THE EFFECT OF LOCALIZED NEW HIGGS DOUBLET ON THE RADIATIVE LEPTON–FLAVOR VIOLATING DECAYS IN THE RANDALL SUNDRUM BACKGROUND

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We study the radiative lepton–flavor violating $l_1 \to l_2\gamma$ decays in the two Higgs doublet model with localized new Higgs doublet in the Randall Sundrum background. We estimate the contributions of the KK modes of new Higgs bosons and left (right) handed charged lepton doublets (singlets) on the branching ratios of the decays considered. We observe that there is an enhancement in the branching ratios with the addition of new Higgs boson and lepton KK modes.

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1. Introduction

The processes with flavor violation (FV) are worthwhile to study since they exist at least in the one loop level in the standard model (SM) and, therefore, they are rich from the theoretical point of view. The lepton–flavor violating (LFV) interactions are among the most exciting candidates of these processes, since they are clean in the sense that they are free from strong interactions. Furthermore, the small numerical values of branching ratios (BRs) of LFV decays stimulate one to search beyond and to study the more fundamental models in order to enhance these numerical values to reach the current experimental upper limits. Among the LFV decays the radiative $l_1 \to l_2\gamma$ processes reach great interest and their current experimental upper limits of the BRs are: BR $(\mu \to e\gamma) = 1.2 \times 10^{-11}$ [1], BR$(\tau \to e\gamma) = 3.9{\times}10^{-7}$ [2] and BR$(\tau \to \mu\gamma) = 1.1{\times}10^{-6}$ $(9.0{\times}10^{-8}; 6.8{\times}10^{-8}, 90\%$ C.L.) [3] ([4,5]), respectively. Furthermore, in order to search the $\mu \to e\gamma$ decay, a new experiment, aiming to reach a sensitivity of BR $\sim 10^{-14}$, at PSI has been described [6]. At present, this experiment (PSI-R-99-05 Experiment) is still running in the MEG [7].

The theoretical values of the BRs of the radiative LFV decays in the framework of the SM are negligible compared to the experimental upper limits and the addition of one more Higgs doublet, which drives the flavor changing neutral currents (FCNCs) and the LFV interactions at tree level, may cause to pull the theoretical values of the BRs near to the experimental upper limits. This is the case that the lepton FV is induced by the internal new neutral Higgs bosons, h^0 and A^0, and the strength of this violation is regulated by the Yukawa couplings, appearing as free parameters which should be restricted by using the experimental data. These decays were examined in the framework of the SM with one more Higgs doublet [8–11], the so called two Higgs doublet model (2HDM), in [12–17]. Besides the theoretical calculations based on the 2HDM, they were studied in the supersymmetric models [18–24], in a model independent way [25], in the framework of 2HDM and the supersymmetric model [26] and, recently, in the SM including effective operators coming from the possible unparticle effects [27, 28].

Another possibility to enhance the numerical values of the BRs of these processes is to consider the extra dimension what results in the additional effects of the KK modes of the particles in the loops, after the compactification. In the present work, we consider the extended Higgs sector, the 2HDM, in the RS1 background [29, 30]. The RS1 model is based on the curved extra dimension and the corresponding metric reads

$$ds^2 = e^{-2\sigma}\, \eta_{\mu\nu}\, dx^\mu\, dx^\nu - dy^2\,, \tag{1}$$

where $\sigma = k\,|y|$, k is the bulk curvature constant, the exponential $e^{-\sigma}$, with $y = R\,|\theta|$, is the warp factor. Here, the extra dimension is compactified onto S^1/Z_2 orbifold and R is the compactification radius. The extra dimension has two boundaries, the hidden (Planck) brane and the visible (TeV) brane, with opposite and equal tensions. This choice leads to the fact that the low energy effective theory has flat 4D spacetime, even if the 5D cosmological constant is non vanishing. In the RS1 background, the gravity is taken to be localized on the hidden brane and to be extended into the bulk with varying strength and the SM fields live in the visible brane. If some of the SM fields are accessible to the extra dimension, the phenomenology becomes richer and there are various work done in the literature respecting such scenarios [31-49]. If fermions are accessible to the extra dimension and there is a Dirac mass term in the Lagrangian, the fermion mass hierarchy can be explained. In this case the fermion mass hierarchy comes from the possible fermion field locations [35, 38–40]. The quark and lepton FV, which is based on the different locations of the fermion fields in the extra dimension, is extensively studied in [45, 46]. In these works, it is considered that the FV is carried by the Yukawa interactions, coming from the SM Higgs-fermion–fermion vertices. In [47], the high precision measurements of top pair production at the ILC is addressed by considering that the fermions are localized in the bulk of RS1 background. In recent works [48, 49], the various experimental FCNC constraints and the electro weak precision tests for the location parameters of the fermions in the extra dimension are discussed. The other possibility is to consider the localization of Higgs field in the extra dimension. The brane localized mass terms for scalar fields are considered in order to get small couplings of KK modes with the boundaries [35] and these mass terms result in the fact that the zero mode localized solution is obtained. In [41] the hierarchy of fermion masses is analyzed by taking that the Higgs field has an exponential profile around the TeV brane. [42] is devoted to an extensive work on the bulk fields in various multi-brane models.

In our work, we assume that the new Higgs doublet is accessible to the extra dimension of RS1 background. First, we study the case that the charged leptons are restricted to the 4D brane and, second, we consider that the charged leptons are also localized in the extra dimension. Notice that, in both cases, the gauge bosons are necessarily accessible to the extra dimension.

The paper is organized as follows: In Sec. 2, we present the BRs of LFV interactions $l_1 \to l_2\gamma$ in the 2HDM, by considering that the new Higgs doublet is localized in the extra dimension of RS1 background. Sec. 3 is devoted to discussion and our conclusions. In Appendix A, we study the construction of new Higgs boson mass matrix. In Appendix B, we present the amplitudes appearing in the calculation of the decay widths of the radiative decays under consideration Appendix C is devoted to calculation of the zero mode lepton fields and their KK modes.

2. LFV $l_1 \to l_2\gamma$ decays in the Randall Sundrum background with localized new Higgs boson

We start with the action for the new Higgs doublet ϕ (see for example [31, 42] for a massive bulk scalar field case),

$$\mathcal{S}_\phi = \frac{1}{2} \int d^4x \int dy \sqrt{g} \left(g^{MN} \left(\partial_M \phi\right)^\dagger \partial_N \phi + m_\phi^2 \, \phi^\dagger \phi \right) , \tag{2}$$

where $g = \mathrm{Det}[g_{MN}] = e^{-8\sigma}$, $M, N = 0, 1, \dots, 4$. The decomposition of the scalar doublet into KK modes

$$\phi(x, y) = \sum_{n=0}^{\infty} \phi^{(n)}(x) \, f_n(y) \tag{3}$$

brings the action Eq. (2) into form

$$\mathcal{S}_{\rm S} = \frac{1}{2} \sum_{n=0}^{\infty} \int d^4x \left(\eta^{\mu\nu} \left(\partial_\mu \phi_n(x)\right)^\dagger \partial_\nu \, \phi_n(x) + m_{n\,S}^2 \left(\phi_n(x)\right)^\dagger \phi_n(x) \right), \quad (4)$$

with the second order differential equation

$$-e^{4\,\sigma} \frac{d}{dy} \left(e^{-4\,\sigma} \frac{d\, f_n(y)}{dy} \right) + m_\phi^2 \, f_n(y) = m_{n\,S}^2 \, e^{2\,\sigma} \, f_n(y) \,, \quad (5)$$

and the orthogonality relation

$$\int_{-\pi R}^{\pi R} dy \, e^{-2\,\sigma} \, f_n^*(y) \, f_m(y) = \delta_{nm} \,. \quad (6)$$

The choice of the mass term in Eq. (2)

$$m_\phi^2 = a \left(\frac{d\,\sigma}{dy} \right)^2 + b \, \frac{d^2\,\sigma}{dy^2} \,, \quad (7)$$

results in the differential equation in the bulk

$$-e^{4\,\sigma} \frac{d}{dy} \left(e^{-4\,\sigma} \frac{d\, f_n(y)}{dy} \right) + a\, k^2 \, f_n(y) = m_{n\,S}^2 \, e^{2\,\sigma} \, f_n(y) \,, \quad (8)$$

where $n = 1, 2, \ldots$. Now, the the boundary mass term[1]

$$m_{\phi,\ \rm bound}^2 = b \, \frac{d^2\,\sigma}{dy^2} \,, \quad (9)$$

is considered in order to obtain zero mode Higgs doublet and this term induces the boundary condition

$$\left(\frac{\partial \phi(x,y)}{\partial y} - b\, k\, \phi(x,y) \right) \Bigg|_{y=0,\pi R} = 0 \,. \quad (10)$$

Notice that the non-vanishing zero mode can be obtained with the fine tuning of the parameters[2] b and a,

$$b = 2 + \sqrt{4+a} \,, \quad (11)$$

[1] Here the boundary mass terms have the same magnitude and the opposite sign on the branes. The idea of brane localized mass terms has been considered for scalar fields in [31, 35].

[2] There is another possibility of fine tuning of the parameters b and a for the non-vanishing zero mode, namely $b = 2 - \sqrt{4+a}$. However, we ignore this choice since it is not appropriate for the brane localized fermion scenario and bulk fermion scenario with the parameter set used (see discussion section for details).

and it reads

$$f_0(y) = \frac{e^{b\,k\,y}}{\sqrt{\frac{e^{2\,(b-1)\,k\,\pi\,R}-1}{(b-1)\,k}}}\,. \tag{12}$$

On the other hand, the KK mode Higgs doublet is obtained as

$$f_n(y) = \frac{e^{2\,\sigma}}{N_{S\,n}} \left(J_{\sqrt{4+a}}(e^{\sigma}\,x_{nS}) + \alpha_n\,Y_{\sqrt{4+a}}(e^{\sigma}\,x_{nS}) \right), \tag{13}$$

where $N_{S\,n}$ is the normalization constant, $x_{nS} = (m_{n\,S})/k$ and α_n reads

$$\alpha_n \;=\; \frac{(2-b)\,J_{\sqrt{4+a}}(x_{nS}) + x_{nS}\,J'_{\sqrt{4+a}}(x_{nS})}{(b-2)\,Y_{\sqrt{4+a}}(x_{nS}) - x_{nS}\,Y'_{\sqrt{4+a}}(x_{nS})}\,. \tag{14}$$

Here, the functions $J_\beta(w)$ and $Y_\beta(w)$ are the Bessel functions of the first kind and of the second kind, respectively. Finally, the mass spectrum of KK modes ($n = 1, 2, ...$) is obtained by using the boundary conditions at $y = 0$ and $y = \pi\,R$ (see Eq. (10)),

$$m_{n\,S} \simeq \left(n + \frac{1}{2}\,(b-2) - \frac{3}{4} \right) \pi\,k\,e^{-k\pi R}\,, \tag{15}$$

for $k\,e^{-k\pi R} \ll m_{n\,S} \ll k$.

At this stage, we consider two possibilities for the charged leptons:

— they are restricted to the 4D brane

— they are localized in the extra dimension.

2.1. The charged leptons restricted to the brane

The LFV interactions are driven by the part of the action

$$\mathcal{S}_{\rm Y} = \int d^5x \sqrt{g} \Big(\xi^{\rm E}_{5ij}\,\bar{l}_{iL}\phi_2 E_{jR} + \text{h.c.} \Big)\,\delta(y-\pi R)\,, \tag{16}$$

where L and R denote chiral projections $L(R) = 1/2(1 \mp \gamma_5)$, ϕ_2 is the new scalar doublet, l_{iL} (E_{jR}) are lepton doublets (singlets), $\xi^{\rm E}_{5\,ij}$, with family indices i, j, are the Yukawa couplings in five dimensions, which are responsible for the flavor violating interactions in the leptonic sector. Here, we assume that the Higgs doublet ϕ_1 lives on the visible brane and it has non-zero vacuum expectation value in order to ensure the ordinary masses of the gauge fields and the fermions. On the other hand the second doublet, which is accessible to the extra dimension, has no vacuum expectation value[3]:

$$\phi_1=\frac{1}{\sqrt{2}}\left[\left(\begin{array}{c} 0\\ v+H^0\end{array}\right)+\left(\begin{array}{c}\sqrt{2}\chi^+\\ i\chi^0\end{array}\right)\right], \qquad \phi_2=\frac{1}{\sqrt{2}}\left(\begin{array}{c}\sqrt{2}H^+\\ H^1+iH^2\end{array}\right), \tag{17}$$

and

$$\langle\phi_1\rangle=\frac{1}{\sqrt{2}}\left(\begin{array}{c}0\\ v\end{array}\right), \qquad \langle\phi_2\rangle=0\,. \tag{18}$$

The new Higgs doublet ϕ_2 is expanded into KK modes after the compactification of the extra dimension as given in Eq. (3) and the zero (KK) mode Higgs fields are obtained by imposing the fine tuning condition in Eq. (11). On the other hand, after the electro weak breaking, the SM Higgs acquires the vacuum expectation value Eq. (18) and there appears mixing between zero mode and KK mode new Higgs bosons. However, we do not take into account the additional effects coming from this mixing since they are suppressed (see Appendix A for detail).

For the effective Yukawa coupling ξ^{E}_{ij} we integrate out the Yukawa interaction Eq. (16) over the fifth dimension by taking the zero mode neutral Higgs fields $S=h^0, A^0$:

$$\xi^{E}_{ij}=V^0_{ij}\,\xi^{E}_{5\,ij}\,, \tag{19}$$

where

$$V^0_{ij} = \int_{-\pi R}^{\pi R} dy\, e^{-4\sigma} f_0(y)\delta(y-\pi R) = \frac{e^{(b-4)\,k\,\pi\,R}}{\sqrt{\frac{e^{2\,(b-1)\,k\,\pi\,R}-1}{k\,(b-1)}}}\,. \tag{20}$$

Here it is assumed that the coupling $\xi^{E}_{5\,ij}$ is flavor dependent and it is regulated in such a way that the overall quantity $V^0_{ij}\,\xi^{E}_{5\,ij}$ is pointed to the chosen numerical value of ξ^{E}_{ij}.

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[3] Here we take the Higgs potential as

$$\begin{aligned} V(\phi_1,\phi_2) &= c_1(\phi_1^+\phi_1-v^2/2)^2+c_2(\phi_2^+\phi_2)^2+c_3[(\phi_1^+\phi_1)(\phi_2^+\phi_2)-(\phi_1^+\phi_2)(\phi_2^+\phi_1)]\\ &\quad +c_4[\mathrm{Re}(\phi_1^+\phi_2)]^2+c_5[\mathrm{Im}(\phi_1^+\phi_2)]^2\,. \end{aligned}$$

This choice leads to no tree level mixing between the CP even neutral Higgs bosons, namely H^0 and H^1. Therefore, the SM particles (new particles) are collected in the first (second) doublet and H^1, H^2 are obtained as the mass eigenstates h^0 and A^0, respectively. Notice that, in general, the mixing between the CP even neutral Higgs bosons can exist in the loop level when one considers the quantum corrections.

The effective Yukawa coupling inducing the tree level interaction among the KK mode Higgs and charged leptons reads

$$\xi^{\mathrm{E}\,n}_{ij} = V^{\mathrm{S}\,n}_{ij}\,\xi^{\mathrm{E}}_{5\,ij}\,, \tag{21}$$

where

$$V^{\mathrm{S}\,n}_{ij} = \frac{e^{-2\,k\,\pi\,R}}{N_{\mathrm{S}\,n}}\left(J_{b-2}\left(e^{k\,\pi\,R}\,x_{nS}\right) + \alpha_n Y_{b-2}\left(e^{k\,\pi\,R}\,x_{nS}\right)\right). \tag{22}$$

Here $N_{S\,n}$ is the normalization constant (see Eq. (13)) and b (α_n) is defined in Eqs. (11), (14). Finally, the effective Yukawa coupling $\xi^{\mathrm{E}\,n}_{ij}$ is obtained as

$$\xi^{\mathrm{E}\,n}_{ij} = \frac{V^n_{S\,ij}}{V^0_{ij}}\,\xi^{\mathrm{E}}_{ij} = \frac{e^{(2-b)\,k\,\pi\,R}\sqrt{\frac{e^{2\,(b-1)\,k\,\pi\,R}-1)}{k\,(b-1)}}}{N_{S\,n}} \times\left(J_{b-2}\left(e^{k\,\pi\,R}\,x_{nS}\right) + \alpha_n\,Y_{b-2}\left(e^{k\,\pi\,R}\,x_{nS}\right)\right)\xi^{\mathrm{E}}_{ij}\,. \tag{23}$$

Now, we present the decay widths of the LFV $l_1 \to l_2\gamma$ decays, including the KK modes of new neutral Higgs fields. Since these decays exist at least in the one loop level, there appear the logarithmic divergences in the calculations. In order to eliminate these divergences, we follow the on-shell renormalization scheme. In this scheme, the self energy diagrams can be written in the form $\sum(p) = (\hat{p} - m_{l_1})\bar{\sum}(p)(\hat{p} - m_{l_2})$, which results in that these diagrams do not contribute for on-shell leptons and, only, the vertex diagrams (see Fig. 1) contribute[4]. Taking only tau lepton for the internal line[5], the decay width Γ reads

$$\Gamma(l_1 \to l_2\gamma) = c_1(|A_1|^2 + |A_2|^2)\,, \tag{24}$$

where

$$\begin{aligned} A_1 &= A^0_1 + A^{\mathrm{S\,KK}}_1\,, \\ A_2 &= A^0_2 + A^{\mathrm{S\,KK}}_2\,. \end{aligned} \tag{25}$$

For the explicit expression of these amplitudes see Appendix B.

[4] This is the case that the divergences can be eliminated by introducing a counter term V^C_μ with the relation $V^{\mathrm{Ren}}_\mu = V^0_\mu + V^C_\mu$, where V^{Ren}_μ (V^0_μ) is the renormalized (bare) vertex and by using the gauge invariance $k^\mu V^{\mathrm{Ren}}_\mu = 0$. Here, k^μ is the four momentum vector of the outgoing photon.

[5] We take into account only the internal tau lepton contribution since we respect the idea that the couplings $\bar{\xi}^{\mathrm{E}}_{\mathrm{N},ij}$ ($i,j = e,\mu$), are small compared to $\bar{\xi}^{\mathrm{E}}_{\mathrm{N},\tau\,i}$ ($i = e,\mu,\tau$), due to the possible proportionality of them to the masses of leptons under consideration in the vertices. Here, we use the dimensionful coupling $\bar{\xi}^{\mathrm{E}}_{\mathrm{N},ij}$ with the definition $\xi^{\mathrm{E}}_{\mathrm{N},ij} = \sqrt{(4G_{\mathrm{F}})/\sqrt{2}}\;\bar{\xi}^{\mathrm{E}}_{\mathrm{N},ij}$ where N denotes the word "neutral".

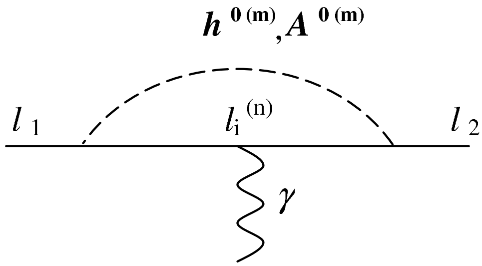

Fig. 1. One loop diagrams contribute to $l_1 \to l_2\gamma$ decay due to the zero mode (KK mode) leptons and Higgs fields in the 2HDM.

2.2. The charged leptons localized in the extra dimension

The part of the action which drives the LFV interactions in this case reads

$$\mathcal{S}_{\rm Y} = \int d^5x \sqrt{g} \left(\xi^{\rm E}_{5\,ij}\, \bar{l}_{iL} \phi_2 E_{jR} + {\rm h.c.} \right), \tag{26}$$

where l_{iL} (E_{jR}) are lepton doublets (singlets) which are localized in the extra dimension. The addition of Dirac mass term to the Lagrangian of bulk fermions causes this localization [32, 34, 35, 37, 38, 40, 41]. Since the combination $\bar{\psi}\psi$ is odd due to the two possible transformation properties of fermions under the orbifold Z_2 symmetry, $Z_2\psi = \pm\gamma_5\psi$, in order to construct the Z_2 invariant mass term, one needs Z_2 odd scalar field to be coupled. This discussion leads to the mass term

$$\mathcal{S}_m = -\int d^4x \int dy \sqrt{g}\, m(y)\, \bar{\psi}\psi\,, \tag{27}$$

where $m(y) = m\sigma'(y)/k$ with $\sigma'(y) = (d\sigma)/dy$. With the help of the given mass term the localized zero mode leptons are obtained. We present the construction of the zero mode and KK mode leptons in the Appendix C extensively.

For the effective Yukawa coupling $\xi^{\rm E}_{ij}$, similar to the previous case, we integrate out the Yukawa interaction Eq. (26) over the fifth dimension. By taking the zero mode lepton doublets, singlets (see Eq. (C.3)) and neutral Higgs fields $S = h^0, A^0$ (see Eq. (12)), we get

$$\xi^{\rm E}_{ij} \left((\xi^{\rm E}_{ij})^\dagger \right) = V^{00}_{S\,RL(LR)\,ij}\, \xi^{\rm E}_{5\,ij} \left((\xi^{\rm E}_{5\,ij})^\dagger \right)\,, \tag{28}$$

where

$$V^{00}_{S\,RL\,ij} = \frac{1}{2\,\pi\,R} \int\limits_{-\pi\,R}^{\pi\,R} dy\,\chi_{iR0}(y)\,\chi_{jL0}(y)\,f_0(y)$$
$$= \frac{\left(1-e^{(b-r_{iR}-r_{jL})\,k\,\pi\,R}\right)}{(r_{iR}+r_{jL}-b)\sqrt{\frac{1-e^{(1-2\,r_{iR})\,k\,\pi\,R}}{(2\,r_{iR}-1)}}\sqrt{\frac{1-e^{(1-2\,r_{jL})\,k\,\pi\,R}}{(2\,r_{jL}-1)}}\sqrt{\frac{1-e^{2\,(b-1)\,k\,\pi\,R}}{k\,(1-b)}}}\,. \quad (29)$$

Here, similar to the previous scenario, the coupling $\xi^{\rm E}_{5\,ij}$ in five dimension is flavor dependent and it is regulated in such a way that the overall quantity $V^0_{RL\,(LR)\,ij}\,\xi^{\rm E}_{5\,ij}$ is pointed to the chosen numerical value of $\xi^{\rm E}_{ij}\,\left((\xi^{\rm E}_{ij})^\dagger\right)$. This is the case that the hierarchy of new Yukawa couplings, describing the tree level Higgs zero mode($S^{(0)}$)-lepton zero mode ($l^{(0)}$)-lepton zero mode ($l^{(0)}$) interaction, is not related to the Higgs field and lepton field locations.

The effective Yukawa coupling which drives the tree level KK mode Higgs ($S^{(n)}$)–$l^{(0)}$–$l^{(0)}$ interaction is

$$\xi^{{\rm E}\,n\,0}_{ij}\left((\xi^{{\rm E}\,n\,0}_{ij})^\dagger\right) = V^{n0}_{S\,RL(LR)\,ij}\,\xi^{\rm E}_{5\,ij}\left((\xi^{\rm E}_{5\,ij})^\dagger\right)\,, \quad (30)$$

where

$$V^{n0}_{S\,RL\,ij} = \frac{\int\limits_{-\pi\,R}^{\pi\,R} dy\,e^{(2-r_{iR}-r_{jL})\,\sigma}\left(J_{b-2}(e^\sigma\,x_{nS})+\alpha_n\,Y_{b-2}(e^\sigma\,x_{nS})\right)}{N_{S\,n}\sqrt{\frac{1-e^{(1-2\,r_{iR})\,k\,\pi\,R}}{k\,(2\,r_{iR}-1)}}\sqrt{\frac{1-e^{(1-2\,r_{jL})\,k\,\pi\,R}}{k\,(2\,r_{jL}-1)}}}\,. \quad (31)$$

Using Eqs. (30) and (31), the effective Yukawa coupling $\xi^{{\rm E}\,n\,0}_{ij}$ is obtained as

$$\xi^{{\rm E}\,n\,0}_{ij} = \frac{V^{n0}_{S\,RL\,ij}}{V^{00}_{S\,RL\,ij}}\,\xi^{\rm E}_{ij} = \frac{(r_{iR}+r_{jL}-b)\sqrt{\frac{k\,(e^{2\,(b-1)\,k\,\pi\,R}-1)}{(b-1)}}}{N_{S\,n}\left(1-e^{(b-r_{iR}-r_{jL})\,k\,\pi\,R}\right)}$$
$$\times \int\limits_{-\pi\,R}^{\pi\,R} dy\,e^{(2-r_{iR}-r_{jL})\,\sigma}\left(J_{b-2}(e^\sigma\,x_{nS})+\alpha_n\,Y_{b-2}(e^\sigma\,x_{nS})\right)\xi^{\rm E}_{ij}\,. \quad (32)$$

$S^{(0)}-l^{(0)}-l^{(n)}$ vertex drives another possible tree level interaction appearing in the loop calculations and the corresponding is effective Yukawa coupling reads

$$\xi^{{\rm E}\,0\,n}_{ij}\left((\xi^{{\rm E}\,0\,n}_{ij})^\dagger\right) = V^{0n}_{S\,RL\,(LR)\,ij}\,\xi^{\rm E}_{5\,ij}\left((\xi^{\rm E}_{5\,ij})^\dagger\right)\,, \quad (33)$$

where

$$V^{0n}_{S\,RL\,(LR)\,ij} = \frac{N_{Ln\,(Rn)} \int\limits_{-\pi R}^{\pi R} dy\, e^{(b-r_{iR\,(iL)}+\frac{1}{2})\,\sigma}}{\pi R \sqrt{\frac{1-e^{(1-2\,r_{iR\,(iL)})\,k\,\pi\,R}}{k\,\pi\,R\,(2\,r_{iR\,(iL)}-1)}} \sqrt{\frac{e^{2\,(b-1)\,k\,\pi\,R}-1}{(b-1)\,k}}} \times \left(J_{\frac{1}{2}\mp r_{jL(jR)}}(e^{\sigma}\, x_{nL(R)}) + c_{L\,(R)} Y_{\frac{1}{2}\mp r_{jL(jR)}}(e^{\sigma}\, x_{nL(R)}) \right). \quad (34)$$

Here the parameters $x_{nR(L)}$, $c_R(L)$, the lepton localization parameters $r_{iR\,(iL)}$ and the normalization constant $N_{R(L)n}$ are given in Appendix C. By using Eqs. (33) and (34) we get the effective Yukawa coupling $\xi^{\mathrm{E}\,0\,n}_{ij}\,((\xi^{\mathrm{E}\,0\,n}_{ij})^{\dagger})$ as:

$$\begin{aligned} \xi^{\mathrm{E}\,0\,n}_{ij}\left((\xi^{\mathrm{E}\,0\,n}_{ij})^{\dagger}\right) &= \frac{V^{0n}_{S\,RL\,(LR)\,ij}}{V^{00}_{S\,RL(LR)\,ij}}\, \xi^{\mathrm{E}}_{ij}\left((\xi^{\mathrm{E}}_{ij})^{\dagger}\right) \\ &= N_{Ln\,(Rn)} \sqrt{\frac{k\left(1-e^{(1-2\,r_{jL(jR)})\,k\pi R}\right)}{\pi\,R\,(2\,r_{jL\,(iR)}-1)}}\, \left(r_{iR\,(iL)}+r_{jL(jR)}-b\right) \\ &\times \frac{\int\limits_{-\pi R}^{\pi R} dy\, e^{(b-r_{iR\,(iL)}+\frac{1}{2})\,\sigma}}{1-e^{(b-r_{iR\,(iL)}-r_{jL\,(jR)})\,k\,\pi\,R}} \\ &\times \left(J_{\frac{1}{2}\mp r_{jL\,(jR)}}\left(e^{\sigma}\, x_{nL(R)}\right) + c_{L\,(R)}\, Y_{\frac{1}{2}\mp r_{jL\,(jR)}}\left(e^{\sigma}\, x_{nL(R)}\right)\right) \xi^{\mathrm{E}}_{ij}\left((\xi^{\mathrm{E}}_{ij})^{\dagger}\right). \end{aligned} \quad (35)$$

Finally, the tree level $S^{(m)}-l^{(0)}-l^{(n)}$ interaction is carried by the effective Yukawa coupling $\xi^{\mathrm{E}\,m\,n}_{ij}$ and it reads

$$\xi^{\mathrm{E}\,m\,n}_{ij}\left((\xi^{\mathrm{E}\,m\,n}_{ij})^{\dagger}\right) = V^{mn}_{S\,RL\,(LR)\,ij}\, \xi^{\mathrm{E}}_{5\,ij}\left((\xi^{\mathrm{E}}_{5\,ij})^{\dagger}\right), \quad (36)$$

with

$$\begin{aligned} V^{mn}_{S\,RL\,(LR)\,ij} &= \frac{N_{Ln\,(Rn)}}{N_{S\,m}\,\pi\,R\sqrt{\frac{1-e^{(1-2\,r_{iR\,(iL)})\,k\,\pi\,R}}{k\,\pi\,R\,(2\,r_{iR\,(iL)}-1)}}} \\ &\times \int\limits_{-\pi R}^{\pi R} dy\, e^{(\frac{5}{2}-r_{iR\,(iL)})\,\sigma} \left(J_{b-2}(e^{\sigma}\, x_{mS}) + \alpha_n\, Y_{b-2}(e^{\sigma}\, x_{mS})\right) \\ &\times \left(J_{\frac{1}{2}\mp r_{jL\,(jR)}}(e^{\sigma}\, x_{nL(R)}) + c_{L\,(R)}\, Y_{\frac{1}{2}\mp r_{jL\,(jR)}}(e^{\sigma}\, x_{nL(R)})\right), \end{aligned} \quad (37)$$

and, therefore, we get

$$\xi^{E\,m\,n}_{ij}\Big((\xi^{E\,m\,n}_{ij})^\dagger\Big) = \frac{V^{mn}_{S\,RL\,(LR)\,ij}}{V^{00}_{S\,RL(LR)\,ij}}\,\xi^{E}_{ij}\,\Big((\xi^{E}_{ij})^\dagger\Big)$$
$$= \frac{N_{Ln\,(Rn)}\sqrt{\frac{1-e^{(1-2\,r_{jL\,(jR)})\,k\,\pi\,R}}{\pi\,R\,(2\,r_{jL\,(jR)}-1)}}\sqrt{\frac{e^{2\,(b-1)\,k\,\pi\,R}-1}{(b-1)}}(r_{iR\,(iL)}+r_{jL\,(jR)}-b)}{N_{S\,m}\Big(1-e^{(b-r_{iR\,(iL)}-r_{jL\,(jR)})\,k\,\pi\,R}\Big)}$$
$$\times \int_{-\pi R}^{\pi R} dy\, e^{(\frac{5}{2}-r_{iR\,(iL)})\,\sigma}\Big(J_{b-2}(e^\sigma\,x_{mS})+\alpha_n\,Y_{b-2}(e^\sigma\,x_{mS})\Big)$$
$$\times\Big(J_{\frac{1}{2}\mp r_{jL\,(jR)}}(e^\sigma\,x_{nL(R)})+c_{L\,(R)}\,Y_{\frac{1}{2}\mp r_{jL\,(jR)}}(e^\sigma\,x_{nL(R)})\Big)\,\xi^{E}_{ij}\,\Big((\xi^{E}_{ij})^\dagger\Big)\,. \quad (38)$$

The decay widths of the LFV $\mu \to e\gamma$, $\tau \to e\gamma$ and $\tau \to \mu\gamma$ decays are calculated by using on-shell renormalization scheme (see Sec. 2.1) and we have

$$\Gamma(l_1 \to l_2\gamma) = c_1(|A_1|^2 + |A_2|^2)\,, \quad (39)$$

with

$$\begin{aligned} A_1 &= A_1^0 + A_1^{\mathrm{S\,KK}} + A_1^{l\,\mathrm{KK}} + A_1^{\mathrm{S},l\,\mathrm{KK}}\,, \\ A_2 &= A_2^0 + A_2^{\mathrm{S\,KK}} + A_2^{l\,\mathrm{KK}} + A_2^{\mathrm{S},l\,\mathrm{KK}}\,. \end{aligned} \quad (40)$$

Notice that we present the explicit expression of the amplitudes $A^0_{1\,(2)}$, $A^{\mathrm{S\,KK}}_{1\,(2)}$ and $A^{\mathrm{S},\,l\,\mathrm{KK}}_{1\,(2)}$ in Appendix B.

3. Discussion

The Yukawa interactions coming from lepton–lepton-S vertices drive the radiative LFV $l_1 \to l_2\gamma$ decays[6], and their strengths are regulated by the Yukawa couplings which are free parameters of the model used. In the present work, we study these LFV decays in the RS1 background and we assume that the new Higgs doublet and the gauge fields are accessible to the extra dimension. Here, in order to obtain zero mode Higgs doublet, one considers the boundary mass term (see Eq. (9)) and impose the fine tuning

[6] Here, we do not take into account the internal neutrino mediation due to their weak contribution to the BRs of the processes we study and, therefore, we assume that the lepton FV comes from the internal new neutral Higgs bosons, h^0 and A^0. Notice that we ignored the possible restrictions coming from the hadronic decays.

$b = 2 + \sqrt{4+a}$ (Eq. (11)) of the parameters b and a which regulates to the boundary and bulk mass terms. Finally, the zero mode new Higgs doublet is obtained as an exponential function of the parameter b (Eq. (12)) and it is highly localized around the visible brane.

The choice $b = 2 - \sqrt{4+a}$ is also possible for the non-vanishing zero mode. However, we do not take this possibility into account because of the following reason: If the fermions are localized on the 4D brane, the overall quantity $V_{ij}^{0}\,\xi_{5\,ij}^{\mathrm{E}}$ is fixed to the chosen numerical value of ξ_{ij}^{E} with the assumption that the coupling $\xi_{5\,ij}^{\mathrm{E}}$ is flavor dependent and appropriately regulated. For $b = 2 - \sqrt{4+a}$ the coupling V_{ij}^{0} is a number of orders smaller compared to $b = 2 + \sqrt{4+a}$ and, since this term appear in the denominator of the coupling $\xi^{\mathrm{E}\,n}$ according to our definition, $\xi^{\mathrm{E}\,n}$ exceeds the range of perturbative calculation. Notice that the coupling $V_{ij}^{S\,n}$ is not so much sensitive to the parameter b. In the case of bulk fermions, the choice $b = 2 - \sqrt{4+a}$ results in extremely large coupling which breaks the perturbative upper limit (negligible coupling which causes weak sensitivity to KK mode contributions) for set II (set I).

As a first attempt we assume that the leptons are restricted to the 4D brane. In this case the contribution of the extra dimension is due to the new Higgs KK modes which appear in the internal line of the loop with the modified Yukawa couplings (Eq. (23)). Second, we consider that the leptons are also localized in the extra dimension. We follow the idea that the localization of the lepton fields in the extra dimension occurs with the addition of a Dirac mass term $m_l = r\sigma'$ with $\sigma = k\,|y|$ (Eq. (27)). In this case, the right and left handed lepton zero modes (Eq. (C.3)) are chosen to locate at different positions in the extra dimension in order to explain different flavor mass hierarchy. In the scenario we choose the contribution of the extra dimension is coming from the new Higgs KK modes and the lepton KK modes appearing in the internal line of the loop. The FV is carried by the new Yukawa couplings which are fixed to an appropriate number,

TABLE I

The values of the input parameters used in the numerical calculations.

Parameter	Value
m_μ	0.106 (GeV)
m_τ	1.78 (GeV)
m_{h^0}	100 (GeV)
m_{A^0}	200 (GeV)
G_{F}	$1.16637 10^{-5}$ (GeV^{-2})

respecting the current measurements and the location parameters of leptons are responsible for the lepton mass hierarchy. This choice makes the constraints coming from various LFV processes to be more relaxed. Here, we consider two different set of locations of charged leptons in order to obtain the masses of different flavors[7]. In the first set (Table II), we consider the left and right handed fields having the same location in the extra dimension. In the second, we choose the left handed charged lepton locations as the same for each flavor, and we estimate the right handed ones by respecting the current charged lepton masses. For the second set, we observe that the BRs of the decays under consideration enhance since the KK mode couplings to the new Higgs scalars, which are highly localized near the visible brane, become stronger if the left handed lepton field is near to this brane. For the effective Yukawa couplings in four dimension we choose that $\bar{\xi}^{E}_{N,ij}$, $i,j = e,\mu$ are smaller compared to $\bar{\xi}^{E}_{N,\tau\, i}\, i = e,\mu,\tau$, since latter ones contain heavy flavor and we assume that, in four dimensions, the couplings $\bar{\xi}^{E}_{N,ij}$ is symmetric with respect to the indices i and j. Furthermore, the curvature parameter k and the compactification radius R are among the free parameters of the theory. Here, we take $k\,R = 10.83$ and consider in the region 10^{17} (GeV)$\leq k \leq 10^{18}$ (GeV) (see the discussion in Appendix C and [40]). Throughout our calculations we use the input values given in Table I.

In the case that the leptons live on the 4D brane the contribution of the Higgs boson KK modes is negligible for the decays under consideration The weakness of the new contribution is due to the tiny ratio z_{Sn} appearing in the expression (Eq. (B.2)) which represents the additional effects to the amplitudes.

Now we analyze the case that the leptons are also accessible to the extra dimension.

Fig. 2 represents the parameter k dependence of the BR of the LFV $\mu \to e\gamma$ decay for $\bar{\xi}^{E}_{N,\tau e} = 0.01$ GeV, $\bar{\xi}^{E}_{N,\tau\mu} = 1.0$ GeV. Here the solid (dashed, short dashed) line represents the BR without KK modes of leptons and new Higgs bosons (with KK modes of leptons and new Higgs bosons for lepton location set II, set I), for $a = 0.01$ and 0.1[8]. It is observed that the BR($\mu \to e\gamma$) is of the order of 10^{-11} without the internal lepton and

[7] The gauge sector is necessarily lives in the extra dimension and their KK modes appear after the compactification of the extra dimension. The different fermion locations can induce additional FCNC effects at tree level due to the couplings of neutral gauge KK modes-leptons and they should be suppressed even for low KK masses, by choosing the location parameters r_L (r_R) appropriately. In the set of location parameters we use (Table II), we verify the various experimental FCNC constraints with KK neutral gauge boson masses as low as few TeVs (see the similar the set of location parameters and the discussion given in [48, 49].)

[8] For $a = 0.01$ and $a = 0.1$ the curves almost coincide.

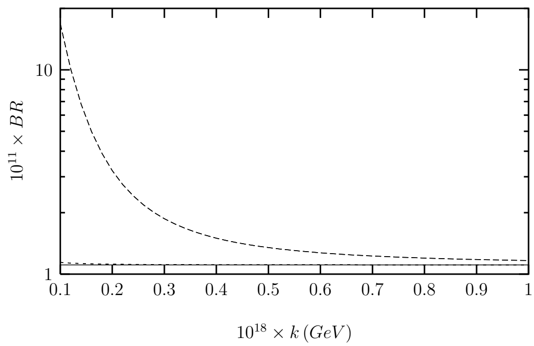

Fig. 2. k dependence of the BR($\mu \to e\gamma$) for $\bar{\xi}^{E}_{N,\tau e} = 0.01\,\mathrm{GeV}$, $\bar{\xi}^{E}_{N,\tau\mu} = 1.0\,\mathrm{GeV}$. Here the solid (dashed, short dashed) line represents the BR without KK modes of leptons and new Higgs bosons (with KK modes of leptons and new Higgs bosons for lepton location set II, set I), for $a = 0.01$ and 0.1.

new Higgs boson KK mode contributions. The addition of these KK modes result in that the BR enhances almost 2× one order, for the lepton location set II, especially for the small values of the parameter k. For the set I, the enhancement of the BR is negligible. On the other hand, the BRs are weakly sensitive to the parameter a which plays a crucial role in the localization of new Higgs bosons. We present the parameter a dependence of the BR ($\mu \to e\gamma$) for the lepton location set II, in Fig. 3 for $k = 10^{18}\,\mathrm{GeV}$. This figure shows that the enhancement of the BR ($\mu \to e\gamma$) is of the order of $\sim 0.1\%$ in the range of a, $0.01 \leq a \leq 1.0$. This is a negligible enhancement which can not be determined. On the other hand the enhancement in the case of set II is due to the fact that the left handed leptons (KK modes) are near to the visible brane and their couplings to the new Higgs bosons, which are localized near the visible brane, become stronger.

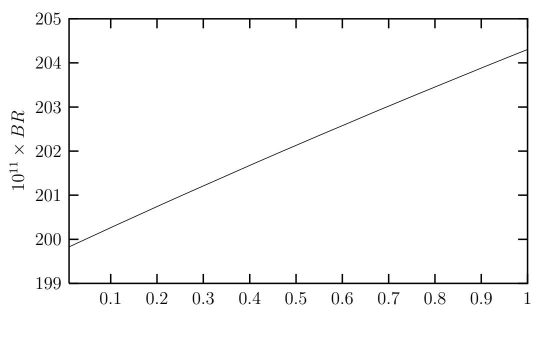

Fig. 3. a dependence of the BR($\mu \to e\gamma$) for $\bar{\xi}^{E}_{N,\tau e} = 0.01\,\mathrm{GeV}$, $\bar{\xi}^{E}_{N,\tau\mu} = 1.0\,\mathrm{GeV}$, for the lepton location set II and $k = 10^{18}\,\mathrm{GeV}$.

Fig. 4 is devoted to the parameter k dependence of the BR of the LFV $\tau \to e\gamma$ decay for $\bar{\xi}^{E}_{N,\tau e} = 0.1\,\mathrm{GeV}$, $\bar{\xi}^{E}_{N,\tau\tau} = 50\,\mathrm{GeV}$. Here the solid (dashed, short dashed) line represents the BR without KK modes of leptons and new Higgs bosons (with KK modes of leptons and new Higgs bosons for lepton location set II, set I), for $a = 0.01$ and 0.1. This figure shows that the BR $(\tau \to e\gamma)$ is of the order of 10^{-12} without the internal lepton and new Higgs boson KK mode contributions. The addition of these KK modes results in that the BR enhances by almost three orders for the small values of the parameter k and the lepton location set II. For the set I, the BR enhances to the value almost two times larger compared to the one without KK modes.

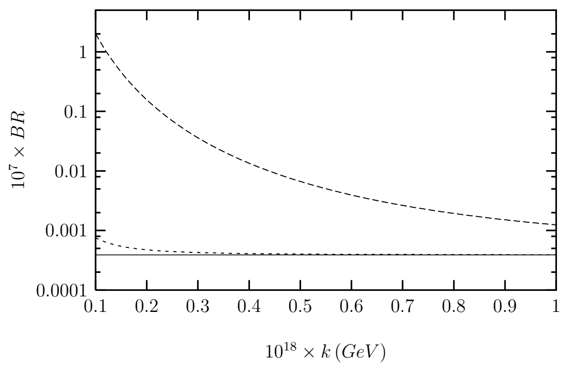

Fig. 4. The same as Fig. 2 but for $\tau \to e\gamma$ decay and for $\bar{\xi}^{E}_{N,\tau e} = 0.1$ GeV, $\bar{\xi}^{E}_{N,\tau\tau} = 50\,\mathrm{GeV}$.

Fig. 5 represents the parameter a dependence of the BR $(\tau \to e\gamma)$ for $k = 10^{17}\,\mathrm{GeV}$, for the lepton location set II. It is observed that the enhancement of the BR $(\tau \to e\gamma)$ is greater than $\sim 2.0\%$ in the range of a, $0.01 \le a \le 1.0$.

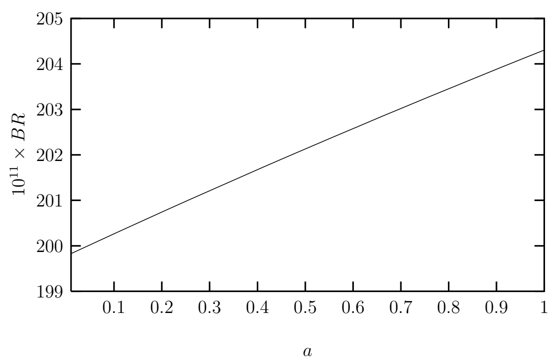

Fig. 5. The same as Fig. 3 but for $\tau \to e\gamma$ decay, for $\bar{\xi}^{E}_{N,\tau e} = 0.1\,\mathrm{GeV}$, $\bar{\xi}^{E}_{N,\tau\tau} = 50\,\mathrm{GeV}$ and $k = 10^{17}\,\mathrm{GeV}$.

Similar to the previous decay, this is a small enhancement which cannot be determined.

Fig. 6 shows is the parameter k dependence of the BR of the LFV $\tau \to \mu\gamma$ decay for $\bar{\xi}^{E}_{N,\tau\mu} = 1.0\,\text{GeV}$, $\bar{\xi}^{E}_{N,\tau\tau} = 50\,\text{GeV}$. Here the solid (dashed, short dashed) line represents the BR without KK modes of leptons and new Higgs bosons (with KK modes of leptons and new Higgs bosons for lepton location set II, set I), for $a = 0.01$ and 0.1. The BR ($\tau \to \mu\gamma$) is at the order of the magnitude of 10^{-10} without the internal lepton and new Higgs boson KK mode contributions. The lepton and Higgs boson KK modes cause more than three order enhancement in the BR for the small values of the parameter k and for the lepton location set II. For the set I, this enhancement is almost two times of the BR without KK modes.

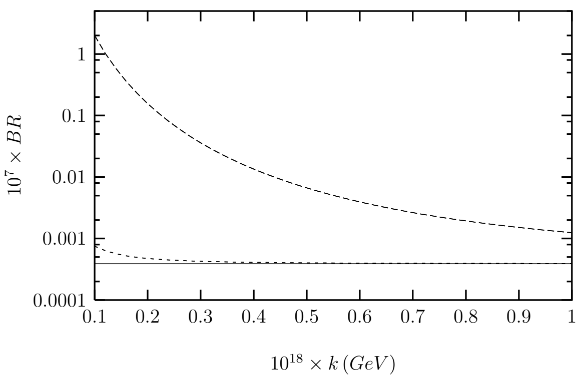

Fig. 6. The same as Fig. 2 but for $\tau \to \mu\gamma$ decay and for $\bar{\xi}^{E}_{N,\tau\mu} = 1.0\,\text{GeV}$, $\bar{\xi}^{E}_{N,\tau\tau} = 50\,\text{GeV}$.

In Fig. 7 we present the parameter a dependence of the BR ($\tau \to \mu\gamma$) for $k = 10^{17}\,\text{GeV}$ and the lepton location set II. We observe that the enhancement of the BR ($\tau \to \mu\gamma$) is more than $\sim 2.0\%$ in the range of a, $0.01 \leq a \leq 1.0$.

At this stage, we would like to summarize our results. For the brane leptons, the contribution of the Higgs boson KK modes to the BRs of the radiative LFV decays is too small to be detected. However, if one considers that the leptons are also accessible to the extra dimension, there exists a considerable enhancement in the BRs, especially for the small values of the parameter k. This enhancement occurs for the lepton location set II and it is due to the fact that the left handed leptons (KK modes), which are near to the visible brane, have enhanced couplings to the new Higgs bosons, which are also localized near the visible brane. Finally, we observe that, the BRs are weakly sensitive to the parameter a which regulates the localization of the new Higgs doublet in the extra dimension. With the more

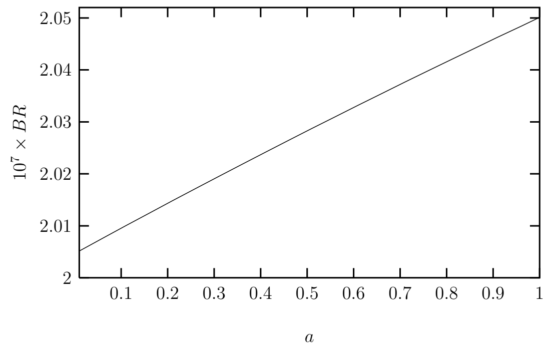

Fig. 7. The same as Fig. 3 but for $\tau \to \mu\gamma$ decay, for $\bar{\xi}^{E}_{N,\tau\mu} = 1.0\,\text{GeV}$, $\bar{\xi}^{E}_{N,\tau\tau} = 50\,\text{GeV}$ and $k = 10^{17}$ GeV.

accurate forthcoming measurements of the BRs of the LFV decays it would be possible to test the existence of the warped extra dimensions and, to get a considerable information which fields are accessible to the extra dimension.

Appendix A

The mass matrix of new Higgs boson

The Higgs potential which creates the masses of neutral CP even and CP odd Higgs bosons, $S = h^0$, A^0 reads

$$V_{m_S} = c'_h[\text{Re}(\phi_1^+\phi_2)]^2 + c'_A[\text{Im}(\phi_1^+\phi_2)]^2 , \qquad \text{(A.1)}$$

where ϕ_1 and ϕ_2 are given in (Eq. (17)). After the electrowek breaking the SM Higgs acquires a vacuum expectation value (see Eq. (18)) and the mass Lagrangian of new CP even Higgs boson (h^0) [9] becomes

$$\mathcal{L}_S = \frac{1}{2}\sum_{n=1}^{\infty} m^2_{n\,S}\left(S^{(n)}(x)\right)^2 + c'_h\frac{v^2}{2}\left(S^{(0)}(x) + \sum_{n=1}^{\infty} S^{(n)}(x)\alpha_n\right)^2 \qquad \text{(A.2)}$$

with $c_h = c'_h\, f_0^2(\pi R)$, $\alpha_n = f_n(\pi R)/f_0(\pi R)$ and $m_S^2 = c_h\, v^2$, $S = h^0$. By using the mass Lagrangian the S boson mass matrix is obtained as (see [50] and [51] for boson mass matrix in the one and two non-universal extra dimensions, [36] for $U(1)_Y$ gauge boson mass matrix.):

[9] The similar mass Lagrangian appears for the CP odd Higgs boson A^0 with the replacement $c'_h \to c'_A$.

$$M_S^2 = \begin{pmatrix} m_S^2 & \alpha_1\, m_S^2 & \alpha_2\, m_S^2 & \cdots \\ \alpha_1\, m_S^2 & m_{1\,S}^2 + m_S^2\, \alpha_1^2 & m_S^2\, \alpha_1\, \alpha_2 & \cdots \\ \alpha_2\, m_S^2 & \alpha_2\, \alpha_1\, m_S^2 & m_{2\,S}^2 + m_S^2\, \alpha_2^2 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \tag{A.3}$$

and the determinant equation reads

$$\mathrm{Det}\Big(M_S^2 - \lambda I\Big) = \left(\prod_{n=1}^{\infty}(m_{n\,S}^2 - \lambda)\right)\left(m_S^2 - \lambda - \lambda m_S^2 \sum_{n=1}^{\infty} \frac{\alpha_n}{m_{n\,S}^2 - \lambda}\right) = 0\,. \tag{A.4}$$

This equation is used to calculate the physical masses of zero and KK modes of S bosons and their eigenstates. To leading order, the physical S boson mass (the zero mode one) reads

$$(m_S^{\rm phys})^2 = m_S^2 \left(1 + \sum_{n=1}^{\infty} \frac{m_S^2\, \alpha_n^2}{m_{n\,S}^2}\right), \tag{A.5}$$

since $m_{n\,S} \gg m_S$. Notice that in our numerical calculations we do not take into account the additional effects coming from the mixing because they are suppressed due to the fact that the KK mode masses are considerably larger compared to the zero mode one.

Appendix B

The amplitudes appearing in the text

Here, we present the amplitudes which appear in the calculation of the decay widths of the radiative decays under consideration.

In the case that the charged leptons are restricted to the brane, the amplitudes A_1^0, A_2^0, $A_1^{\rm S\,KK}$ and $A_2^{\rm S\,KK}$ in Eq. (25) read

$$\begin{aligned} A_1^0 \;=\; & Q_\tau \frac{1}{4\, m_\tau^2} \Bigg\{ m_{l_1}\, \bar{\xi}^{\rm E}_{N,\tau l_2}\, \bar{\xi}^{\rm E}_{N,\tau l_1} \int_0^1 dx \int_0^{1-x} dy\; x\,(x+y-1) \left(\frac{z_{h^0}}{L_{h^0}} + \frac{z_{A^0}}{L_{A^0}}\right) \\ & - m_{l_2} \bar{\xi}^{\rm E}_{N,l_2\tau} \bar{\xi}^{\rm E}_{N,l_1\tau} \int_0^1 dx \int_0^{1-x} dy\, x\, y \left(\frac{z_{h^0}}{L_{h^0}} + \frac{z_{A^0}}{L_{A^0}}\right) \\ & + m_\tau\, \bar{\xi}^{\rm E}_{N,\tau l_2} \bar{\xi}^{\rm E}_{N,l_1\tau} \int_0^1 dx \int_0^{1-x} dy\, (x-1) \left(\frac{z_{h^0}}{L_{h^0}} - \frac{z_{A^0}}{L_{A^0}}\right) \Bigg\}, \end{aligned}$$

$$A_2^0 = Q_\tau \frac{1}{4\,m_\tau^2}\Big\{ - m_{l_1}\, \bar{\xi}^{\rm E}_{N,l_2\tau}\bar{\xi}^{\rm E}_{N,l_1\tau} \int_0^1 dx \int_0^{1-x} dy\, x\,(x+y-1)\left(\frac{z_{h^0}}{L_{h^0}}+\frac{z_{A^0}}{L_{A^0}}\right)$$
$$+ m_{l_2}\, \bar{\xi}^{\rm E}_{N,\tau l_2}\, \bar{\xi}^{\rm E}_{N,\tau l_1} \int_0^1 dx \int_0^{1-x} dy\, x\, y\, (\frac{z_{h^0}}{L_{h^0}}+\frac{z_{A^0}}{L_{A^0}})$$
$$- m_\tau\, \bar{\xi}^{\rm E}_{N,l_2\tau}\, \bar{\xi}^{\rm E}_{N,\tau l_1} \int_0^1 dx \int_0^{1-x} dy\, (x-1)\, (\frac{z_{h^0}}{L_{h^0}}-\frac{z_{A^0}}{L_{A^0}})\Big\}, \qquad \text{(B.1)}$$

$$A_1^{\rm S\,KK} = Q_\tau \frac{1}{4\,m_\tau^2}\sum_{n=1}^{\infty}\Big\{ m_{l_1}\, \bar{\xi}^{{\rm E}\,n}_{N,\tau l_2}\, \bar{\xi}^{{\rm E}\,n}_{N,\tau l_1} \int_0^1 dx \int_0^{1-x} dy x\,(x+y-1)$$
$$\times \left(\frac{z_{h^0\,n}}{L_{h^0\,n}}+\frac{z_{A^0\,n}}{L_{A^0\,n}}\right)$$
$$m_{l_2}\, \bar{\xi}^{{\rm E}\,n}_{N,l_2\tau}\, \bar{\xi}^{{\rm E}\,n}_{N,l_1\tau} \int_0^1 dx \int_0^{1-x} dy\, x\, y\left(\frac{z_{h^0\,n}}{L_{h^0\,n}}+\frac{z_{A^0\,n}}{L_{A^0\,n}}\right)$$
$$+ m_\tau\, \bar{\xi}^{{\rm E}\,n}_{N,\tau l_2}\, \bar{\xi}^{{\rm E}\,n}_{N,l_1\tau} \int_0^1 dx \int_0^{1-x} dy\, (x-1)\left(\frac{z_{h^0\,n}}{L_{h^0\,n}}-\frac{z_{A^0\,n}}{L_{A^0\,n}}\right)\Big\},$$

$$A_2^{\rm S\,KK} = Q_\tau \frac{1}{4\,m_\tau^2}\sum_{n=1}^{\infty}\Big\{ - m_{l_1}\, \bar{\xi}^{{\rm E}\,n}_{N,l_2\tau}\, \bar{\xi}^{{\rm E}\,n}_{N,l_1\tau} \int_0^1 dx \int_0^{1-x} dy\, x\,(x+y-1)$$
$$\times \left(\frac{z_{h^0\,n}}{L_{h^0\,n}}+\frac{z_{A^0\,n}}{L_{A^0\,n}}\right)$$
$$+ m_{l_2}\, \bar{\xi}^{{\rm E}\,n}_{N,\tau l_2}\, \bar{\xi}^{{\rm E}\,n}_{N,\tau l_1} \int_0^1 dx \int_0^{1-x} dy\, x\, y(\frac{z_{h^0\,n}}{L_{h^0\,n}}+\frac{z_{A^0\,n}}{L_{A^0\,n}})$$
$$- m_\tau\, \bar{\xi}^{{\rm E}\,n}_{N,l_2\tau}\, \bar{\xi}^{{\rm E}\,n}_{N,\tau l_1} \int_0^1 dx \int_0^{1-x} dy\, (x-1)\left(\frac{z_{h^0\,n}}{L_{h^0\,n}}-\frac{z_{A^0\,n}}{L_{A^0\,n}}\right)\Big\}, \qquad \text{(B.2)}$$

where

$$L_S = z_S + x^2\, z_S + x\Big(1+(y-2)\, z_S\Big)\,,$$
$$L_{S\,n} = z_{S\,n} + x^2\, z_{S\,n} + x\Big(1+(y-2)\, z_{S\,n}\Big)\,, \qquad \text{(B.3)}$$

with $z_S = m_\tau^2/m_S^2$, $z_{S\,n} = m_\tau^2/m_{n\,S}^2$. Here, $l_1\,(l_2) = \tau;\mu\,(\mu$ or $e;e)$, $c_1 = (G_F^2 \alpha_{em} m_{l_1}^3)/32\pi^4$, A_1 (A_2) is the left (right) chiral amplitude, Q_τ is the charge of tau lepton and $m_{n\,S}$ is the internal Higgs KK mode mass (see Eq. (15)). Notice that we take the Yukawa couplings real.

If the charged leptons are also accessible to the extra dimension the amplitudes $A^0_{1(2)}$, $A^{\mathrm{S\,KK}}_{1\,(2)}$ and $A^{\mathrm{S},l\,\mathrm{KK}}_{1(2)}$ (see Eq. (40)) are

$$
\begin{aligned}
A_1^0 &= Q_\tau \frac{1}{4\,m_\tau^2}\Bigg\{ m_{l_1}\,\bar{\xi}^{\mathrm{E}}_{N,\tau l_2}\,\bar{\xi}^{\mathrm{E}}_{N,\tau l_1}\int_0^1 dx \int_0^{1-x} dy\, x\,(x+y-1)\left(\frac{z_{h^0}}{L_{h^0}}+\frac{z_{A^0}}{L_{A^0}}\right) \\
&\quad - m_{l_2}\,\bar{\xi}^{\mathrm{E}}_{N,l_2\tau}\,\bar{\xi}^{\mathrm{E}}_{N,l_1\tau}\int_0^1 dx \int_0^{1-x} dy\, x\,y\left(\frac{z_{h^0}}{L_{h^0}}+\frac{z_{A^0}}{L_{A^0}}\right) \\
&\quad + m_\tau\,\bar{\xi}^{\mathrm{E}}_{N,\tau l_2}\,\bar{\xi}^{\mathrm{E}}_{N,l_1\tau}\int_0^1 dx \int_0^{1-x} dy\,(x-1)\left(\frac{z_{h^0}}{L_{h^0}}-\frac{z_{A^0}}{L_{A^0}}\right)\Bigg\}, \\
A_2^0 &= Q_\tau \frac{1}{4\,m_\tau^2}\Bigg\{ - m_{l_1}\bar{\xi}^{\mathrm{E}}_{N,l_2\tau}\bar{\xi}^{\mathrm{E}}_{N,l_1\tau}\int_0^1 dx \int_0^{1-x} dy\, x\,(x+y-1)\left(\frac{z_{h^0}}{L_{h^0}}+\frac{z_{A^0}}{L_{A^0}}\right) \\
&\quad + m_{l_2}\,\bar{\xi}^{\mathrm{E}}_{N,\tau l_2}\,\bar{\xi}^{\mathrm{E}}_{N,\tau l_1}\int_0^1 dx \int_0^{1-x} dy\, x\,y\left(\frac{z_{h^0}}{L_{h^0}}+\frac{z_{A^0}}{L_{A^0}}\right) \\
&\quad - m_\tau\,\bar{\xi}^{\mathrm{E}}_{N,l_2\tau}\,\bar{\xi}^{\mathrm{E}}_{N,\tau l_1}\int_0^1 dx \int_0^{1-x} dy\,(x-1)\left(\frac{z_{h^0}}{L_{h^0}}-\frac{z_{A^0}}{L_{A^0}}\right)\Bigg\}, \qquad \text{(B.4)}
\end{aligned}
$$

$$
\begin{aligned}
A_1^{\mathrm{S\,KK}} &= Q_\tau \frac{1}{4\,m_\tau^2}\sum_{n=1}^{\infty}\Bigg\{ m_{l_1}\,\bar{\xi}^{\mathrm{E}\,n0}_{N,\tau l_2}\,\bar{\xi}^{\mathrm{E}\,n0}_{N,\tau l_1}\int_0^1 dx \int_0^{1-x} dy\, x\,(x+y-1) \\
&\quad \times\left(\frac{z_{h^0\,n}}{L_{h^0\,n}}+\frac{z_{A^0\,n}}{L_{A^0\,n}}\right) \\
&\quad - m_{l_2}\,\bar{\xi}^{\mathrm{E}\,n0}_{N,l_2\tau}\,\bar{\xi}^{\mathrm{E}\,n0}_{N,l_1\tau}\int_0^1 dx \int_0^{1-x} dy\, x\,y\left(\frac{z_{h^0\,n}}{L_{h^0\,n}}+\frac{z_{A^0\,n}}{L_{A^0\,n}}\right) \\
&\quad + m_\tau\,\bar{\xi}^{\mathrm{E}\,n0}_{N,\tau l_2}\,\bar{\xi}^{\mathrm{E}\,n0}_{N,l_1\tau}\int_0^1 dx \int_0^{1-x} dy\,(x-1)\left(\frac{z_{h^0\,n}}{L_{h^0\,n}}-\frac{z_{A^0\,n}}{L_{A^0\,n}}\right)\Bigg\},
\end{aligned}
$$

$$
\begin{aligned}
A_2^{\mathrm{S\,KK}} &= Q_\tau \frac{1}{4\,m_\tau^2} \sum_{n=1}^{\infty} \Bigg\{ - m_{l_1} \bar{\xi}^{\mathrm{E}\,n0}_{N,l_2\tau}\, \bar{\xi}^{\mathrm{E}\,n0}_{N,l_1\tau} \int\limits_0^1 dx \int\limits_0^{1-x} dy\, x\,(x+y-1) \\
&\quad \times \left(\frac{z_{h^0\,n}}{L_{h^0\,n}} + \frac{z_{A^0\,n}}{L_{A^0\,n}} \right) \\
&\quad + m_{l_2}\, \bar{\xi}^{\mathrm{E}\,n0}_{N,\tau l_2}\, \bar{\xi}^{\mathrm{E}\,n0}_{N,\tau l_1} \int\limits_0^1 dx \int\limits_0^{1-x} dy\, x\, y \left(\frac{z_{h^0\,n}}{L_{h^0\,n}} + \frac{z_{A^0\,n}}{L_{A^0\,n}} \right) \\
&\quad - m_\tau\, \bar{\xi}^{\mathrm{E}\,n0}_{N,l_2\tau}\, \bar{\xi}^{\mathrm{E}\,n0}_{N,\tau l_1} \int\limits_0^1 dx \int\limits_0^{1-x} dy\,(x-1) \left(\frac{z_{h^0\,n}}{L_{h^0\,n}} - \frac{z_{A^0\,n}}{L_{A^0\,n}} \right) \Bigg\}, \qquad \text{(B.5)}
\end{aligned}
$$

$$
\begin{aligned}
A_1^{l\,\mathrm{KK}} &= \frac{Q_\tau}{48\,m_\tau^2} \sum_{n=1}^{\infty} \Bigg\{ \frac{m_\tau^2}{m_{nR}^2}\, m_{l_1}\, (\bar{\xi}^{\mathrm{E}\,0n}_{N,l_2\tau})^\dagger\, (\bar{\xi}^{\mathrm{E}\,0n}_{N,l_1\tau})^\dagger \Big(G(z_{nR,h^0}) + G(z_{nR,A^0}) \Big) \\
&\quad + \frac{m_\tau^2}{m_{nL}^2}\, m_{l_2}\, \bar{\xi}^{\mathrm{E}\,0n}_{N,l_2\tau}\, \bar{\xi}^{\mathrm{E}\,0n}_{N,l_1\tau} \Big(G(z_{nL,h^0}) + G(z_{nL,A^0}) \Big) \Bigg\}, \\
A_2^{l\,\mathrm{KK}} &= \frac{-Q_\tau}{48\,m_\tau^2} \sum_{n=1}^{\infty} \Bigg\{ \frac{m_\tau^2}{m_{nR}^2}\, m_{l_2}\, (\bar{\xi}^{\mathrm{E}\,0n}_{N,l_2\tau})^\dagger\, (\bar{\xi}^{\mathrm{E}\,0n}_{N,l_1\tau})^\dagger \Big(G(z_{nR,h^0}) + G(z_{nR,A^0}) \Big) \\
&\quad + \frac{m_\tau^2}{m_{nL}^2}\, m_{l_1}\, \bar{\xi}^{\mathrm{E}\,0n}_{N,l_2\tau}\, \bar{\xi}^{\mathrm{E}\,0n}_{N,l_1\tau} \Big(G(z_{nL,h^0}) + G(z_{nL,A^0}) \Big) \Bigg\}, \qquad \text{(B.6)}
\end{aligned}
$$

$$
\begin{aligned}
A_1^{\mathrm{S},l\,\mathrm{KK}} &= \frac{Q_\tau}{48\,m_\tau^2} \sum_{n,m=1}^{\infty} \Bigg\{ \frac{m_\tau^2}{m_{nR}^2}\, m_{l_1}\, (\bar{\xi}^{\mathrm{E}\,mn}_{N,l_2\tau})^\dagger\, (\bar{\xi}^{\mathrm{E}\,mn}_{N,l_1\tau})^\dagger \\
&\quad \times \Big(G(z_{nR,h^0\,m}) + G(z_{nR,A^0\,m}) \Big) \\
&\quad + \frac{m_\tau^2}{m_{nL}^2}\, m_{l_2}\, \bar{\xi}^{\mathrm{E}\,mn}_{N,l_2\tau}\, \bar{\xi}^{\mathrm{E}\,mn}_{N,l_1\tau} \Big(G(z_{nL,h^0\,m}) + G(z_{nL,A^0\,m}) \Big) \Bigg\}, \\
A_2^{\mathrm{S},l\,\mathrm{KK}} &= \frac{-Q_\tau}{48\,m_\tau^2} \sum_{n,m=1}^{\infty} \Bigg\{ \frac{m_\tau^2}{m_{nR}^2}\, m_{l_2}\, (\bar{\xi}^{\mathrm{E}\,mn}_{N,l_2\tau})^\dagger\, (\bar{\xi}^{\mathrm{E}\,mn}_{N,l_1\tau})^\dagger \\
&\quad \times \Big(G(z_{nR,h^0\,m}) + G(z_{nR,A^0\,m}) \Big) \\
&\quad + \frac{m_\tau^2}{m_{nL}^2}\, m_{l_1}\, \bar{\xi}^{\mathrm{E}\,mn}_{N,l_2\tau} \bar{\xi}^{\mathrm{E}\,mn}_{N,l_1\tau} \Big(G(z_{nL,h^0\,m}) + G(z_{nL,A^0\,m}) \Big) \Bigg\}. \qquad \text{(B.7)}
\end{aligned}
$$

Here A_1 (A_2) is the left (right) chiral amplitude, $l_1\,(l_2) = \tau;\mu\,(\mu$ or $e;e)$, the functions $F(w)$, $G(w)$ are

$$\begin{aligned} F(w) &= \frac{w(3-4\,w+w^2+2\,ln\,w)}{(1-w)^3}\,, \\ G(w) &= -\frac{w(2+3\,w-6\,w^2+w^3+6\,w\,ln\,w)}{(1-w)^4}\,, \end{aligned} \tag{B.8}$$

$c_1 = (G_{\rm F}^2\alpha_{em}m_{l_1}^3)/32\pi^4$, $z_S = (m_\tau^2)/m_S^2$, $z_{Sn} = (m_\tau^2)/(m_{n\,S}^2)$, $z_{nL(nR),S} = (m_{nL\,(nR)}^2)/(m_S^2)$, $z_{nL(nR),S\,m} = (m_{nL\,(nR)}^2)/(m_{m\,S}^2)$ with left (right) handed internal lepton KK mode mass $m_{nL\,(nR)}$ (Eq. (C.14)). In Eqs. (B.4) and (B.5) the functions L_S and $L_{S\,n}$ are given in (Eq. (B.3)). In the case that the incoming and outgoing lepton masses are ignored in the functions L_S and $L_{S\,n}$ one gets the integrated form of $A_{1(2)}^0$ as

$$\begin{aligned} A_1^0 &= Q_\tau \frac{1}{48\,m_\tau^2}\Big\{6\,m_\tau\,\bar{\xi}^{\rm E}_{N,\tau l_2}\,\bar{\xi}^{\rm E}_{N,l_1\tau}\Big(F(z_{h^0})-F(z_{A^0})\Big) \\ &\quad + m_{l_1}\,\bar{\xi}^{\rm E}_{N,\tau l_2}\,\bar{\xi}^{\rm E}_{N,\tau l_1}\Big(G(z_{h^0})+G(z_{A^0})\Big)\Big\}\,, \\ A_2^0 &= -Q_\tau \frac{1}{48\,m_\tau^2}\Big\{6\,m_\tau\,\bar{\xi}^{\rm E}_{N,l_2\tau}\,\bar{\xi}^{\rm E}_{N,\tau l_1}\Big(F(z_{h^0})-F(z_{A^0})\Big) \\ &\quad + m_{l_1}\,\bar{\xi}^{\rm E}_{N,l_2\tau}\,\bar{\xi}^{\rm E}_{N,l_1\tau}\Big(G(z_{h^0})+G(z_{A^0})\Big)\Big\}\,. \end{aligned} \tag{B.9}$$

Notice that, for the amplitudes $A_{1(2)}^{l\,\rm KK}$ and $A_{1(2)}^{{\rm S},l\,\rm KK}$, the incoming and outgoing lepton masses are ignored in the functions $L_{\rm S}$ and $L_{{\rm S}\,n}$ since the internal KK leptons are heavy.

Appendix C

The construction of zero mode and KK mode leptons

This appendix is devoted to the construction of the zero mode and KK mode leptons in the case that the leptons are localized in the extra dimension with the help of the Dirac mass term given in (Eq. (27)). We start with the expansion of the bulk fermion as

$$\psi(x^\mu,y) = \frac{1}{\sqrt{2\,\pi\,R}}\sum_{n=0}^{\infty}\psi^{(n)}(x^\mu)\,e^{2\,\sigma}\,\chi_n(y)\,. \tag{C.1}$$

By using the normalization

$$\frac{1}{2\pi R}\int_{-\pi R}^{\pi R} dy\, e^{\sigma}\,\chi_n(y)\chi_m(y) = \delta_{nm}\,, \tag{C.2}$$

and the Dirac equation the zero mode fermion is obtained as

$$\chi_0(y) = N_0\, e^{-r\,\sigma}\,, \tag{C.3}$$

where $r = m/k$ and N_0 is the normalization constant:

$$N_0 = \sqrt{\frac{k\pi R\,(1-2\,r)}{e^{k\,\pi R(1-2\,r)}-1}}\,. \tag{C.4}$$

The appropriately normalized solution

$$\chi_0'(y) = e^{-\sigma/2}\,\chi_0(y) \tag{C.5}$$

is localized in the extra dimension where the localization is regulated by the parameter r. For $r > \frac{1}{2}$ ($r < \frac{1}{2}$) this solution is localized near the hidden (visible) brane and it has a constant profile for $r = \frac{1}{2}$.

Now, we are ready construct the SM leptons. What we need is to consider $\mathrm{SU}(2)_L$ doublet ψ_L and singlet ψ_R with separate Z_2 projection conditions: $Z_2\psi_R = \gamma_5\psi_R$ and $Z_2\psi_L = -\gamma_5\psi_L$ (see for example [32]). Finally we get the leptons accessible to the extra dimension as

$$\begin{aligned} l_{iL}(x^\mu, y) &= \frac{1}{\sqrt{2\,\pi\,R}}\, e^{2\,\sigma}\, l_{iL}^{(0)}(x^\mu)\,\chi_{i\,L0}(y) \\ &\quad + \frac{1}{\sqrt{2\,\pi\,R}}\sum_{n=1}^{\infty} e^{2\,\sigma}\left(l_{iL}^{(n)}(x^\mu)\,\chi^l_{i\,Ln}(y) + l_{iR}^{(n)}(x^\mu)\,\chi^l_{i\,Rn}(y)\right), \\ E_{jR}(x^\mu, y) &= \frac{1}{\sqrt{2\,\pi\,R}}\, e^{2\,\sigma}\, E_{jR}^{(0)}(x^\mu)\,\chi_{j\,R0}(y) \\ &\quad + \frac{1}{\sqrt{2\,\pi\,R}}\sum_{n=0}^{\infty} e^{2\sigma}\left(E_{jR}^{(n)}(x^\mu)\chi^E_{j\,Rn}(y) + E_{jL}^{(n)}(x^\mu)\chi^E_{j\,Ln}(y)\right). \end{aligned} \tag{C.6}$$

Here the zero mode leptons $\chi_{i\,L0}(y)$ and $\chi_{j\,R0}(y)$ are given in Eq. (C.3) with the replacements $r \to r_{iL}$ and $r \to r_{jR}$, respectively.

The zero mode fermions can get mass through the Z_2 invariant left handed fermion-right handed fermion–Higgs interaction, $\bar{\psi}_R\,\psi_L H$[10]. If the SM Higgs field lives on the visible brane as in our choice, the masses of fermions are calculated by using the integral

$$m_i = \frac{1}{2\pi R}\int\limits_{-\pi R}^{\pi R} dy\,\lambda_5\,\chi_{iL0}(y)\,\chi_{iR0}(y)\,\langle H\rangle\,\delta(y-\pi R)\,, \qquad \text{(C.7)}$$

where λ_5 is the coupling in five dimensions and it can be parametrized in terms of the one in four dimensions, the dimensionless coupling λ, $\lambda_5 = \lambda/\sqrt{k}$. Here the expectation value of the Higgs field $\langle H\rangle$ reads $\langle H\rangle = v/\sqrt{k}$ where v is the vacuum expectation value[11]. Now, we choose two different sets of location of charged lepton fields in order to obtain the masses of different flavors. In set I, the left and right handed fields of the same flavor have the same location, however, in set II, we choose the left handed charged lepton locations the same for each flavor. For both cases we estimate the left–right handed charged lepton locations by respecting their current masses.

TABLE II

Two possible locations of charged lepton fields. Here r_L and r_R are left handed and right handed lepton field location parameters, respectively.

	SET I		SET II	
	r_L	r_R	r_L	r_R
e	0.6710	0.6710	−0.4900	0.8800
μ	0.5826	0.5826	−0.4900	0.7160
τ	0.5273	0.5273	−0.4900	0.6249

The Z_2 the projection condition $Z_2\psi = -\gamma_5\psi$, used in constructing the left handed fields on the branes, results in that the left handed zero mode appears, the left (right) handed KK modes appear (disappear) on the branes, with the boundary conditions due to the Dirac mass term in the action Eq. (27):

$$\left(\frac{d}{dy} - m\right)\chi^l_{iLn}(y_0) = 0\,, \qquad \chi^l_{iRn}(y_0) = 0\,, \qquad \text{(C.8)}$$

[10] Here, we consider different location parameters r for each left handed and right handed part of different flavors. The location parameters for fermion fields are chosen so that this interaction creates the current masses of fermions.

[11] We take $v = 0.043\,M_{\rm Pl}$ to provide the measured gauge boson masses [40] and choose $k\,R = 10.83$ in order to get the correct effective scale on the visible brane, *i.e.*, $M_W = e^{-\pi kR}M_{\rm Pl}$ is of the order of TeV.

where $y_0 = 0$ or πR. Using the Dirac equation for KK mode leptons one gets the left handed lepton $\chi^l_{i\,Ln}(y)$ that lives on the visible brane as

$$\chi^l_{iLn}(y) = N_{Ln}\, e^{\sigma/2} \left(J_{\frac{1}{2}-r_{iL}}(e^{\sigma}\, x_{nL}) + c_L\, Y_{\frac{1}{2}-r_{iL}}(e^{\sigma}\, x_{nL}) \right), \qquad \text{(C.9)}$$

with the constant

$$c_L = -\frac{J_{-r_{iL}-\frac{1}{2}}(x_{nL})}{Y_{-r_{iL}-\frac{1}{2}}(x_{nL})}\,. \qquad \text{(C.10)}$$

Here, N_{Ln} is the normalization constant and $x_{nL} = m_{Ln}/k$. The functions $J_\beta(w)$ and $Y_\beta(w)$ appearing in Eq. (C.9) are the Bessel function of the first kind and of the second kind, respectively. On the other hand, the Z_2 projection condition $Z_2\psi = \gamma_5\psi$ is used in order to construct the right handed fields on the branes and this ensures that the right handed zero mode appears, the right (left) handed KK modes appear (disappear) on the branes with the boundary conditions:

$$\left(\frac{d}{dy} + m\right) \chi^E_{iRn}(y_0) = 0\,, \qquad \chi^E_{iLn}(y_0) = 0\,. \qquad \text{(C.11)}$$

Again, using the Dirac equation for KK mode leptons, one gets the right handed lepton $\chi^E_{i\,Rn}(y)$ that lives on the visible brane as

$$\chi^E_{iRn}(y) = N_{Rn}\, e^{\sigma/2} \left(J_{\frac{1}{2}+r_{iR}}(e^{\sigma}\, x_{nR}) + c_R\, Y_{\frac{1}{2}+r_{iR}}(e^{\sigma}\, x_{nR}) \right), \qquad \text{(C.12)}$$

with

$$c_R = -\frac{J_{r_{iR}-\frac{1}{2}}(x_{nR})}{Y_{r_{iR}-\frac{1}{2}}(x_{nR})}\,, \qquad \text{(C.13)}$$

where N_{Rn} is the normalization constant and $x_{nR} = m_{Rn}/k$. Notice that the constant c_L, the n^{th} KK mode mass m_{Ln} in Eq. (C.9) and the constant c_R, the n^{th} KK mode mass m_{Rn} in Eq. (C.12) are obtained by using the boundary conditions Eq. (C.8) and Eq. (C.11), respectively. For $m_{L(R)n} \ll k$ and $kR \gg 1$ they are approximated as:

$$\begin{aligned} m_{Ln} &\simeq k\,\pi \left(n - \frac{\frac{1}{2} - r}{2} + \frac{1}{4} \right) e^{-\pi\, k\, R}\,, \\ m_{Rn} &\simeq k\,\pi \left(n - \frac{\frac{1}{2} + r}{2} + \frac{1}{4} \right) e^{-\pi\, k\, R} \qquad \text{for } r < 0.5\,, \\ m_{Rn} &\simeq k\,\pi \left(n + \frac{\frac{1}{2} + r}{2} - \frac{3}{4} \right) e^{-\pi\, k\, R} \qquad \text{for } r > 0.5\,. \end{aligned} \qquad \text{(C.14)}$$

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