p}N$ threshold. This is a consequence of binding given by the nucleon separation energy $E_{\rm S}$ and $E_{\rm recoil}$ — the recoil of the $\bar{p}N$ subsystem with respect to the rest of the nucleus. Thus, the elementary $\bar{p}N$ scattering amplitudes, involved in the atomic-system interactions, become functions of energy, *viz.*: $a(-E_{\rm S} - E_{\rm recoil})$ and $b(-E_{\rm S} - E_{\rm recoil})$.

The main problem now is to express the nuclear scattering parameters A_L in terms of the elementary scattering amplitudes $a(E)$ and $b(E)$. The next step is the extraction of $a(E)$ and $b(E)$ from the experimental values of A_L. The first step is a lengthy calculation. The optical potential method is not applicable and the method used here consists in summation of the multiple scattering series.

For an illustration we present the nuclear scattering parameter A_L given by a single $\bar{p}N$ collision in S wave. Assuming contact interactions one obtains

$$A_L = \frac{\mu_{\bar{p},AN}}{\mu_{\bar{p},N}} \frac{\langle R^{2L} \rangle}{[(2L+1)!!]^2} \,\Sigma_i \, \bar{a}_i(E)\,, \tag{4}$$

where

$$\bar{a}(E) = \int d\boldsymbol{p}\, a\left(-E_{\rm S} - E_{\rm rec}(p)\right) F_L(p) \tag{5}$$

is the appropriate $\bar{p}N$ scattering amplitude averaged over the subthreshold energies. In Eq. (4) $\mu_{\bar{p},AN}$ and $\mu_{\bar{p},N}$ represent the $\bar{p}$-nucleus and $\bar{p}$-nucleon reduced masses, respectively. Weighting functions F_L describe the distribution of $\bar{p}N$ total momentum, and are calculable with $\bar{p}$ and N wave functions. The average energies are given in Table II.

The coordinate R which enters Eq. (4) is the distance of the antiproton to the center of mass of the nucleus. Thus the scattering parameters involve the $2L$-th moments of the nuclear density which are fairly well known. The sum extends over all nucleons in the system. The equation (4) is satisfactory only in higher angular momentum states. For lower L one has to sum the multiple scattering series. The summation method is described in Ref. [11].

TABLE II

Average subthreshold energies $E_{\rm S} + E_{\rm recoil}$ involved in the $\bar{p}N$ amplitudes in the atomic systems (rounded off to one MeV). Numbers in parentheses indicate approximate half-widths of the corresponding energy distributions.

Atom	$1S$	$2P$	$3D$
$\bar{p}\ ^1$H	0	0	0
$\bar{p}\ ^2$H	11(5)	4(2)	3(1)
$\bar{p}\ ^2$He	19(7)	8(2)	7(1)
$\bar{p}\ ^4$He	36(10)	36(10)	35(10)

With no fine structure data we operate with spin and isospin averaged $a(E)$ and $b(E)$. As there are less shifts than widths only the absorptive parts of these amplitudes may be extracted by a best fit procedure. The corresponding $a(\langle E\rangle)$ attributed to the average energies are given in Fig. 1.

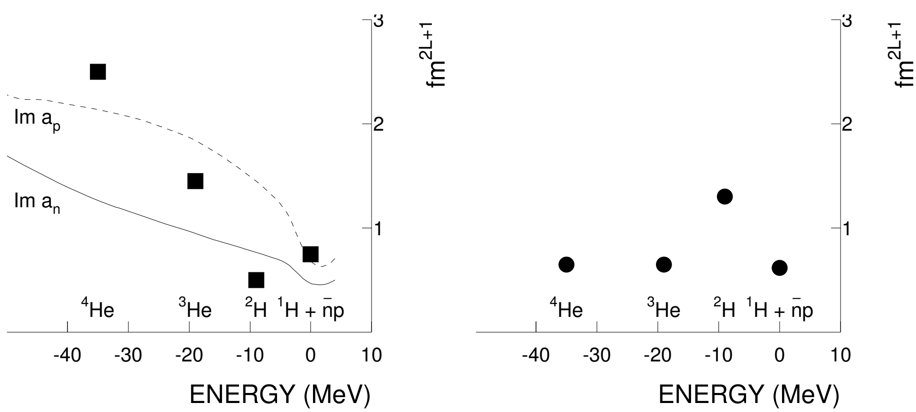

Fig. 1. Left panel: the absorptive parts of $\bar{p}N$ S-wave amplitudes in the subthreshold region extracted from the atomic level shifts and widths. The errors are of the size of the squares. The curves are calculated with the Paris $N\bar{N}$ potential, [6]. The dotted line refers to $p\bar{p}$ and the continuous line to $n\bar{p}$ pairs. Right panel: the absorptive parts of $\bar{p}N$ P-wave scattering volumes. The enhancement in deuteron is consistent with a P-wave bound state predicted by the Paris potential at 9 MeV.

3. Other experiments

Atomic experiments in heavy atoms indicate some increase of the absorption down the subthreshold region, [12], but a continuation of the plot in Fig. 1 cannot be obtained this way. On the other hand, the Paris potential generates a resonant like shape of the $\mathrm{Im}\, a(E)$ due to a 50 MeV broad quasi-bound state in the $^{1,1}S_0$ state. This state dominates $\mathrm{Im}\, a(E)$ and is apparently related to the $X(1835)$. Figure 2 shows the experimental invariant mass of $\eta'\pi^+\pi^-$ representing the X. The related curve is calculated with the Paris potential assuming the $J/\psi \to \gamma\eta'\pi^+\pi^-$ reaction going via the $\gamma +^{1,1} S_0\ \bar{N}N$ state.

It would be interesting to check the conclusion that $X(1835)$ is related to $\bar{N}N$ quasi-bound state. One possibility is opened by the PAX proposal [8] to construct a source of polarized antiprotons. The inclusive reaction $\bar{p}p \to \gamma + X$ performed with parallel or anti-parallel initial spins could pinpoint the shape of the X. Rough estimates give the cross section (integrated over the resonance line) of the order of few microbarns.

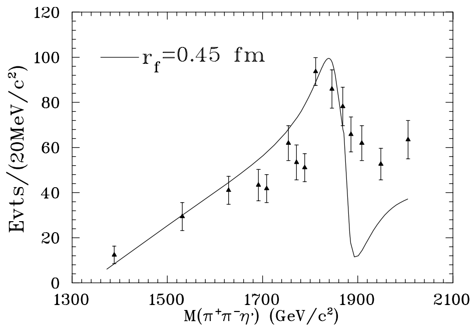

Fig. 2. The spectral function representing the $X(1835)$ shape, from BES. The curve calculated with the Paris model results as an interference of two decay amplitudes.

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