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NONEXTENSIVE CRITICAL EFFECTS IN THE NAMBU–JONA-LASINIO MODEL*

Jacek Rożynek, Grzegorz Wilk

Theoretical Physics Division, The A. Sołtan Institute for Nuclear Studies
Hoża 69, 00-681 Warsaw, Poland

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The critical phenomena in strongly interaction matter are generally investigated using the mean-field model and are characterized by well defined critical exponents. However, such models provide only average properties of the corresponding order parameters and neglect altogether their possible fluctuations. Also the possible long range effect are neglected in the mean field approach. Here we investigate the critical behavior in the nonextensive version of the Nambu–Jona-Lasinio model (NJL). It allows to account for such effects in a phenomenological way by means of a single parameter q, the nonextensivity parameter. In particular, we show how the nonextensive statistics influence the region of the critical temperature and chemical potential in the NJL mean field approach.

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1. Introduction

Critical phenomena in strongly interaction matter are of great interest nowadays, *cf.*, for example, [1, 2]. They are usually described by a mean field type of theories[1]. Such theories are based on the usual Boltzmann–Gibbs (BG) statistical mechanics and reflect only behavior of the mean field, *i.e.*, are not able to accommodate effects of possible fluctuations and/or correlations caused, among others, by the smallness of the sample of matter under consideration, by its rapid evolution, or by limitations of the available phase space. All these factors (both separately and taken together) render the spatial configuration of the system being far from uniform and prevent the global equilibrium from being established. Nevertheless, it is known

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[1] See, for example, [3] or [4] mentioned in this presentation, we refer to them for the most recent literature on this subject.

in the literature that it is possible to maintain simplicity of the statistical description and, at the same time, to account for these effects provided one uses a nonextensive version of the statistical mechanics, for example the one proposed by Tsallis [5] (which we shall use here). In this approach action of all the above mentioned factors is summarily accounted for by one additional parameter q, the nonextensivity parameter. It does not differentiate between the particular dynamical phenomena responsible for departure from the BG picture. In this approach the usual BG exponent, $\exp(-X/T)$, is deformed into the so called q-exponent (Tsallis distribution), $\exp_q(-X/T) = [1-(1-q) X/T]^{1/(1-q)}$, such that for $q \to 1$ one recovers the BG picture again.

The applications of the nonextensive statistical mechanics to nuclear and particle physics are numerous and we refer to [6] for details. In what follows we shall present the nonextensive version of the NJL model, the q-NJL model [7] [2]. Details of the q-NJL model and most of results were already presented in [7], here we shall concentrate on the influence of dynamical factors causing nonextensivity and represented by parameter q on the vicinity of the *critical end point* (CEP).

2. Results

Let us present first the basic elements of the q-NJL model introduced in [7] (to which we refer for more details). It is a q-version of standard SU(3) NJL model with $U(1)_A$ symmetry described in [4], with the usual Lagrangian of the NJL model used in a form suitable for the bosonization procedure (with four quarks interactions only), from which we obtain the gap equations for the constituent quark masses M_i:

$$M_i = m_i - 2g_S \langle \bar{q}_i q_i \rangle - 2g_D \langle \bar{q}_j q_j \rangle \langle \bar{q}_k q_k \rangle \,, \tag{1}$$

with cyclic permutation of $i, j, k = u, d, s$ and with the quark condensates given by $\langle \bar{q}_i q_i \rangle = -i\mathrm{Tr}[S_i(p)]$ ($S_i(p)$ is the quark Green function); m_i denotes the current mass of quark of flavor i. We consider a system of volume V, temperature T and the i-th quark chemical potential μ_i characterized by the baryonic thermodynamic potential of the grand canonical ensemble (with quark density equal to $\rho_i = N_i/V$, the baryonic chemical potential $\mu_B = \frac{1}{3}(\mu_u + \mu_d + \mu_s)$ and the baryonic matter density as $\rho_B = \frac{1}{3}(\rho_u + \rho_d + \rho_s)$),

$$\Omega(T, V, \mu_i) = E - TS - \sum_{i=u,d,s} \mu_i N_i \,. \tag{2}$$

[2] Nonextensive calculations using the Walecka model for dense nuclear matter has been done in [8].

The internal energy, E, the entropy, S, and the particle number, N_i, are given by [4] (here $E_i = \sqrt{M_i^2 + p^2}$):

$$\begin{aligned} E &= -\frac{N_c}{\pi^2} V \sum_{i=u,d,s} \left[\int p^2 dp \frac{p^2 + m_i M_i}{E_i} (1 - n_i - \bar{n}_i) \right] \\ &\quad - g_S V \sum_{i=u,d,s} \left(\langle \bar{q}_i q_i \rangle \right)^2 - 2 g_D V \langle \bar{u} u \rangle \langle \bar{d} d \rangle \langle \bar{s} s \rangle , \qquad (3) \\ S &= -\frac{N_c}{\pi^2} V \sum_{i=u,d,s} \int p^2 dp \, \tilde{S} , \qquad (4) \end{aligned}$$

where $\tilde{S} = [n_i \ln n_i + (1 - n_i) \ln(1 - n_i)] + [n_i \to 1 - \bar{n}_i]$,

$$N_i = \frac{N_c}{\pi^2} V \int p^2 dp \, (n_i - \bar{n}_i) \; . \qquad (5)$$

The $n_i = 1/\left\{\exp\left[\beta\left(E_i - \mu_i\right)\right] + 1\right\}$ and $\bar{n}_i = 1/\left\{\exp\left[(\beta(E_i + \mu_i)\right] + 1\right\}$ are, respectively, quark and antiquark occupation numbers with which one calculates values of the quark condensates present in Eq. (1),

$$\langle \bar{q}_i q_i \rangle = -\frac{N_c}{\pi^2} \sum_{i=u,d,s} \left[\int \frac{p^2 M_i}{E_i} (1 - n_i - \bar{n}_i) \right] dp \, . \qquad (6)$$

Eqs. (1) and (6) form a self consistent set of equations from which one gets the effective quark masses M_i and values of the corresponding quark condensates.

The values of the pressure, P, and the energy density, ε, are defined as:

$$P(\mu_i, T) = -\frac{\Omega(\mu_i, T)}{V} \, , \quad \varepsilon(\mu_i, T) = \frac{E(\mu_i, T)}{V} \quad \text{with} \quad P(0,0) = \varepsilon(0,0) = 0 \, . \qquad (7)$$

The q-statistics is introduced by using the q-form of quantum distributions for fermions $(+1)$ and bosons (-1), namely, $n_{qi} = 1/\left\{\tilde{e}_q(\beta(E_i - \mu_i)) \pm 1\right]\}$ where $(x = \beta(E - \mu))$ $\tilde{e}_q(x) = [1 + |(q-1)x|]^{x/|(q-1)x|}$ [7]. With such choice one can treat consistently on the same footing quarks and antiquarks, which should show the particle-hole symmetry observed in the q-Fermi distribution in plasma containing both particles and antiparticles, namely that $n_q(E, \beta, \mu, q) = 1 - n_{2-q}(-E, \beta, -\mu)$ [3]. The q-NJL model is obtained by re-

[3] In a system containing both particles and antiparticles both q and $2 - q$ occur (*i.e.*, one can encounter both $q > 1$ and $q < 1$ at the same time). It means that not only the $q > 1$ but also $q < 1$ (or $(2 - q) > 1$ have physical meaning in the systems we are considering what differs our q-NJL model from the q-version of the QHD-I model presented in [8]. Notice that for $q \to 1$ one recovers the standard FD distribution, $n(\mu, T)$. It is important to realize that for $T \to 0$ one always gets $n_q(\mu, T) \to n(\mu, T)$, irrespectively of the value of q [8], *i.e.*, we can expect any nonextensive signature only for high enough temperatures.

placing the formulas of Section 1 with their q-counterparts in what concerns the form of the FD distributions. Additionally, when calculating energies and condensates we follow [9] and use the q-versions of energies and quark condensates replacing Eqs. (3) and (6) by:

$$E_q = -\frac{N_c}{\pi^2} V \sum_{i=u,d,s} \left[\int p^2 dp \frac{p^2 + m_i M_i}{E_i} \left(1 - n_{qi}^q - \bar{n}_{qi}^q \right) \right] - g_S V \sum_{i=u,d,s} \left(\langle \bar{q}_i q_i \rangle_q \right)^2 - 2 g_D V \langle \bar{u} u \rangle_q \langle \bar{d} d \rangle_q \langle \bar{s} s \rangle_q \,, \tag{8}$$

and

$$\langle \bar{q}_i q_i \rangle_q = -\frac{N_c}{\pi^2} \sum_{i=u,d,s} \left[\int \frac{p^2 M_i}{E_i} \left(1 - n_{qi}^q - \bar{n}_{qi}^q \right) \right] dp \,. \tag{9}$$

On the other hand, again following [9], densities which are given by the q-version of Eq. (5) are calculated with n_qs (not with n_q^q, as in (8) and in (9)). The pressure for given q is calculated using the above E_q and the q-entropy version of Eq. (4) with (*cf.* [10])

$$\tilde{S}_q = \left[n_{qi}^q \ln_q n_{qi} + (1 - n_{qi})^q \ln_q (1 - n_{qi}) \right] + \{ n_{qi} \to 1 - \bar{n}_{qi} \} \,. \tag{10}$$

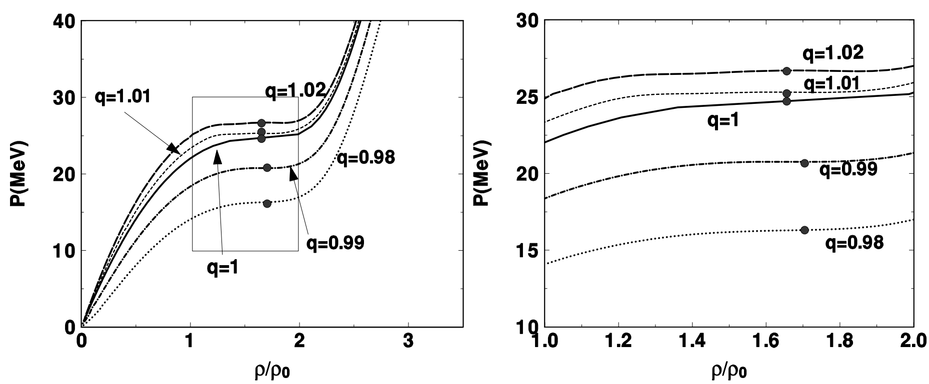

Fig. 1. The pressure at critical temperature $T_{\rm cr}$ as a function of compression ρ/ρ_0 calculated for different values of the nonextensivity parameter q (the area marked at the left panel is shown in detail at the right panel). The dots indicate positions of the inflection points for which first derivative of pressure in compression vanishes. As in [4] for $q = 1$ the corresponding compression is $\rho/\rho_0 = 1.67$ (and this leads to $\mu = 318.5$ MeV); it remains the same for $q > 1$ considered here (but now $\mu = 321$ MeV for $q = 1.01$ and $\mu = 326.1$ MeV for $q = 1.02$) whereas it is shifted to $\rho/\rho = 1.72$ for $q < 1$ ($\mu = 313$ MeV for $q = 0.99$ and $\mu = 307.7$ MeV for $q = 0.98$).

As example of how such approach works, we present in Fig. 1 the pressure at critical temperature $T_{\rm cr}$ as a function of compression ρ/ρ_0 calculated for different values of the nonextensivity parameter q (see [7] for more details). We see that for $q < 1$ the critical pressure is smaller but for $q > 1$ it is bigger than the critical pressure for BG distribution. According to [7] it is directly connected to specific correlation for $q < 1$ and fluctuations for $q > 1$. The role of these factors is shown in more detail in Fig. 2. Notice the remarkable difference for the density derivative at the critical point: from the smooth transition through the critical point for $q < 1$ to a big jump in density for critical value of chemical potential for $q > 1$. It reflects the infinite values of the baryon number susceptibility, χ_B:

$$\chi_B = \frac{1}{V}\sum_{i=u,d,s}\left(\frac{\partial \rho_i}{\partial \mu_B}\right)_T = -\frac{1}{V}\sum_{i=u,d,s}\frac{\partial^2 \Omega}{\partial^2 \mu_B}\bigg|_T . \qquad (11)$$

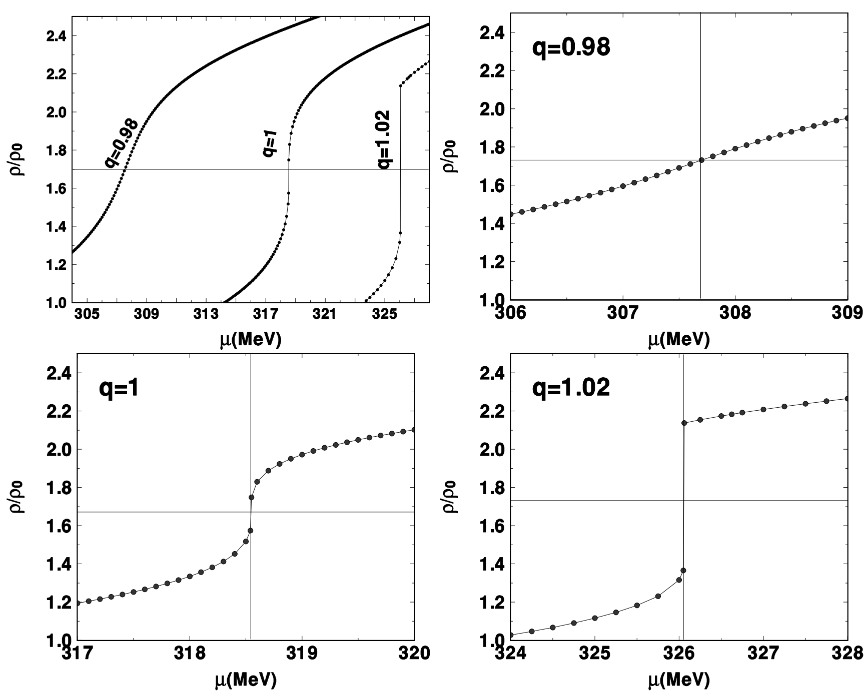

Fig. 2. The baryon compression ρ/ρ_0 (calculated in the vicinity of the critical values of temperature and density indicated by the corresponding dotted lines) as function of the chemical potential μ for different values of the nonextensivity parameter, $q = 0.98, 1.00, 1.02$. The summary presented in the top-left panel is detailed in the three consecutive panels.

The transition between confined and deconfined phases and/or chiral phase transition [1] can be seen by measuring, event by event, the difference in the magnitude of local fluctuation of the net baryon number in heavy ion collision [11]. They are initiated and driven mainly by the quark number fluctuation, described here by χ_B, and can survive through the freezout [11, 12]. Consequently, our q-NJL model allows to make the fine tuning for the magnitude of baryon number fluctuations (measured, for example, by the charge fluctuations of protons) and to find the characteristic for this system value of the parameter q. However, it does not allow to differentiate between possible dynamical mechanisms of baryon fluctuation. We close by noticing that using q dependent χ_B leads to q-dependent parameter ε of the critical exponents which describe the behavior of baryon number susceptibilities near the critical point [13]. Whereas in the mean field universality class one has $\varepsilon = \varepsilon' = 2/3$, our preliminary results using q-NJL model show smaller value of this parameter for $q > 1$, ($\varepsilon \sim 0.6$ for $q = 1.02$) and greater for $q < 1$ ($\varepsilon \sim 0.8$ for $q = 0.98$). It would be interesting to deduce the corresponding values of q from different models and compare them with results on lattice which, by definition, correspond to $q = 1$.

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