THE OPTICAL MODEL POTENTIAL OF THE Σ HYPERON IN NUCLEAR MATTER*

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We present our attempts to determine the optical model potential $U_\Sigma = V_\Sigma - iW_\Sigma$ of the Σ hyperon in nuclear matter. We analyze the following sources of information on U_Σ: ΣN scattering, Σ^- atoms, and final state interaction of Σ hyperons in the (π, K^+) and $(K^-.\pi)$ reactions on nuclear targets. We conclude that V_Σ is repulsive inside the nucleus and has a shallow attractive pocket at the nuclear surface. These features of V_Σ are consistent with the Nijmegen model F of the hyperon–nucleon interaction.

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1. Introduction

The interaction of the Σ hyperon with nuclear matter may be represented by the complex single particle (s.p.) optical model potential $U_\Sigma = V_\Sigma - iW_\Sigma$. In this paper we present our attempts to determine V_Σ and W_Σ. We also point out the most realistic two-body ΣN interaction among the available OBE models of the baryon–baryon interaction.

In the present paper we discuss the following sources of information on U_Σ: ΣN scattering data in Sec. 2, Σ^- atoms in Sec. 3, associated production reactions in Sec. 4, and strangeness exchange reactions in Sec. 5. Our conclusions are presented in Sec. 6.

2. ΣN scattering

The way from the ΣN scattering data to U_Σ consists of two steps: first, we determine the two-body ΣN interaction $\mathcal{V}_{\Sigma N}$, and second, with this $\mathcal{V}_{\Sigma N}$ we calculate U_Σ. The scarcity of the two-body ΣN data makes the first step very difficult. A way of overcoming these difficulties was followed

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by de Swart and his collaborators in Nijmegen: they assumed the mechanism of one-boson exchange (OBE) and the SU(3) symmetry which enabled them to employ the numerous NN data in determining the parameters of their two-body interaction. In this way they produced a number of the Nijmegen models of the baryon–baryon interaction: models D [1], F [2], soft core (SC) model [3], and the new soft-core (NSC) model [4].

2.1. The real potential V_Σ

In calculating V_Σ we use the real part of the effective ΣN interaction YNG [5] in nuclear matter. The YNG interaction is the configuration space representation of the G matrix calculated in the low order Brueckner approximation with the Nijmegen models of the baryon–baryon interaction. Our results obtained for V_Σ as function of the nucleon density ρ are shown in Fig. 1. As the dependence of V_Σ on the Σ momentum k_Σ is not very

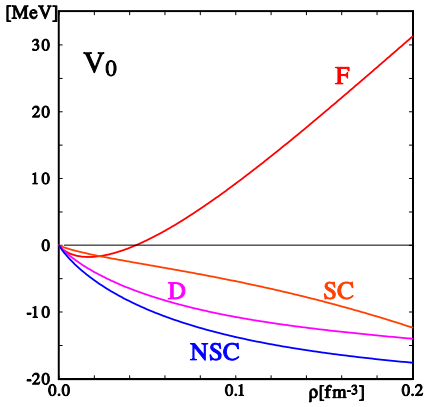

Fig. 1. The isoscalar potential V_Σ as a function of the nucleon density ρ at $k_\Sigma = 0$ for the indicated Nijmegen models of the ΣN interaction.

strong in the relevant interval of k_Σ [6], we use for V_Σ its value calculated at $k_\Sigma = 0$. We see that all the Nijmegen interaction models, except for model F, lead to pure attractive V_Σ which implies the existence of bound states of Σ hyperons in the nuclear core, *i.e.*, Σ hypernuclei. Since no Σ hypernuclei have been observed[1], we conclude that among the Nijmegen interaction models model F is the only realistic representation of the ΣN interaction.

[1] The observed bound state of ${}^4_\Sigma$He [7] is an exception. In the theoretical description of this state, Harada and his collaborators [8] apply phenomenological ΣN interactions, in particular, the interaction SAP-F simulating at low energies the Nijmegen model F interaction. They show that essential for the existence of the bound state of ${}^4_\Sigma$He is a strong Lane component V_τ in V_Σ, and among the Nijmegen models the strongest V_τ is implied by model F [9].

2.2. The absorptive potential W_Σ

As pointed out in [5], the imaginary part of the YNG interaction is very sensitive to the choice of the intermediate state energies in the G matrix equation. In this situation we decided to use for W_Σ the semi-classical expression in terms of the total cross-sections (modified by the exclusion principle) for ΣN scattering, described in [10]. We denote by W_c the contribution to the absorptive potential of the $\Sigma\Lambda$ conversion process $\Sigma N \to \Lambda N'$ and by W_e the contribution of the ΣN elastic scattering, and have $W_\Sigma = W_\mathrm{t} = W_\mathrm{c} + W_\mathrm{e}$[2].

Our results obtained for $W_\mathrm{c}, W_\mathrm{e}, W_\mathrm{t}$ for nuclear matter (with $N = Z$) at equilibrium density $\rho = \rho_0 = 0.166$ fm^{-3} are shown in Fig. 2. With increasing momentum k_Σ the $\Sigma\Lambda$ conversion cross-section decreases, on the other hand the suppression of W_c by the exclusion principle weakens. As the net result W_c does not change very much with k_Σ. The same two mechanisms act in the case of W_e. Here, however, the action of the exclusion principle is much more pronounced: at $k_\Sigma = 0$ the suppression of W_e is complete. At higher momenta, where the Pauli blocking is not important, the total elastic cross-section is much bigger than the conversion cross-section, and we have $W_\mathrm{e} \gg W_\mathrm{c}$, and consequently $W_\Sigma \gg W_\mathrm{c}$.

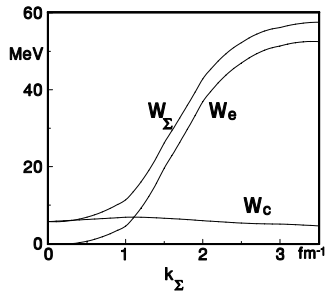

Fig. 2. The component $W_\mathrm{c}, W_\mathrm{e}$, and W_t of the Σ absorptive potential in nuclear matter of density ρ_0 as functions of k_Σ.

3. Σ^- atoms

The available data on strong interaction effects in Σ^- atoms consist of 23 data points: strong interaction shifts ε and widths Γ of the observed levels. These shifts and widths can be measured directly only in the lowest Σ^- atomic levels. The widths of the next to the last level can be obtained indirectly from measurements of the relative yields of X-rays.

[2] Notice that in the case of the nucleon optical potential in nuclear matter (for nucleon energies below the threshold for pion production), $V_N - iW_N$, only the elastic NN scattering contributes to W_N, and the situation is similar as in the case of the contribution W_e to W_Σ.

In [11], we have estimated the 23 values of ε and Γ from the difference between the eigenvalues of the Schrödinger equation of Σ^- in Σ^- atoms with the strong Σ^--atomic nucleus interaction and without this interaction. To obtain this strong interaction, we applied the local density approximation, and used our optical model of Sec. 2. The agreement of our results, calculated with the optical potentials (obtained with the 4 Nijmegen ΣN interaction models) with the 23 empirical data points is characterized by the following values of χ^2: $\chi^2(\text{model D}) > 130$, $\chi^2(\text{model F}) = 38.1$, $\chi^2(\text{model SC}) = 55.0$, $\chi^2(\text{model NSC}) > 904$, and we conclude that the Σ^- atomic data point out at model F as the best representation of the ΣN interaction[3].

4. The associated production reactions

The first associated Σ production reaction (π^-, K^+) was observed at KEK on ^{28}Si target at pion momentum of 1.2 GeV/c ([12, 13]), and this reaction is the subject of the present analysis. We consider the reaction (π^-, K^+) in which the pion π^- with momentum $\boldsymbol{k}_\pi$ hits a proton in the ^{28}Si target in the state ψ_P and emerges in the final state as kaon K^+ moving in the direction $\hat{k}_K$ with energy E_K, whereas the hit proton emerges in the final state as a Σ^- hyperon with momentum $\boldsymbol{k}_\Sigma$. We apply the simple impulse approximation described in [14], with K^+ and π^- plane waves, and obtain:

$$d^3\sigma/d\hat{k}_\Sigma d\hat{k}_K dE_K \sim \left| \int d\boldsymbol{r} \exp(-i\boldsymbol{q}r)\psi_{\Sigma,\boldsymbol{k}_\Sigma}(\boldsymbol{r})^{(-)*}\psi_P(\boldsymbol{r}) \right|^2, \qquad (1)$$

where the momentum transfer $\boldsymbol{q} = \boldsymbol{k}_K - \boldsymbol{k}_\pi$, and $\psi_{\Sigma,\boldsymbol{k}_\Sigma}(\boldsymbol{r})^{(-)}$ is the Σ scattering wave function which is the solution of the s.p. Schrödinger equation with the s.p. potential

$$U_\Sigma(r) = (V_\Sigma - iW_\Sigma)\theta(R - r)\,, \qquad (2)$$

where for V_Σ and W_Σ we use the nuclear matter results discussed in Sec. 2, calculated at $\rho = n/[(4\pi/3)R^3]$, where $n = 27$ is the number of nucleons in the final state.

For the ^{28}Si target nucleus we assume a simple shell model with a square well s.p. potential $V_P(r)$ (which determines ψ_P) with the radius R_P (and with a spin-orbit term). The parameters of $V_P(r)$ are adjusted to the proton separation energies (in particular $R_P = 3.756\,\text{fm}$). For R we make the simple and plausible assumption: $R = R_P$.

In the inclusive KEK experiments [12, 13] only the energy spectrum of kaons at fixed $\hat{k}_\Sigma$ was measured. To obtain this energy spectrum, we have to integrate the cross-section (1) over $\hat{k}_\Sigma$.

[3] Notice that the positive sign of the measured values of ε requires an attractive Σ potential at the nuclear surface, *i.e.* at low densities.

We present our results for the inclusive cross-section as a function of B_Σ, the separation (binding) energy of Σ from the hyper nuclear system produced. Our model F and D results[4] for kaon spectrum from (π^-, K^+) reaction on ^{28}Si at $\theta_K = 6^o$ at $p_\pi = 1.2$ GeV/c are shown in Fig. 3. We see that the best fit to the data points is obtained for V_Σ derived from model F and with $W_\Sigma = W_{\rm t} = W_{\rm c} + W_{\rm e}$. The fit would improve if we considered the distortion of kaon and especially of pion waves (it was noticed already in Ref. [12] that this distortion pushes the kaon spectrum down). Inclusion into the absorptive potential of the contribution $W_{\rm e}$ of the elastic ΣN scattering is essential for obtaining this result with $V_\Sigma\,(\text{model F}) = 17.25$ MeV. Earlier estimates of the kaon spectrum without this contribution suggested a repulsive V_Σ with an unexpected strength of about 100 MeV. Notice that the action of the absorptive potential W_Σ on the Σ wave function (decrease of this wave function) is similar as the action of a repulsive V_Σ. Therefore, we achieve with strong absorption the same final effect with a relatively weaker repulsion.

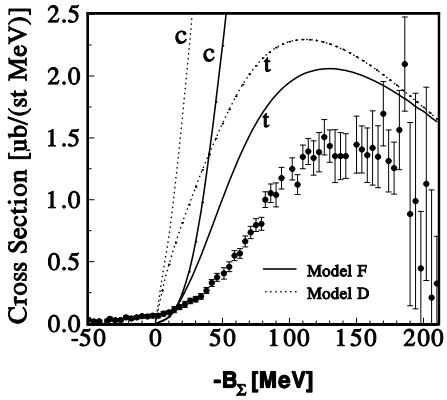

Fig. 3. Kaon spectrum from (π^-, K^+) reaction on ^{28}Si at $\theta_K = 6^\circ$ at $p_\pi = 1.2\,\text{GeV}/c$ obtained with V_Σ determined by models F and D of the ΣN interaction. Curves denoted by $c(t)$ were obtained with $W_\Sigma = W_{\rm c}(W_{\rm t})$. Data points are taken from [13].

5. The strangeness exchange reactions

First observations of the strangeness exchange (K^-, π) reactions with a reliable accuracy were performed at BNL. Here, we shall discuss the $(K-, \pi^+)$ reaction observed at BNL on Be^9 target with 600 MeV/c kaons [15].

[4] The remaining models SC and NSC are similar to model D: they all lead to attractive V_Σ in contradistinction to model F leading to repulsive V_Σ (at densities inside nuclei — see Fig. 1). Consequently, the results for the kaon spectrum for models SC and NSC are expected to be similar as in the case of model D.

Proceeding similarly as in the case of the associated production described in Sec. 4, we get the results shown in Fig. 4. We see that similarly as in Sect. 4 the fit to the data points obtained for V_Σ derived from model F is much better than the fit obtained with model D.

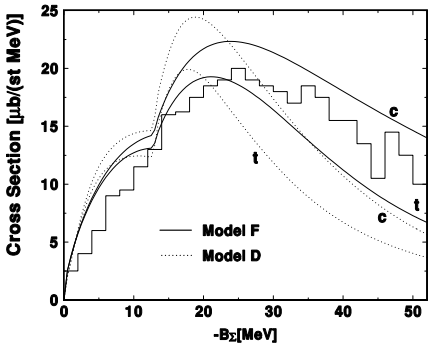

Fig. 4. Pion spectrum from (K^-, π^+) reaction on ^{9}Be at $\theta_\pi = 4^\circ$ at $p_K = 0.6\,\mathrm{GeV}/c$ obtained with V_Σ determined by models F and D of the ΣN interaction. Curves denoted by $c(t)$ were obtained with $W_\Sigma = W_\mathrm{c}(W_\mathrm{t})$. Data points are taken from [15].

6. Conclusions

• The real part V_Σ of the Σ optical potential is repulsive inside the nucleus and has a shallow attractive pocket at the nuclear surface.

• Among the Nijmegen models of the baryon–baryon interaction only model F leads to this form of V_Σ.

• The contribution of the elastic ΣN scattering to the absorptive part W_Σ of the Σ optical potential is essential in the analysis of Σ production processes.

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