K^-pp STUDIED WITH COUPLED-CHANNEL COMPLEX SCALING METHOD*

Akinobu Doté

High Energy Accelerator Research Organization (IPNS/KEK)
1-1 Oho Tsukuba Ibaraki, 305-0801 Japan

Takashi Inoue

Graduate School of Pure and Applied Sciences, University of Tsukuba
Tsukuba, 305-8577 Ibaraki, Japan

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We introduce the recent status of theoretical studies of the most essential kaonic nucleus K^-pp. The calculated binding energy and decay width are strongly dependent on theoretical models. In particular, there is a large discrepancy between the results of a variational calculation and a Faddeev calculation, in spite that $\bar{K}N$ potentials used in both calculations are constrained by chiral SU(3) theory. This discrepancy is expected to be caused by $\pi\Sigma N$ three-body dynamics which might be missing in the variational calculation. In order to consider this issue, we propose "Coupled-channel Complex Scaling Method" which can be regarded as an extended method of the variational scheme.

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1. Introduction

$\bar{K}$-nuclear system (nuclei with anti-kaon, kaonic nuclei) has been expected to have interesting properties, because of strong $\bar{K}N$ attraction. Kaonic nuclei have been a hot topic in nuclear and hadron physics, since possible existence of deeply bound kaonic nuclei was predicted. According to studies with a phenomenological $\bar{K}N$ potential reproducing low-energy $\bar{K}N$ data, a K^- meson is found to be deeply bound in light nuclei with more than 100 MeV binding energy, and such $\bar{K}$-nuclear systems can exist as a quasi-bound state with narrow decay width [1]. The strong $\bar{K}N$ attraction makes a nucleus shrunk to be highly dense (up to 4 times normal density in average) involving so strange structures that we have never seen in the

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study of ordinary nuclei [2]. However, such exotic features of kaonic nuclei have not been established yet in both theoretical and experimental studies. These years, researchers have focused on a prototype of kaonic nuclei K^-pp, before going to more complicated systems. We expect that essential properties of kaonic nuclei are clarified by investigating such a simple system; the K^-pp is indeed a three-body system composed of two protons and a K^- meson.

TABLE I

Summary of recent results of theoretical calculations of K^-pp. DHW *etc.* is a label indicating each calculation. The binding energy and decay width ($K^-pp \to \pi Y$) obtained in each calculation are shown as B.E. and width (πY) in unit of MeV, respectively. Method indicates the method of each calculation. The type of employed $\bar{K}N$ potential is given in the line of $\bar{K}N$ int.: Chiral means a potential based on chiral SU(3) theory, while Phenom. means a phenomenological potential.

	DHW [3]	AY [4]	IS [5]	SG [6]
B.E.	20 ± 3	48	60–95	50–70
Width (πY)	40–70	61	45–80	90–110
Method	Variational	Variational	Faddeev	Faddeev
$\bar{K}N$ int.	Chiral	Phenom.	Chiral	Phenom.

2. Recent status of theoretical studies of K^-pp

We summarize results of recent calculations of K^-pp with s-wave $\bar{K}N$ potential in Table I[1]. All calculations listed in this table show that the K^-pp system can be bound. However, the predicted binding energy and decay width are strongly dependent on employed methods and used interactions, although all these studies are constrained by the existing experimental data on $\bar{K}N$ system; the low energy scattering data, kaonic hydrogen atom data and the observed $\Lambda(1405)$.

Difference between the results of two variational calculations, DHW [3] and AY [4], is clearly caused by different interpretation of the $\Lambda(1405)$ which is an important building block for kaonic nuclei. In the phenomenological AY study, the experimentally observed $\Lambda(1405)$ is described as a quasi-bound state of $I = 0$ $\bar{K}N$ system with 27 MeV binding energy (corresponding to 1405 MeV), embedded in $\pi\Sigma$ continuum state. On the other hand, according to the chiral SU(3) model the DHW study based on, the $\pi\Sigma$ state also forms a broad resonant state, in which the $I = 0$ $\bar{K}N$ bound state is embedded. In this model, the $\bar{K}N$ resonance in $I = 0$ channel appears at 1420 MeV,

[1] A study including p-wave $\bar{K}N$ potential is reported in Ref. [7].

not the nominal position at 1405 MeV. Because of such a different picture of $\Lambda(1405)$, the attraction of the chiral-based $\bar{K}N$ potential is about half of that of a phenomenological one. As a result, the DHW calculation shows shallow binding of $K^{-}pp$, compared to the AY calculation.

There is a large discrepancy also between DHW and IS [5] results, although the $\bar{K}N$ potentials used in both calculations are based on the chiral SU(3) theory. The binding energy of $K^{-}pp$ is 20 MeV in the DHW study, while it is 80 MeV in the IS study. As origin of this difference, several reasons are considered: finite-range potential in r-space or separable potential in p-space, non-relativistic or semi-relativistic treatment, and so on. Here, we focus on a possible origin of three-body $\pi\Sigma N$ dynamics [8].

In the DHW study, the variational calculation is performed in a single $\bar{K}N$ channel with an effective $\bar{K}N$ potential which is constructed so as to reproduce the $\bar{K}N$ scattering amplitude predicted by the coupled-channel chiral dynamics. [9] The $\pi\Sigma$ channel is eliminated and incorporated into the effective $\bar{K}N$ potential, as explained diagrammatically in the left panel of Fig. 1. This procedure can be performed exactly in case of two-body system. Applying the effective $\bar{K}N$ potential to a three-body system, we came across a problem: Certainly the $\pi\Sigma$ channel is involved in the effective potential, but the $\pi\Sigma$ energy is assumed to be always equal to the energy of the initial $\bar{K}N$ state. This is correct for the case of the isolated two-body system. However, this is not true for the two-body subsystem in the three-body system. In the three-body system of $K^{-}pp$, the total energy is conserved but the energy of the intermediate $\pi\Sigma$ state can differ from the initial $\bar{K}N$ energy, due to the existence of the third nucleon. (See the right panel of Fig. 1.)

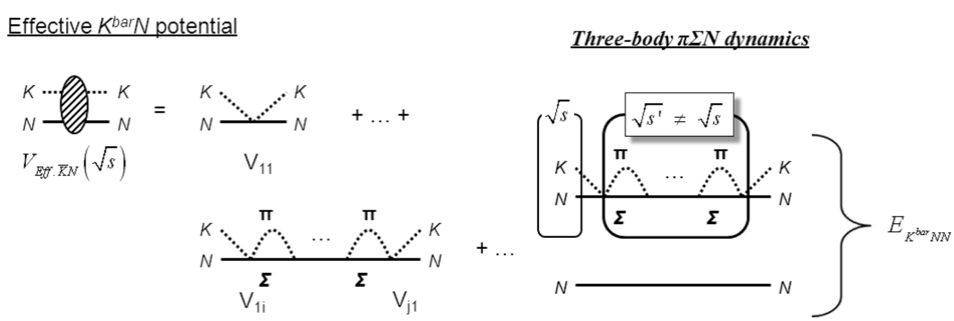

Fig. 1. Diagrammatic explanation of the effective $\bar{K}N$ potential used in DHW (left panel) and three-body $\pi\Sigma N$ dynamics (right panel).

Such three-body $\pi\Sigma N$ dynamics is obviously taken into account in the Faddeev calculation, because the $\pi\Sigma$ degree of freedom is explicitly treated, differently from the variational calculation. This dynamics is expected to bring extra attraction to the K^-pp system.

3. Complex scaling method

To investigate the effect of the $\pi\Sigma N$ dynamics on the variational calculation, explicit treatment of the $\pi\Sigma$ channel is needed as done in the Faddeev calculation. However, such a coupled-channel calculation seems impossible in a naive variational scheme used for bound-states calculation[2]. From recent studies, the K^-pp state is expected to exist between $\bar{K}NN$ and $\pi\Sigma N$ thresholds as shown in the left panel of Fig. 2. If this is the case, the K^-pp is surely a bound state for the $\bar{K}NN$ channel, but it is a resonant state for the $\pi\Sigma N$ channel. The variational calculation cannot be applied to this case, because the variational principle is useless to search for resonant states. Here, we employ Complex Scaling Method (CSM) [10] to deal with resonant states, because this method is considered to be an extension of variational method as explained later.

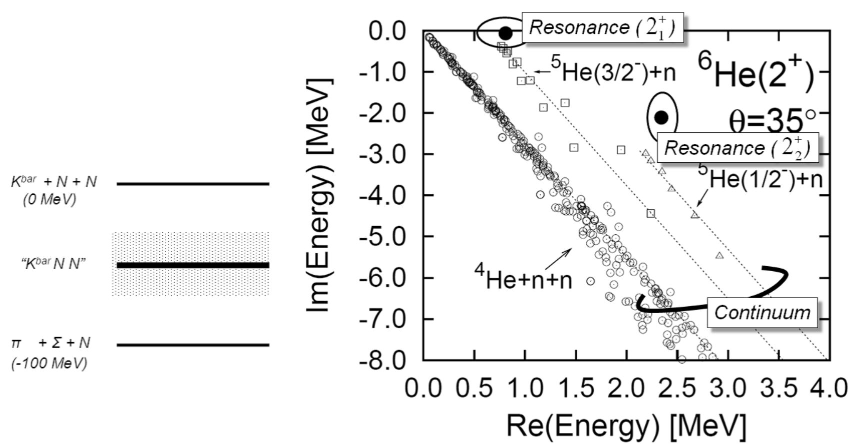

Fig. 2. (Left) Threshold relation. (Right) Result of ^{6}He calculated with CSM at $\theta = 35°$. (Given by Dr. Myo [11].)

In the CSM, Hamiltonian H and a wave function $|\Phi\rangle$ are transformed by complex scaling for a coordinate $\boldsymbol{r}$:

$$U(\theta) : \boldsymbol{r} \to \boldsymbol{r}e^{i\theta}\,,\ \boldsymbol{k} \to \boldsymbol{k}e^{-i\theta} \Longrightarrow H_\theta \equiv U(\theta)HU^{-1}(\theta)\,, \quad |\Phi_\theta\rangle \equiv U(\theta)|\Phi\rangle\,. \tag{1}$$

[2] With the aid of multiple scattering theory, such a coupled-channel calculation in the variational scheme is realized in Ref. [7].

Energies of bound and resonant states are independent of the scaling angle θ while those of continuum states, which are complex value due to the complex scaling, change with θ (*ABC theorem* [10]). Therefore, investigating the θ-dependence of the energy of each state, we can find resonant states.

An advantage of the CSM is that resonant states can be calculated quite in the same way as bound states. Here, let us consider a case of two-body system. A wave function of a resonant state is expressed as

$$\Phi_R(r) \sim e^{ik_R r} = e^{i(\kappa - i\gamma)r} , \tag{2}$$

where κ and γ are real numbers and γ is positive. By applying the complex scaling to this wave function it is modified as

$$U(\theta)\Phi_R(r) \sim e^{ik_R r e^{i\theta}} = \exp[(\gamma \cos\theta - \kappa \sin\theta)r] \exp[i(\gamma \sin\theta + \kappa \cos\theta)r] . \tag{3}$$

When a condition $\theta > \tan^{-1}(\gamma/\kappa)$ is satisfied, the first exponential term becomes a dumping function. In other words, the boundary condition for resonant states is modified to the same one for bound states. Therefore, we can obtain resonant states by diagonalizing the complex-scaled Hamiltonian H_θ with Gaussian base as done in usual studies of bound states.

The CSM has often been applied to nuclear physics. In particular, it has succeeded in the study of resonant states of unstable nuclei. As an example, we show the result of ^{6}He studied with CSM [11]. The right panel in Fig. 2 shows the distribution of eigenvalues of the complex-scaled Hamiltonian on a complex-energy plane $(E, \Gamma/2)$. The eigenvalues indicating continuum states necessarily appear on the line of $\tan^{-1}(\Gamma/2E) = -2\theta$, even if we change the scaling angle θ. On the other hand, the eigenvalues for resonant states keep staying at the same position for any θ, separately from the 2θ line. In the present example, two resonant states, 2_1^+ and 2_2^+, are found at $(E, \Gamma) = (0.81, 0.13)$ MeV and $(2.35,\ 4.22)$ MeV, respectively. The energy and width of the obtained 2_1^+ are in good agreement with the experimental observation, (E, Γ)=(0.822, 0.113) MeV.

4. Summary and future plans

In this article, we propose the application of Coupled-channel Complex Scaling Method to the study of the most essential kaonic nucleus, K^-pp, in order to consider the three-body $\pi\Sigma N$ dynamics which might be a reason of large discrepancy between results of the variational and Faddeev calculations with $\bar{K}N$ potentials constrained by chiral SU(3) theory.

We are going to investigate K^-pp with coupled-channel CSM using novel NN and $\bar{K}N$ potentials that are employed in the previous variational calculations [3, 4]. It is expected that we can obtain more reliable result on K^-pp. Furthermore, detailed properties of the K^-pp system can be clarified by analyzing the obtained wave function.

Recently, DISTO group reported a clear peak structure in the Λp invariant-mass spectrum just on the $\pi\Sigma N$ threshold [12]. Currently, we do not know what the observed state is, although it might be the deeply bound state of K^-pp. The $\pi\Sigma N$ dynamics is considered to play an important role in such energy region. Our study will contribute to clarify the reported state.

We have not reached conclusive result in experimental study as well as in theoretical study, although several experimental results for kaonic nuclei have been so far reported [13]. This year, J-PARC (Japan Proton Accelerator Research Complex) has started the operation. In J-PARC, two experiments related to kaonic nuclear physics are planned as Day-1 experiments. One experiment is the precise measurement of $3d \to 2p$ X-ray on kaonic ^{3}He atom. This experiment is now undergoing and is expected to give an important constraint to the $\bar{K}N$ interaction. The other experiment is a experiment to search for K^-pp with a reaction ^{3}He (in-flight K^-, n) K^-pp decaying to Λp. This experiment is so-called a *perfect experiment* because all particles will be measured. These J-PARC experiments will provide us helpful hints to resolve controversial situation of today on kaonic nuclei.

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