

# THE $SU(2) \otimes U(1)$ ELECTROWEAK MODEL BASED ON THE NONLINEARLY REALIZED GAUGE GROUP. II. FUNCTIONAL EQUATIONS AND THE WEAK POWER-COUNTING\*

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In the present paper, that is the second part devoted to the construction of an electroweak model based on a nonlinear realization of the gauge group  $SU(2) \otimes U(1)$ , we study the tree-level vertex functional with all the sources necessary for the functional formulation of the relevant symmetries (Local Functional Equation, Slavnov–Taylor identity, Landau Gauge Equation) and for the symmetric removal of the divergences. The Weak Power Counting criterion is proven in the presence of the novel sources. The local invariant solutions of the functional equations are constructed in order to represent the counterterms for the one-loop subtractions. The bleaching technique is fully extended to the fermion sector. The neutral sector of the vector mesons is analyzed in detail in order to identify the physical fields for the photon and the  $Z$  boson. The identities necessary for the decoupling of the unphysical modes are fully analyzed. These latter results are crucially bound to the Landau gauge used throughout the paper.

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## 1. Introduction

In Ref. [1] a consistent formulation of the electroweak model based on a nonlinear realization of the  $SU(2) \otimes U(1)$  gauge group has been presented

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by giving the tools required for the computation of radiative corrections in the loop expansion (Feynman rules and the functional identities necessary in order to guarantee physical unitarity and to carry out the subtraction procedure while respecting the locality of the counterterms to every loop order).

In the nonlinear realization there is no Higgs field [2] in the perturbative spectrum.

In the present paper we consider the tree-level vertex functional by including all the required external sources necessary for the functional formulation of the relevant symmetries of the theory and for the symmetric subtraction of the divergences.

The Local Functional Equation (LFE) [3, 4], which fixes the 1-PI amplitudes involving at least one Goldstone boson (descendant amplitudes) in terms of Goldstone-independent 1-PI amplitudes (ancestor amplitudes), provides a hierarchy among 1-PI Green functions. Once the ancestor amplitudes have been subtracted, the LFE uniquely fixes the descendant amplitudes. The LFE holds together with the Slavnov–Taylor (ST) identity, which guarantees the fulfillment of physical unitarity [5], and the Landau Gauge Equation (LGE), which encodes the stability of the Landau gauge-fixing under radiative corrections.

In the present paper the hierarchy is obtained by using the LFE and a set of external sources that ought to be complete in order to obtain all the descendant amplitudes. The Weak Power-Counting (WPC) is derived in the Landau gauge and used in the presence of this complete set of sources. The peculiar behavior of the fermion UV dimension in the nonlinearly realized theory is analyzed. The method of bleaching is used for all fields and sources with the aim of obtaining the most general local solution of the functional equations (STI, LFE and LGE). Finally the construction of the complete effective action is performed with the use of the WPC. The subtraction procedure is then reconsidered in the presence of the whole set of sources. The Ward–Takahashi identity (WTI) associated to the electric charge is discussed in its consequences, as the self-energy of the  $\gamma$ – $Z$  system and the description of the photon field in physical amplitudes. The identities necessary for the decoupling of the unphysical modes in the Landau gauge are fully analyzed.

We find that the requirement of the validity of the WPC imposes strong constraints on the classical action of the nonlinearly realized electroweak Standard Model. In fact, all possible symmetric anomalous couplings are forbidden by the WPC. Moreover two independent mass invariants appear in the vector meson sector (thus relaxing the tree-level Weinberg relation between the masses of the  $Z$  and  $W$ ).

The symmetric finite subtractions which are mathematically allowed at higher orders in the loop expansion cannot be reinserted back into the tree-

level vertex functional without violating either the symmetries or the WPC. Therefore their interpretation as physical parameters is not possible [6]. One possible *Ansatz* is to perform Minimal Subtraction of properly normalized 1-PI amplitudes [1, 4]. We finally prove that this *Ansatz* guarantees the fulfillment of all the relevant functional identities, order by order in the loop expansion.

The proof is based on a double grading expansion of the 1-PI amplitudes in the number of loops and in the loop order of the counterterms.

The paper is organized as follows. In Sec. 2 we introduce our notation and provide a systematic construction of  $SU(2)_L$ -invariant variables (bleaching procedure) in one-to-one correspondence with the original gauge and matter fields. The Feynman rules for the nonlinearly realized electroweak model are given in Sec. 2.1.

In Sec. 2.2 the gauge-fixing is performed in the Landau gauge. The BRST symmetry of the nonlinearly realized theory is presented and the STI is obtained by introducing the necessary anti-field external sources. The LGE and the associated ghost equation are also derived. In Sec. 2.3 the LFE is obtained as a consequence of the invariance of the path-integral Haar measure under local  $SU(2)_L$  transformations. The sources required in order to define at the renormalized level the operators necessary for the LFE are also introduced. In Sec. 3 we show that the symmetry content of the model allows for additional (anomalous) tree-level couplings.

The WPC is discussed in Sec. 4. In Sec. 5 we study the algebraic properties of the linearized ST operator  $\mathcal{S}_0$  and of the linearized LFE operator  $\mathcal{W}_0$ . In Sec. 5.1 the bleaching procedure is extended to generate  $\mathcal{S}_0$ -invariant variables. These are relevant for the algebraic classification of the counterterms order by order in the loop expansion. Moreover we discuss the subtraction procedure and the symmetric normalization of the 1-PI amplitudes. In Sec. 6 we consider the neutral sector of the vector boson. A detailed study of the STI and of the LGE allows the identification of the physical fields of the photon and of the  $Z$  boson. Useful identities are derived in order to verify the decoupling of the unphysical modes. Finally conclusions are given in Sec. 7.

Appendix A collects the propagators in the Landau gauge, while Appendix B is devoted to the technical proof of the WPC. Appendix C contains the details of the study of the neutral sector of the vector bosons.

## 2. Classical symmetries and bleached variables

The field content of the electroweak model based on the nonlinearly realized  $SU(2)_L \otimes U(1)$  gauge group includes (leaving aside for the moment

the ghosts and the Nakanishi–Lautrup fields) the  $SU(2)_L$  connection  $A_\mu = A_{a\mu} \frac{\tau_a}{2}$  ( $\tau_a$ ,  $a = 1, 2, 3$  are the Pauli matrices), the  $U(1)$  connection  $B_\mu$ , the fermionic left doublets collectively denoted by  $L$  and the right singlets, *i.e.*

$$\begin{aligned} L &\in \left\{ \begin{pmatrix} l_{Lj}^u \\ l_{Lj}^d \end{pmatrix}, \begin{pmatrix} q_{Lj}^u \\ V_{jk} q_{Lk}^d \end{pmatrix}, \quad j, k = 1, 2, 3 \right\}, \\ R &\in \left\{ \begin{pmatrix} l_{Rj}^u \\ l_{Rj}^d \end{pmatrix}, \begin{pmatrix} q_{Rj}^u \\ q_{Rj}^d \end{pmatrix}, \quad j = 1, 2, 3 \right\}. \end{aligned} \quad (1)$$

In the above equation the quark fields  $(q_j^u, j = 1, 2, 3) = (u, c, t)$  and  $(q_j^d, j = 1, 2, 3) = (d, s, b)$  are taken to be the mass eigenstates in the tree-level Lagrangian;  $V_{jk}$  is the CKM matrix. Similarly we use for the leptons the notation  $(l_j^u, j = 1, 2, 3) = (\nu_e, \nu_\mu, \nu_\tau)$  and  $(l_j^d, j = 1, 2, 3) = (e, \mu, \tau)$ . The single left doublets are denoted by  $L_j^l$ ,  $j = 1, 2, 3$  for the leptons,  $L_j^q$ ,  $j = 1, 2, 3$  for the quarks. Color indexes are not displayed.

One also introduces the  $SU(2)$  matrix  $\Omega$

$$\Omega = \frac{1}{v}(\phi_0 + i\phi_a \tau_a), \quad \Omega^\dagger \Omega = 1 \Rightarrow \phi_0^2 + \phi_a^2 = v^2. \quad (2)$$

The mass scale  $v$  gives  $\phi$  the canonical dimension at  $D = 4$ . We fix the direction of Spontaneous Symmetry Breaking by imposing the tree-level constraint

$$\phi_0 = \sqrt{v^2 - \phi_a^2}. \quad (3)$$

The condition  $\langle \Omega \rangle = 1$  cannot be imposed at a generic order of perturbation theory.

The  $SU(2)$  flat connection is defined by

$$F_\mu = i\Omega \partial_\mu \Omega^\dagger. \quad (4)$$

The transformation properties under the local  $SU(2)_L$  transformations are ( $g$  is the  $SU(2)_L$  coupling constant)

$$\begin{aligned} \Omega' &= U \Omega, & B'_\mu &= B_\mu, \\ A'_\mu &= U A_\mu U^\dagger + \frac{i}{g} U \partial_\mu U^\dagger, & L' &= U L, \\ F'_\mu &= U F_\mu U^\dagger + i U \partial_\mu U^\dagger, & R' &= R. \end{aligned} \quad (5)$$

Under local  $U(1)_R$  transformations one has

$$\begin{aligned}\Omega' &= \Omega V^\dagger, & B'_\mu &= B_\mu + \frac{1}{g'} \partial_\mu \alpha, \\ A'_\mu &= A_\mu, & L' &= \exp\left(i\frac{\alpha}{2} Y_L\right) L, \\ F'_\mu &= F_\mu + i\Omega V^\dagger \partial_\mu V \Omega, & R' &= \exp\left(i\frac{\alpha}{2} (Y_L + \tau_3)\right) R,\end{aligned}\quad (6)$$

where  $V(\alpha) = \exp(i\alpha \frac{\tau_3}{2})$ .

The electric charge is defined according to the Gell-Mann–Nishijima relation

$$Q = I_3 + Y, \quad (7)$$

where the hypercharge operator  $Y$  is the generator of the  $U(1)_R$  transformations (6) and  $I_3$  is an abstract object. The introduction of the matrix  $\Omega$  allows to perform an invertible change of variables from the original set of fields to a new set of  $SU(2)_L$ -invariant ones (bleaching procedure). For that purpose we define

$$\begin{aligned}w_\mu &= w_{a\mu} \frac{\tau_a}{2} = g\Omega^\dagger A_\mu \Omega - g' B_\mu \frac{\tau_3}{2} + i\Omega^\dagger \partial_\mu \Omega, \\ \tilde{L} &= \Omega^\dagger L.\end{aligned}\quad (8)$$

Both  $w_\mu$  and  $\tilde{L}$  are  $SU(2)_L$ -invariant, while under  $U(1)_R$  they transform as

$$w'_\mu = V w_\mu V^\dagger, \quad \tilde{L}' = \exp\left(i\frac{\alpha}{2} (\tau_3 + Y_L)\right) \tilde{L}. \quad (9)$$

*I.e.* the electric charge coincides with the hypercharge on the bleached fields, as it is apparent from the comparison of Eqs. (6), (7) and (9).

### 2.1. Classical action

Two mass invariants are expected for the vector mesons, as a consequence of the breaking of the global  $SU(2)_R$  invariance induced by the hypercharge. We introduce the charged combinations

$$w_\mu^\pm = \frac{1}{\sqrt{2}}(w_{1\mu} \mp iw_{2\mu}), \quad w_\mu^{\pm'} = \exp(\pm i\alpha) w_\mu^\pm. \quad (10)$$

The neutral component  $w_{3\mu}$  is invariant. Thus one obtains two independent mass terms which can be parameterized as

$$M^2 \left( w^+ w^- + \frac{1}{2} w_3^2 \right), \quad \frac{M^2 \kappa}{2} w_3^2. \quad (11)$$

Discarding the neutrino mass terms, the classical action for the nonlinearly realized  $SU(2) \otimes U(1)$  gauge group with two independent mass parameters for the vector mesons can be written as follows, where the dependence on  $\Omega$  is explicitly shown:

$$\begin{aligned}
S = & \Lambda^{(D-4)} \int d^D x \left( 2\text{Tr} \left\{ -\frac{1}{4} G_{\mu\nu} G^{\mu\nu} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right\} \right. \\
& + M^2 \text{Tr} \left\{ \left( g A_\mu - \frac{g'}{2} \Omega \tau_3 B_\mu \Omega^\dagger - F_\mu \right)^2 \right\} \\
& + M^2 \frac{\kappa}{2} \left( \text{Tr} \left\{ \left( g \Omega^\dagger A_\mu \Omega - g' B_\mu \frac{\tau_3}{2} + i \Omega^\dagger \partial_\mu \Omega \right) \tau_3 \right\} \right)^2 \\
& + \sum_L \left[ \bar{L} \left( i \not{\partial} + g \not{A} + \frac{g'}{2} Y_L \not{B} \right) L \right] + \sum_R \left[ \bar{R} \left( i \not{\partial} + \frac{g'}{2} (Y_L + \tau_3) \not{B} \right) R \right] \\
& + \sum_j \left[ m_{l_j} \bar{R}_j^l \frac{1 - \tau_3}{2} \Omega^\dagger L_j^l - m_{q_j^u} \bar{R}_j^q \frac{1 + \tau_3}{2} \Omega^\dagger L_j^q \right. \\
& \left. + m_{q_k^d} V_{kj}^\dagger \bar{R}_k^q \frac{1 - \tau_3}{2} \Omega^\dagger L_j^q + \text{h.c.} \right] \Bigg). \tag{12}
\end{aligned}$$

In  $D$  dimensions the doublets  $L$  and  $R$  obey

$$\gamma_D L = -L, \quad \gamma_D R = R \tag{13}$$

being  $\gamma_D$  a gamma matrix that anti-commutes with every other  $\gamma^\mu$ .

The non-Abelian field strength  $G_{\mu\nu}$  is defined by

$$G_{\mu\nu} = G_{a\mu\nu} \frac{\tau_a}{2} = (\partial_\mu A_{a\nu} - \partial_\nu A_{a\mu} + g \epsilon_{abc} A_{b\mu} A_{c\nu}) \frac{\tau_a}{2}, \tag{14}$$

while the Abelian field strength  $F_{\mu\nu}$  is

$$F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu. \tag{15}$$

In the above equation the phenomenologically successful structure of the couplings has been imposed by hand. The discussion of the possible anomalous couplings and of the stabilization mechanism induced by the WPC is deferred to Sec. 3.

## 2.2. Gauge-fixing and BRST symmetry

In order to set up the framework for the perturbative quantization of the model, the classical action in Eq. (12) needs to be gauge-fixed. The ghosts

associated with the  $SU(2)_L$  symmetry are denoted by  $c_a$ . Their anti-ghosts are denoted by  $\bar{c}_a$ , the Nakanishi–Lautrup fields by  $b_a$ . It is also useful to adopt the matrix notation

$$c = c_a \frac{\tau_a}{2}, \quad b = b_a \frac{\tau_a}{2}, \quad \bar{c} = \bar{c}_a \frac{\tau_a}{2}. \quad (16)$$

The Abelian ghost is  $c_0$ , the Abelian anti-ghost  $\bar{c}_0$  and the Abelian Nakanishi–Lautrup field  $b_0$ .

For the sake of simplicity we deal here with the Landau gauge. We also include the anti-fields for the  $SU(2)_L$  BRST transformation (those for the  $U(1)_R$  BRST transformation are not required since the Abelian ghost is free in the Landau gauge).

$$\begin{aligned} \Gamma_{\text{GF}}^{(0)} = & \Lambda^{(D-4)} \int d^D x \left( b_0 \partial_\mu B^\mu - \bar{c}_0 \square c_0 + 2Tr \left\{ b \partial_\mu A^\mu - \bar{c} \partial^\mu D[A]_\mu c \right. \right. \\ & + V^\mu \left( D[A]_\mu b - ig \bar{c} D[A]_\mu c - ig (D[A]_\mu c) \bar{c} \right) + \Theta^\mu D[A]_\mu \bar{c} \left. \right\} + K_0 \phi_0 \\ & + A_{a\mu}^* \mathfrak{s} A_a^\mu + \phi_0^* \mathfrak{s} \phi_0 + \phi_a^* \mathfrak{s} \phi_a + c_a^* \mathfrak{s} c_a + \sum_L \left( L^* \mathfrak{s} L + \bar{L}^* \mathfrak{s} \bar{L} \right) \Big). \end{aligned} \quad (17)$$

The full tree-level vertex functional is

$$\Gamma^{(0)} = S + \Gamma_{\text{GF}}^{(0)}. \quad (18)$$

The  $SU(2)_L$  BRST symmetry is generated by the differential  $\mathfrak{s}$ :

$$\begin{aligned} \mathfrak{s} A_\mu &= D[A]_\mu c, & \mathfrak{s} \Omega &= igc\Omega, & \mathfrak{s} \bar{c} &= b, & \mathfrak{s} \bar{c}_0 &= 0, \\ \mathfrak{s} c &= igcc, & \mathfrak{s} B_\mu &= 0, & \mathfrak{s} b &= 0, & \mathfrak{s} b_0 &= 0, \\ \mathfrak{s} L &= igcL, & \mathfrak{s} R &= 0, & \mathfrak{s} c_0 &= 0. \end{aligned} \quad (19)$$

The source  $K_0$  is required in order to define the nonlinear constraint  $\phi_0$ . This implies the inclusion of the source  $\phi_0^*$ , coupled to the BRST variation of  $\phi_0$ . The resulting STI is

$$\begin{aligned} \mathcal{S}\Gamma \equiv & \int d^D x \left[ \Lambda^{-(D-4)} \left( \Gamma_{A_{a\mu}^*} \Gamma_{A_a^\mu} + \Gamma_{\phi_a^*} \Gamma_{\phi_a} + \Gamma_{c_a^*} \Gamma_{c_a} \right. \right. \\ & \left. \left. + \Gamma_{L^*} \Gamma_L + \Gamma_{\bar{L}^*} \Gamma_{\bar{L}} \right) + b_a \Gamma_{\bar{c}_a} + \Theta_{a\mu} \Gamma_{V_{a\mu}} - K_0 \Gamma_{\phi_0^*} \right] = 0. \end{aligned} \quad (20)$$

In the above equation the background connection  $V_{a\mu}$  is paired into a doublet with  $\Theta_{a\mu}$ . This is a standard procedure in order to guarantee the independence of the physics on the background sources [7].  $(\phi_0^*, -K_0)$  are also

arranged into doublets in the above STI. This is required in order to preserve the STI in the presence of the source  $K_0$  and signals that  $K_0$  is not a physical variable. This feature has been addressed in [4] in the context of massive SU(2) Yang–Mills theory.

Moreover the following Abelian STI holds:

$$\begin{aligned}
& -\frac{2}{g'}\Lambda^{(D-4)}\Box b_0 - \frac{2}{g'}\partial^\mu \frac{\delta\Gamma}{\delta B^\mu} - \Lambda^{(D-4)}\phi_3 K_0 + \phi_2 \frac{\delta\Gamma}{\delta\phi_1} - \phi_1 \frac{\delta\Gamma}{\delta\phi_2} \\
& - \frac{1}{\Lambda^{(D-4)}} \frac{\delta\Gamma}{\delta K_0} \frac{\delta\Gamma}{\delta\phi_3} - \phi_3^* \frac{\delta\Gamma}{\delta\phi_0^*} + \phi_2^* \frac{\delta\Gamma}{\delta\phi_1^*} - \phi_1^* \frac{\delta\Gamma}{\delta\phi_2^*} + \phi_0^* \frac{\delta\Gamma}{\delta\phi_3^*} \\
& + iY_L L \frac{\delta\Gamma}{\delta L} - iY_L \bar{L} \frac{\delta\Gamma}{\delta \bar{L}} + i(Y_L + \tau_3)R \frac{\delta\Gamma}{\delta R} - i\bar{R}(Y_L + \tau_3) \frac{\delta\Gamma}{\delta \bar{R}} \\
& - iY_L L^* \frac{\delta\Gamma}{\delta L^*} + iY_L \bar{L}^* \frac{\delta\Gamma}{\delta \bar{L}^*} = 0.
\end{aligned} \tag{21}$$

The transformations of the fields in the above equation are generated by the  $U(1)_R$  BRST symmetry

$$\begin{aligned}
\mathfrak{s}_1 A_\mu &= 0, & \mathfrak{s}_1 \Omega &= -\frac{i}{2}g'\Omega c_0 \tau_3, & \mathfrak{s}_1 \bar{c} &= 0, & \mathfrak{s}_1 \bar{c}_0 &= b_0, \\
\mathfrak{s}_1 c &= 0, & \mathfrak{s}_1 B_\mu &= \partial_\mu c_0, & \mathfrak{s}_1 b &= 0, & \mathfrak{s}_1 b_0 &= 0, \\
\mathfrak{s}_1 L &= g'c_0 Y_L L, & \mathfrak{s}_1 R &= \frac{i}{2}g'c_0(Y_L + \tau_3)R, & \mathfrak{s}_1 c_0 &= 0.
\end{aligned} \tag{22}$$

By construction

$$\{\mathfrak{s}, \mathfrak{s}_1\} = 0. \tag{23}$$

Eq. (21) can be derived from the invariance under the  $U(1)_R$  transformations in Eq. (6) supplemented by the following transformations on the additional variables (we set  $\Omega^* = \phi_0^* - i\phi_a^* \tau_a$ )

$$\begin{aligned}
V'_\mu &= V_\mu, & \Omega^{*'} &= V\Omega^*, & L^{*'} &= \exp\left(-i\frac{\alpha}{2}Y_L\right)L^*, & K'_0 &= K_0, \\
\Theta'_\mu &= \Theta_\mu, & b' &= b, & \bar{L}^{*'} &= \exp\left(i\frac{\alpha}{2}Y_L\right)\bar{L}^*, & b'_0 &= b_0, \\
c' &= c, & \bar{c}' &= \bar{c}, & c^{*'} &= c^*, \\
c'_0 &= c_0, & \bar{c}'_0 &= \bar{c}_0, & A_{\mu}^{*'} &= A_{\mu}^*.
\end{aligned} \tag{24}$$

The ghost number is defined as follows:  $A_{a\mu}^*, \phi_a^*, \phi_0^*, L^*, \bar{L}^*, \bar{c}_a, \bar{c}_0$  have ghost number -1,  $c^*$  has ghost number -2,  $c_a, c_0$  and  $\Theta_{a\mu}$  have ghost number +1, while all the other fields and external sources have ghost number zero.

The LGE is

$$\Gamma_{b_a} = \Lambda^{(D-4)} (D^\mu[V](A_\mu - V_\mu))_a \tag{25}$$



which implies the ghost equation

$$\Gamma_{\bar{c}_a} = \left( -D_\mu[V] \Gamma_{A_\mu^*} + \Lambda^{(D-4)} D_\mu[A] \Theta^\mu \right)_a, \quad (26)$$

by using the STI (20).

### 2.3. The Local Functional Equation

The dependence of the vertex functional on the Goldstone fields is controlled by the LFE associated to the invariance of the path-integral Haar measure under the  $SU(2)_L$  transformations in Eq. (5), extended to the ghost, anti-ghost, Nakanishi–Lautrup fields and to the external sources according to

$$\begin{aligned} V'_\mu &= UV_\mu U^\dagger + \frac{i}{g} U \partial_\mu U^\dagger, & \Omega^{*'} &= \Omega^* U^\dagger, & L^{*'} &= L^* U^\dagger, & K'_0 &= K_0, \\ \Theta'_\mu &= U \Theta_\mu U^\dagger, & b' &= U b U^\dagger, & \bar{L}^{*'} &= U \bar{L}^*, & b'_0 &= b_0, \\ c' &= U c U^\dagger, & \bar{c}' &= U \bar{c} U^\dagger, & c^{*'} &= c^*, \\ c'_0 &= c_0, & \bar{c}'_0 &= \bar{c}_0, & A_{\mu}^{*'} &= U A_\mu^* U^\dagger. \end{aligned} \quad (27)$$

Thus the resulting identity associated to the  $SU(2)_L$  local transformations is ( $x$ -dependence is not shown)

$$\begin{aligned} (\mathcal{W}\Gamma)_a &\equiv -\frac{1}{g} \partial_\mu \Gamma_{V_{a\mu}} + \epsilon_{abc} V_{c\mu} \Gamma_{V_{b\mu}} - \frac{1}{g} \partial_\mu \Gamma_{A_{a\mu}} \\ &+ \epsilon_{abc} A_{c\mu} \Gamma_{A_{b\mu}} + \epsilon_{abc} b_c \Gamma_{b_b} + \frac{\Lambda^{(D-4)}}{2} K_0 \phi_a + \frac{1}{2\Lambda^{(D-4)}} \Gamma_{K_0} \Gamma_{\phi_a} \\ &+ \frac{1}{2} \epsilon_{abc} \phi_c \Gamma_{\phi_b} + \epsilon_{abc} \bar{c}_c \Gamma_{\bar{c}_b} + \epsilon_{abc} c_c \Gamma_{c_b} \\ &+ \frac{i}{2} \tau_a L \Gamma_L - \frac{i}{2} \bar{L} \tau_a \Gamma_{\bar{L}} - \frac{i}{2} L^* \tau_a \Gamma_{L^*} + \frac{i}{2} \tau_a \bar{L}^* \Gamma_{\bar{L}^*} \\ &+ \epsilon_{abc} \Theta_{c\mu} \Gamma_{\Theta_{b\mu}} + \epsilon_{abc} A_{c\mu}^* \Gamma_{A_{b\mu}^*} + \epsilon_{abc} c_c^* \Gamma_{c_b^*} - \frac{1}{2} \phi_0^* \Gamma_{\phi_a^*} \\ &+ \frac{1}{2} \epsilon_{abc} \phi_c^* \Gamma_{\phi_b^*} + \frac{1}{2} \phi_a^* \Gamma_{\phi_0^*} = 0, \end{aligned} \quad (28)$$

where the nonlinearity of the realization of the  $SU(2)_L$  gauge group is revealed by the presence of the bilinear term  $\Gamma_{K_0} \Gamma_{\phi_a}$ . Since in the loopwise expansion  $\Gamma_{K_0}$  is invertible, Eq. (28) entails that every amplitude with  $\phi$ -external leg (descendant amplitudes) can be obtained from those without.

This is a crucial property in order to tame the divergences of the model. In fact, already at one loop level the Feynman rules in Eq. (18) give rise to divergent Feynman diagrams with an arbitrary number of external  $\phi$ -legs. However at every loop order there is only a finite number of ancestor

amplitudes, *i.e.* amplitudes which do not involve external Goldstone fields. This property is referred to as the WPC. Consequently a finite number of subtractions is required in order to make the theory finite at each loop order.

### 3. Anomalous couplings

Any  $U(1)_R$ -invariant local functional built out of the components of  $w_\mu, \tilde{L}, R$ , the Abelian field strength  $F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$  and derivatives thereof (covariant derivatives w.r.t.  $B_\mu$  for  $U(1)_R$ -charged fields, ordinary derivatives for the neutral fields) is allowed on symmetry grounds.

We discuss here those invariants with dimension  $\leq 4$ .

Many possibilities arise for the interaction terms. For the gauge bosons self-interactions

$$\begin{aligned} a_1 (w^+ w^-)^2, & \quad a_2 (w^+)^2 (w^-)^2, & \quad a_3 (w^+ w^-) w_3^2, \\ a_4 w_{3\nu} \partial^\mu w^{+\nu} w_\mu^-, & \quad a_5 w_{3\nu} w^{+\nu} \partial w^-, \\ a_6 w_3^\nu w_\mu^+ \partial^\mu w_\nu^-, & \quad a_7 w_{3\nu} \partial w^+ w^{-\nu}, \\ a_8 w_{3\nu} \partial^\nu w_\mu^+ w^{-\mu}, & \quad a_9 w_{3\nu} w_\mu^+ \partial^\nu w^{-\mu}. \end{aligned} \quad (29)$$

Hermiticity requires  $a_4^* = a_6$ ,  $a_5^* = a_7$  and  $a_8^* = a_9$ . For the leptonic neutral currents

$$\begin{aligned} g_{kj}^{Lu,0} \bar{l}_L^u \psi_3 l_{Lj}^u, & \quad g_{kj}^{Ld,0} \bar{l}_L^d \psi_3 l_{Lj}^d, \\ g_{kj}^{Ru,0} \bar{l}_R^u \psi_3 l_{Rj}^u, & \quad g_{kj}^{Rd,0} \bar{l}_R^d \psi_3 l_{Rj}^d. \end{aligned} \quad (30)$$

A similar pattern applies to the quark neutral currents:

$$\begin{aligned} h_{kj}^{Lu,0} \bar{q}_L^u \psi_3 q_{Lj}^u, & \quad h_{kj}^{Ld,0} \bar{q}_L^d \psi_3 q_{Lj}^d, \\ h_{kj}^{Ru,0} \bar{q}_R^u \psi_3 q_{Rj}^u, & \quad h_{kj}^{Rd,0} \bar{q}_R^d \psi_3 q_{Rj}^d. \end{aligned} \quad (31)$$

For the charged currents one has in the leptonic sector

$$g_{kj}^{Lu,+} \bar{l}_L^u \psi^+ l_{Lj}^d + \text{h.c.}, \quad g_{kj}^{Ru,+} \bar{l}_R^u \psi^+ l_{Rj}^d + \text{h.c.}, \quad (32)$$

and in the hadronic sector

$$h_{kj}^{Lu,+} \bar{q}_L^u \psi^+ q_{Lj}^d + \text{h.c.}, \quad h_{kj}^{Ru,+} \bar{q}_R^u \psi^+ q_{Rj}^d + \text{h.c.} \quad (33)$$

The anomalous gauge bosons couplings in Eq. (29) are not forbidden on symmetry grounds, as well as the flavor-changing neutral currents generated by the off-diagonal elements of the couplings matrices in Eqs. (30) and (31). They are excluded by hand in Eq. (18) on phenomenological grounds. In Sec. 4 we show that this choice is unique if one requires the weak power-counting formula (34).

#### 4. The Weak Power-Counting

In the massive nonlinearly realized  $SU(2)$  Yang–Mills theory [8] and in the Electroweak model based on the nonlinear representation of the  $SU(2) \otimes U(1)$  gauge group [1] the number of divergent 1–PI amplitudes involving the Goldstone fields is infinite already at one loop. However these amplitudes are uniquely fixed order by order in the loop expansion by the LFE in Eq. (28) once the 1–PI amplitudes not involving the Goldstone fields (ancestor amplitudes) are known. We call this property hierarchy among 1–PI Green functions. It holds in the nonlinear sigma model in the flat connection formalism [3]. The tools for the integration of the LFE have been developed in [9]. Hierarchy among 1–PI Green functions has been studied for the massive nonlinearly realized  $SU(2)$  Yang–Mills theory in [4].

The WPC [10] amounts to the request that only a finite number of divergent ancestor amplitudes exists at each loop order. This restricts the number of allowed tree-level interaction vertexes.

Let  $\mathcal{G}$  be an arbitrary  $n$ -loop 1–PI ancestor graph with  $I$  internal lines,  $V$  vertexes and a given set  $\{N_A, N_B, N_F, N_{\bar{F}}, N_c, N_V, N_\Theta, N_{\phi_0^*}, N_{K_0}, N_{\phi_a^*}, N_{A^*}, N_{c^*}, N_{L^*}, N_{\bar{L}^*}\}$  of external legs.  $F, \bar{F}$  are a collective notation for the fermion and anti-fermion matter fields, which can be treated in a unified manner. Then the superficial degree of divergence of the graph  $\mathcal{G}$  is bounded by

$$d(\mathcal{G}) \leq (D-2)n + 2 - N_A - N_B - N_c - N_F - N_{\bar{F}} - N_V - N_{\phi_a^*} - 2(N_\Theta + N_{A^*} + N_{\phi_0^*} + N_{L^*} + N_{\bar{L}^*} + N_{c^*} + N_{K_0}) . \quad (34)$$

The proof of this formula is given in Appendix B by exploiting the symmetric formalism where the original fields  $(A_{a\mu}, B_\mu)$  are used instead of the mass eigenstates  $W_\mu^\pm, Z_\mu, A_\mu$ . The propagators in the symmetric formulation are summarized in Appendix A.

The validity of the WPC formula forbids the appearance of the anomalous gauge bosons self-interactions in Eq. (29) into the tree-level vertex functional  $\Gamma^{(0)}$  in Eq. (18). In fact, the terms in Eq. (29) would give rise upon expansion in powers of the Goldstone fields to quadrilinear interaction vertexes with two gauge bosons, two Goldstone legs and two derivatives. Therefore at one loop level there would exist an infinite number of divergent amplitudes with external gauge boson legs, associated to graphs like the one in Fig. 1. Therefore the WPC would be maximally violated already at one loop level.

The only allowed combination is the Yang–Mills action, as was pointed out in [4]. On the other hand, the WPC does not put any constraint on the gauge boson mass invariants. In the nonlinearly realized electroweak model

the hypercharge  $U(1)_R$  invariance allows for the two independent mass terms in Eq. (11).

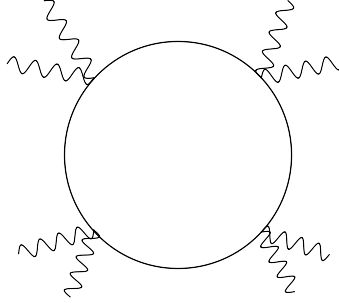


Fig. 1. Logarithmically divergent one-loop graphs with an arbitrary number of external gauge boson legs (solid lines denote Goldstone propagators).

According to the WPC formula in Eq. (34) the fermionic fields have UV degree 1 (instead of  $3/2$  as in power-counting renormalizable theories). This is a peculiar feature of the electroweak model based on the nonlinearly realized gauge group  $SU(2) \otimes U(1)$ . It is easy to see that the UV degree of massive chiral fermions in the nonlinearly realized theory cannot be greater than 1. In fact, the invariant fermionic mass terms in Eq. (12) contain couplings generated by the expansion of the nonlinear constraint  $\phi_0$  with the following structure

$$\frac{m_f}{v} \bar{f} f \phi_0 \sim m_f \bar{f} f \left[ 1 - \sum_{k=1}^{\infty} \frac{1}{k!} \frac{(2k-3)!!}{2^k} \left( \frac{\phi^2}{v^2} \right)^k \right]. \quad (35)$$

The first interaction term on the r.h.s. contains a quadrilinear coupling giving rise to graphs like the one in Fig. 2. Thus there are one loop logarithmically divergent graphs with four external fermion legs and therefore the UV degree of massive chiral fermions can be at most one. For massless neutrinos the bond of Eq. (34) still works but one cannot associate their UV dimension on the basis of the degree of divergence of the graphs in Fig. 2.

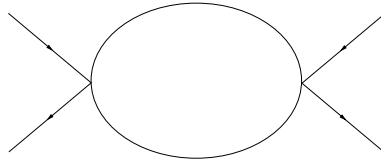


Fig. 2. Logarithmically divergent one-loop graphs with four fermion legs (solid lines denote Goldstone propagators).

If the symmetric interactions in Eqs. (30)–(33) are turned on, the UV degree of the fermions is downgraded to one half. This is readily established by expanding the invariants in powers of the Goldstone fields and by looking at the graphs arising from the interaction vertices involving two Goldstone legs. An example is displayed in Fig. 3.

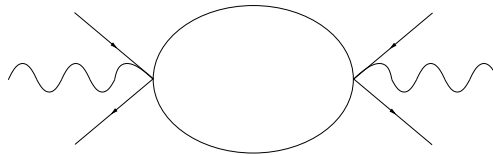


Fig. 3. Logarithmically divergent one-loop graphs with four fermionic external legs and two gauge bosons legs generated by  $\bar{l}_{Lk}^u \not{u}_3 l_{Lj}^u$  (solid lines denote Goldstone propagators).

It is interesting to notice that fermions with UV degree equal to one half are compatible with four fermion interactions generated in a symmetric way by using invariant bleached variables, like for instance

$$\bar{l}_{Rj}^u \tilde{l}_{Lj}^u \bar{l}_{Rj}^u \tilde{l}_{Lj}^u + \text{h.c.} \quad (36)$$

which would generate the quadratically divergent one loop graph in Fig. 4.

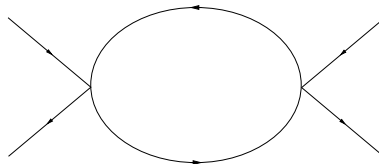


Fig. 4. Quadratically divergent one-loop graphs with four fermionic external legs generated by four-fermion interactions.

In the nonlinearly realized theory it turns out that one is the UV degree for the fermion fields compatible with the invariant mass terms for chiral fermions. As a consequence one recovers via the WPC the phenomenologically successful structure of the SM couplings in Eq. (12).

## 5. Functional identities and minimal subtraction procedure

Perturbation theory is carried out in the loop-wise expansion. Accordingly the functional identities in Eq. (20), (21), (25), (26) and (28) are developed order by order in  $\hbar$ . We denote by  $\Gamma^{(n)}$  the  $n$ -th loop vertex

functional, *i.e.*

$$\Gamma = \sum_{n=0}^{\infty} \Gamma^{(n)}. \quad (37)$$

By Eq. (25)  $\Gamma^{(n)}$ ,  $n \geq 1$  is independent of  $b_a$ . By Eq. (26) the dependence of  $\Gamma^{(n)}$ ,  $n \geq 1$  on  $\bar{c}_a$  only happens via the combination

$$\hat{A}_{a\mu}^* = A_{a\mu}^* + (D_\mu[V]\bar{c})_a. \quad (38)$$

At order  $n \geq 1$  in the loop expansion the STI in Eq. (20) is

$$\mathcal{S}_0 \left( \Gamma^{(n)} \right) + \sum_{j=1}^{n-1} \left( \Gamma^{(n-j)}, \Gamma^{(j)} \right) = 0, \quad (39)$$

where the classical linearized ST operator  $\mathcal{S}_0$  is given by

$$\begin{aligned} \mathcal{S}_0 \Gamma \equiv & \int d^D x \left[ \Lambda^{-(D-4)} \left( \Gamma_{A_a^\mu}^{(0)} \frac{\delta}{\delta A_{a\mu}^*} + \Gamma_{A_{a\mu}^*}^{(0)} \frac{\delta}{\delta A_a^\mu} + \Gamma_{\phi_a^*}^{(0)} \frac{\delta}{\delta \phi_a} + \Gamma_{\phi_a}^{(0)} \frac{\delta}{\delta \phi_a^*} \right. \right. \\ & + \Gamma_{c_a^*}^{(0)} \frac{\delta}{\delta c_a} + \Gamma_{c_a}^{(0)} \frac{\delta}{\delta c_a^*} + \Gamma_{L^*}^{(0)} \frac{\delta}{\delta L} + \Gamma_L^{(0)} \frac{\delta}{\delta L^*} + \Gamma_{\bar{L}^*}^{(0)} \frac{\delta}{\delta \bar{L}} + \Gamma_{\bar{L}}^{(0)} \frac{\delta}{\delta \bar{L}^*} \Big) \\ & \left. + b_a \frac{\delta}{\delta \bar{c}_a} + \Theta_{a\mu} \frac{\delta}{\delta V_{a\mu}} - K_0 \frac{\delta}{\delta \phi_0^*} \right] \Gamma. \end{aligned} \quad (40)$$

The bracket in Eq. (39) is

$$(X, Y) = \int d^D x \Lambda^{-(D-4)} \sum_j \frac{\delta X}{\delta \varphi_j^*} \frac{\delta Y}{\delta \varphi_j}, \quad (41)$$

where  $\varphi \in \{A_{a\mu}, \phi_a, c_a, L, \bar{L}\}$  and  $\varphi_j^*$  stands for the anti-field associated to  $\varphi_j$ .

At order  $n \geq 1$  the LFE in Eq. (28) yields

$$\left( \mathcal{W}_0 \Gamma^{(n)} \right)_a + \frac{1}{2\Lambda^{(D-4)}} \sum_{j=1}^{n-1} \frac{\delta \Gamma^{(n-j)}}{\delta K_0(x)} \frac{\delta \Gamma^{(j)}}{\delta \phi_a(x)} = 0, \quad (42)$$

where  $\mathcal{W}_0$  is the classical linearized version of  $\mathcal{W}$ :

$$(\mathcal{W}_0 \Gamma)_a \equiv \left( -\frac{1}{g} \partial_\mu \frac{\delta}{\delta V_{a\mu}} + \epsilon_{abc} V_{c\mu} \frac{\delta}{\delta V_{b\mu}} - \frac{1}{g} \partial_\mu \frac{\delta}{\delta A_{a\mu}} \right.$$

$$\begin{aligned}
 & + \epsilon_{abc} A_{c\mu} \frac{\delta}{\delta A_{b\mu}} + \epsilon_{abc} b_c \frac{\delta}{\delta b_b} + \frac{1}{2\Lambda^{(D-4)}} \frac{\delta \Gamma^{(0)}}{\delta K_0} \frac{\delta}{\delta \phi_a} \\
 & + \frac{1}{2\Lambda^{(D-4)}} \frac{\delta \Gamma^{(0)}}{\delta \phi_a} \frac{\delta}{\delta K_0} + \frac{1}{2} \epsilon_{abc} \phi_c \frac{\delta}{\delta \phi_b} + \epsilon_{abc} \bar{c}_c \frac{\delta}{\delta \bar{c}_b} + \epsilon_{abc} c_c \frac{\delta}{\delta c_b} \\
 & + \frac{i}{2} \tau_a L \frac{\delta}{\delta L} - \frac{i}{2} \bar{L} \tau_a \frac{\delta}{\delta \bar{L}} - \frac{i}{2} L^* \tau_a \frac{\delta}{\delta L^*} + \frac{i}{2} \tau_a \bar{L}^* \frac{\delta}{\delta \bar{L}^*} \\
 & + \epsilon_{abc} \Theta_{c\mu} \frac{\delta}{\delta \Theta_{b\mu}} + \epsilon_{abc} A_{c\mu}^* \frac{\delta}{\delta A_{b\mu}^*} + \epsilon_{abc} c_c^* \frac{\delta}{\delta c_b^*} - \frac{1}{2} \phi_0^* \frac{\delta}{\delta \phi_a^*} \\
 & + \frac{1}{2} \epsilon_{abc} \phi_c^* \frac{\delta}{\delta \phi_b^*} + \frac{1}{2} \phi_a^* \frac{\delta}{\delta \phi_0^*} \Big) \Gamma.
 \end{aligned} \tag{43}$$

It is straightforward to prove that

$$[\mathcal{S}_0, \mathcal{W}_0] = 0. \tag{44}$$

### 5.1. Bleached variables

The LFE in Eq. (42) can be explicitly integrated (with no locality restrictions) by using the techniques developed in [9].

The first step is to extend the bleaching technique in order to generate variables invariant under  $\mathcal{W}_0$ . This has been done for massive  $SU(2)$  Yang–Mills theory in [4]. Here we provide the extension to the case of chiral fermions.

Along the lines of [4] we introduce the bleached partners of  $c$  and of the external sources:

$$\begin{aligned}
 v_\mu &= g\Omega^\dagger V_\mu \Omega - g' B_\mu \frac{\tau_3}{2} + i\Omega^\dagger \partial_\mu \Omega, & \tilde{\Theta}_\mu &= \Omega^\dagger \Theta_\mu \Omega, \\
 \tilde{\Omega}^* &= \Omega^\dagger \Omega^*, & \tilde{c} &= \Omega^\dagger c \Omega, \\
 \widetilde{\hat{A}_\mu^*} &= \Omega^\dagger \hat{A}_\mu^* \Omega, & \tilde{c}^* &= \Omega^\dagger c^* \Omega, \\
 \tilde{L}^* &= L^* \Omega, & \tilde{\bar{L}}^* &= \Omega^\dagger \bar{L}^*.
 \end{aligned} \tag{45}$$

The invariance of the above variables under  $\mathcal{W}_0$  follows directly from Eq. (27). Moreover it can be proved [4] that the following combination is  $\mathcal{W}_0$ -invariant:

$$\tilde{K}_0 = \frac{1}{v} \left( \Lambda^{D-4} \frac{v^2 K_0}{\phi_0} - \phi_a \frac{\delta}{\delta \phi_a} \left( \Gamma^{(0)} \Big|_{K_0=0} \right) \right). \tag{46}$$

The bleached variables in Eq. (8) are  $\mathcal{W}_0$ -invariant. The operator  $\mathcal{W}_0$  takes a particularly simple form in the bleached variables:

$$(\mathcal{W}_0 \Gamma)_a = \Theta_{ab} \frac{\delta}{\delta \phi_b} \Gamma, \tag{47}$$

where the matrix  $\Theta_{ab}$  is defined as

$$\Theta_{ab} = \frac{1}{2}\phi_0\delta_{ab} + \frac{1}{2}\epsilon_{abc}\phi_c. \quad (48)$$

At one loop order the LFE reads

$$\Theta_{ab} \frac{\delta\Gamma^{(1)}}{\delta\phi_b} = 0. \quad (49)$$

Since the matrix  $\Theta_{ab}$  is invertible the above equation implies that the dependence on the Goldstone fields is only via the bleached variables. At higher orders one has to take into account the inhomogeneous term in Eq. (42). In addition to the dependence through the bleached variables (implicit dependence), an additional explicit dependence of  $\Gamma^{(n)}$  on  $\phi_a$  arises [9]. The integration can be explicitly carried out in an elegant way by introducing the homotopy operator associated with  $\mathcal{W}_0$ , as discussed in [9].

The bleached variables  $w_\mu, \tilde{L}, \tilde{\bar{L}}$  as well as  $R, \bar{R}$  and the U(1) connection  $B_\mu$  are both  $\mathcal{W}_0$ - and  $\mathcal{S}_0$ -invariant. Moreover, by Eq. (44) the  $\mathcal{S}_0$ -transforms of bleached variables are bleached.

The solution of the linearized STI can thus be studied in the space spanned by the bleached variables. Since the theory is non-anomalous, the dependence on the bleached ghost  $\tilde{c}$ , on the bleached anti-fields, on the bleached background gauge source  $v_\mu$  and its BRST partner  $\tilde{\Theta}_\mu$  in Eq. (45) and on  $\tilde{K}_0$  in Eq. (46) is confined to the cohomologically trivial sector of  $\mathcal{S}_0$ -invariants which are of the form  $\mathcal{S}_0(X)$ , where  $X$  is a local functional with ghost number  $-1$  [11].

This allows us to classify the possible invariant solutions by the same technique developed in [4] for the SU(2) case. This strategy has been applied in order to obtain the complete set of one loop counterterms for the massive nonlinearly realized SU(2) Yang–Mills theory in [12].

We briefly illustrate the procedure at the one loop level (the full algebraic analysis is beyond the scope of the present paper and will be developed elsewhere). By the WPC formula in Eq. (34) the one-loop invariants can have at most dimension 4. According to the classification described above, they fall into two categories: the first (cohomologically non-trivial sector) is spanned by the Lorentz-invariant electrically neutral monomials in  $w_\mu, \tilde{L}, \tilde{\bar{L}}, R, \bar{R}$  and ordinary derivatives thereof with dimension  $\leq 4$ .

The second class contains the cohomologically trivial electrically neutral invariants with dimension  $\leq 4$ . As an example, we write the allowed cohomologically trivial invariants involving the bleached anti-field  $\widetilde{\hat{A}}^*_{a\mu}$

$$\mathcal{J}_1 = \int d^D x \mathcal{S}_0 \left( \widetilde{\hat{A}}^*_{a\mu} w_a^\mu \right), \quad \mathcal{J}_2 = \int d^D x \mathcal{S}_0 \left( \widetilde{\hat{A}}^*_{3\mu} w_3^\mu \right),$$



$$\mathcal{J}_3 = \int d^D x \mathcal{S}_0 \left( \widetilde{\hat{A}}^*_{a\mu} v_a^\mu \right), \quad \mathcal{J}_4 = \int d^D x \mathcal{S}_0 (\widetilde{\hat{A}}^*_{3\mu} v_3^\mu). \quad (50)$$

Notice that each invariant of the form  $\int d^D x \mathcal{S}_0 (M_{ab} \widetilde{\hat{A}}^*_{a\mu} w_b^\mu)$  and  $\int d^D x \mathcal{S}_0 (N_{ab} \widetilde{\hat{A}}^*_{a\mu} v_b^\mu)$ , with  $M_{ab}, N_{ab}$  real matrices, would be allowed on the basis of the STI in Eq. (20). The requirement of invariance under the Abelian STI in Eq. (21) leaves only the four invariants in Eq. (50).

### 5.2. Minimal Subtraction procedure

The superficial degree of divergence in Eq. (34) shows that the number of divergent amplitudes increases order by order in the loop expansion, though it remains finite at each order. Therefore the theory is not power-counting renormalizable even if we restrict to ancestor amplitudes. This item has been considered at length by the present authors. The extensive discussion is in Ref. [6], where we argue in favor of a particular *Ansatz* for the subtraction procedure which respects locality and unitarity (at variance with the algebraic renormalization which in the present case leads to finite symmetric renormalizations which cannot be reinserted back into the tree-level vertex functional).

In this approach Eq. (28) is used as a guide in order to work out the procedure of the removal of divergences. Dimensional regularization provides the most natural environment. Let us denote by

$$\Gamma^{(n,k)} \quad (51)$$

the vertex functional for 1-PI amplitudes at  $n$ -order in loops where the counterterms enter with a total power  $k$  in  $\hbar$ . In dimensional regularization we can perform a grading in  $k$  of Eq. (28). Thus if we have successfully performed the subtraction procedure satisfying Eq. (28) up to order  $n - 1$  the next order effective action

$$\Gamma^{(n)} = \sum_{k=0}^{n-1} \Gamma^{(n,k)} \quad (52)$$

violates Eq. (28) since the counterterm  $\hat{\Gamma}^{(n)}$  is missing. The breaking term can be determined by writing Eq. (28) at order  $n$  at the grade  $k \leq n - 1$  and then by summing over  $k$ . One gets

$$\begin{aligned} \mathcal{W}_0 \Gamma^{(n)} + \frac{1}{2\Lambda^{(D-4)}} \sum_{n'=1}^{n-1} \left( \frac{\delta \Gamma^{(n-n')}}{\delta K_0} \right) \left( \frac{\delta \Gamma^{(n')}}{\delta \phi_a} \right) \\ = \frac{1}{2\Lambda^{(D-4)}} \sum_{n'=1}^{n-1} \left( \frac{\delta \Gamma^{(n-n',n-n')}}{\delta K_0} \right) \left( \frac{\delta \Gamma^{(n',n')}}{\delta \phi_a} \right). \end{aligned} \quad (53)$$

The first term in the l.h.s. of Eq. (53) has pole parts in  $D-4$  while the second is finite, since the factors are of order less than  $n$ , thus already subtracted. The breaking term contains only counterterms  $\hat{\Gamma}^j = \Gamma^{(j,j)}$ ,  $j < n$ . This suggests the Ansatz that the finite part of the Laurent expansion at  $D = 4$

$$\frac{1}{\Lambda^{(D-4)}} \Gamma^{(n)} \quad (54)$$

gives the correct prescription for the subtraction of the divergences; *i.e.* one has to divide both members of Eq. (53) by  $\Lambda^{(D-4)}$  and remove only the pole parts (Minimal Subtraction). Thus the counterterms have the form

$$\hat{\Gamma}^{(n)} = \Lambda^{(D-4)} \int \frac{d^D x}{(2\pi)^D} \mathcal{M}^{(n)}(x), \quad (55)$$

where the integrand is a local power series in the fields, the external sources and their derivatives (a local polynomial as far as ancestor monomials are concerned) and it possesses only pole parts in its Laurent expansion at  $D = 4$ .

A similar argument applies to the STI in Eq. (39) since the bracket in Eq. (41) has the same prefactor  $\Lambda^{-(D-4)}$ . The U(1) identity in Eq. (59), being linear in  $\Gamma$ , does not pose any problem. Compatibility of the STI and the LFE follows from Eq. (44).

In this subtraction scheme one extra free parameter enters, *i.e.* the overall mass scale  $\Lambda$  for the radiative corrections.

In this scheme the  $\gamma_5$  problem is treated in a pragmatic approach (for a similar treatment see *e.g.* [13]). The matrix  $\gamma_5$  is replaced by a new  $\gamma_D$  which anti-commutes with every  $\gamma_\mu$ . No statement is made on the analytical properties of the traces involving  $\gamma_D$ . Since the theory is not anomalous such traces never meet poles in  $D - 4$  and therefore we can evaluate at the end the traces at  $D = 4$ .

In practice there are two ways to proceed in the regularization procedure. One can use the forest formula and use Minimal Subtraction for every (properly normalized) subgraph. It is possible, as alternative, to evaluate the counterterms for the ancestor amplitudes and then obtain from those all the necessary counterterms involving the Goldstone boson fields  $\vec{\phi}$ .

## 6. The neutral sector

The existence of two equations (STI and LFE), together with the LGE, allows to derive a surprisingly rich set of results for the neutral sector. We focus on those that are relevant for the identification of the photon field after radiative corrections. In this section and in the attached Appendix C

we use a simplified notation

$$W_{A_1 \dots A_n} = \frac{\delta^n W}{\delta J_{A_1} \dots \delta J_{A_n}} = i^{(n-1)} \langle 0 | T(A_1 \dots A_n) | 0 \rangle, \quad (56)$$

where  $J_{A_1}$  is the source for  $A_1$ . Moreover we use the conventions

$$\mathfrak{M}_A \quad (57)$$

for an  $S$ -matrix element on which the functional derivative with respect to  $J_A$  has been taken and all external sources have been put to zero. The states resulting from the reduction formulas are not displayed, if not necessary. Finally the  $\hat{A}$  indicates that the external leg attached to  $A$  has been removed. For instance

$$\mathfrak{M}_{\hat{A}}. \quad (58)$$

By taking the appropriate linear combination of the Abelian STI in Eq. (21) and the third component of the LFE in Eq. (28), the bilinear term  $\Gamma_{K_0} \Gamma_{\phi_3}$  can be removed. This yields

$$\begin{aligned} & -\frac{1}{g'} A^{(D-4)} \square b_0 + \left( -\frac{1}{g'} \partial^\mu \frac{\delta}{\delta B^\mu} - \frac{1}{g} \partial_\mu \frac{\delta}{\delta A_{3\mu}} - \frac{1}{g} \partial_\mu \frac{\delta}{\delta V_{3\mu}} \right. \\ & + A_{2\mu} \frac{\delta}{\delta A_{1\mu}} - A_{1\mu} \frac{\delta}{\delta A_{2\mu}} + iQL \frac{\delta}{\delta L} - i\bar{L}Q \frac{\delta}{\delta \bar{L}} + iQR \frac{\delta}{\delta R} - i\bar{R}Q \frac{\delta}{\delta \bar{R}} \\ & + \phi_2 \frac{\delta}{\delta \phi_1} - \phi_1 \frac{\delta}{\delta \phi_2} + b_2 \frac{\delta}{\delta b_1} - b_1 \frac{\delta}{\delta b_2} + c_2 \frac{\delta}{\delta c_1} - c_1 \frac{\delta}{\delta c_2} \\ & + \bar{c}_2 \frac{\delta}{\delta \bar{c}_1} - \bar{c}_1 \frac{\delta}{\delta \bar{c}_2} + V_{2\mu} \frac{\delta}{\delta V_{1\mu}} - V_{1\mu} \frac{\delta}{\delta V_{2\mu}} + \Theta_{2\mu} \frac{\delta}{\delta \Theta_{1\mu}} - \Theta_{1\mu} \frac{\delta}{\delta \Theta_{2\mu}} \\ & + A_{2\mu}^* \frac{\delta}{\delta A_{1\mu}^*} - A_{1\mu}^* \frac{\delta}{\delta A_{2\mu}^*} + \phi_2^* \frac{\delta}{\delta \phi_1^*} - \phi_1^* \frac{\delta}{\delta \phi_2^*} + c_2^* \frac{\delta}{\delta c_1^*} - c_1^* \frac{\delta}{\delta c_2^*} \\ & \left. - iQL^* \frac{\delta}{\delta L^*} + i\bar{L}^*Q \frac{\delta}{\delta \bar{L}^*} \right) \Gamma = 0, \quad (59) \end{aligned}$$

where  $Q$  is the electric charge of the component of the multiplet. In terms of the fields

$$\begin{aligned} Z_\mu &= \frac{1}{\sqrt{g^2 + g'^2}} (gA_{3\mu} - g'B_\mu), \\ A_\mu &= \frac{1}{\sqrt{g^2 + g'^2}} (g'A_{3\mu} + gB_\mu), \end{aligned} \quad (60)$$

the neutral boson part in Eq. (59) takes the form

$$-\frac{1}{g'}\partial^\mu\frac{\delta}{\delta B^\mu}-\frac{1}{g}\partial^\mu\frac{\delta}{\delta A_{3\mu}}=-\frac{\sqrt{g^2+g'^2}}{gg'}\partial^\mu\frac{\delta}{\delta A^\mu}. \quad (61)$$

The term  $-\frac{1}{g}\partial^\mu\frac{\delta}{\delta V_{3\mu}}$  takes into account that the fields of the photon and of the  $Z_0$ , as superposition of  $(A_{3\mu}, B_\mu)$ , are modified by the perturbative corrections.

In the generic  $S$ -matrix elements the insertion of  $V_{a\mu}$  is zero for physical states. The proof makes use of the STI in Eq. (20) written for the connected amplitude

$$\begin{aligned} SW \equiv \int d^Dx \Big[ \Lambda^{-(D-4)} \Big( -W_{A_{a\mu}^*} J_{a\mu} - W_{\phi_a^*} K_a + W_{c_a^*} \bar{\eta}_a \\ + W_{L^*} \bar{\xi} + W_{\bar{L}^*} \xi \Big) + \eta_a W_{ba} + \Theta_{a\mu} W_{V_{a\mu}} - K_0 W_{\phi_0^*} \Big] = 0, \end{aligned} \quad (62)$$

where  $J_{a\mu} A_a^\mu + K_a \phi_a + \bar{\eta}_a c_a + \bar{c}_a \eta_a + \bar{L} \xi + \bar{\xi} L$  are the source terms. One takes the functional derivative with respect to  $\Theta_{a\mu}$  and subsequently applies the procedure of deriving with respect to the field sources and finally applies the reduction formulas. On the physical states one obtains

$$\mathfrak{M}_{V_{a\mu}\dots} = 0, \quad (63)$$

where the dots  $\dots$  indicate the physical state variables. Consequently from the WTI (59) (written for the connected amplitudes) we get

$$\square \mathfrak{M}_{b_0\dots} = 0. \quad (64)$$

A further important identity can be derived from Eq. (62). By differentiating with respect to  $\eta_3$  and by constructing a physical  $S$ -matrix element, one gets

$$\mathfrak{M}_{b_3\dots} = 0. \quad (65)$$

### 6.1. The two-point functions

In this subsection we determine the most general form of the two-point functions in the Landau gauge. For this purpose we use the STI, LFE, and LGE, where we drop all the terms that cannot produce any contributions. Moreover we impose the condition

$$\Gamma W = -\mathbb{I}. \quad (66)$$

The explicit calculation is given in Appendix C and the results can be displayed in a matrix form both for  $\Gamma$  and  $W$ .

Summary for the two-point function  $\Gamma$

$$\left( \begin{array}{ccccc} A_3^\mu & B^\mu & b_3 & b_0 & \phi_3 \\ A_3^\nu & \Gamma_T^{AA} \Pi^{\mu\nu} + \Gamma_L^{AA} \frac{p^\mu p^\nu}{p^2} & \Gamma_T^{AB} \Pi^{\mu\nu} + \Gamma_L^{AB} \frac{p^\mu p^\nu}{p^2} & i\Lambda^{D-4} p^\nu & 0 \\ B^\nu & \Gamma_T^{BA} \Pi^{\mu\nu} + \Gamma_L^{BA} \frac{p^\mu p^\nu}{p^2} & \Gamma_T^{BB} \Pi^{\mu\nu} + \Gamma_L^{BB} \frac{p^\mu p^\nu}{p^2} & 0 & i\Lambda^{D-4} p^\nu \\ b_3 & -i\Lambda^{D-4} p^\mu & 0 & 0 & 0 \\ b_0 & 0 & -i\Lambda^{D-4} p^\mu & 0 & 0 \\ \phi_3 & i\frac{2p^\mu}{v'g'} \Gamma_L^{BA} & i\frac{2p^\mu}{v'g'} \Gamma_L^{BB} & 0 & p^2 \left( \frac{2}{v'g'} \right)^2 \Gamma_L^{BB} \end{array} \right). \quad (67)$$

Summary for the propagator  $W$

$$\left( \begin{array}{ccccc} A_3^\mu & B^\mu & b_3 & b_0 & \phi_3 \\ A_3^\nu & -\frac{\Gamma_T^{BB}}{\Delta_T} \Pi^{\mu\nu} & \frac{\Gamma_T^{AB}}{\Delta_T} \Pi^{\mu\nu} & -i\frac{p^\nu}{\Lambda^{D-4} p^2} & 0 \\ B^\nu & \frac{\Gamma_T^{AB}}{\Delta_T} \Pi^{\mu\nu} & -\frac{\Gamma_T^{AA}}{\Delta_T} \Pi^{\mu\nu} & 0 & -i\frac{p^\nu}{\Lambda^{D-4} p^2} \\ b_3 & i\frac{p^\mu}{\Lambda^{D-4} p^2} & 0 & 0 & 0 \\ b_0 & 0 & i\frac{p^\mu}{\Lambda^{D-4} p^2} & 0 & 0 \\ \phi_3 & 0 & 0 & -\frac{v'g' \Gamma_L^{AB}}{2\Lambda^{D-4} p^2 \Gamma_L^{BB}} & -\frac{v'g'}{2p^2 \Lambda^{D-4}} \end{array} \right), \quad (68)$$

where

$$\Delta_T = \Gamma_T^{AA} \Gamma_T^{BB} - \Gamma_T^{AB} \Gamma_T^{BA}, \quad \Pi^{\mu\nu} = g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}, \quad v' = \Lambda^{-D+4} \Gamma_{K_0}. \quad (69)$$

We see from Eq. (67) that the field

$$A^\mu \equiv \frac{1}{\sqrt{(\Gamma_L^{BB})^2 + (\Gamma_L^{AB})^2}} \left( -\Gamma_L^{BB} A_3^\mu + \Gamma_L^{AB} B^\mu \right) \quad (70)$$

decouples from  $\phi_3$ . While the corresponding orthogonal combination

$$Z^\mu \equiv \frac{1}{\sqrt{(\Gamma_L^{BB})^2 + (\Gamma_L^{AB})^2}} \left( \Gamma_L^{AB} A_3^\mu + \Gamma_L^{BB} B^\mu \right) \quad (71)$$

remains coupled to  $\phi_3$ . Moreover again from Eq. (67) we see that the longitudinal part of the 1-PI two-point function of  $A_\mu$  is zero while it remains non zero for  $Z_\mu$ . This is due to the fact that  $\Delta_L = 0$ . In fact, from Appendix C, Eq. (C.31) we have

$$\frac{\Gamma_L^{AA}}{\Gamma_L^{AB}} = \frac{\Gamma_L^{BA}}{\Gamma_L^{BB}} = -\frac{2p^2}{v'g'} \frac{\Gamma_{c_3\phi_3^*}}{\Gamma_{c_3\bar{c}_3}}. \quad (72)$$

The above equation (72) shows also that the Nakanishi–Lautrup Lagrange multiplier for  $A_\mu$

$$b_A \equiv \frac{1}{\sqrt{(\Gamma_L^{BB})^2 + (\Gamma_L^{AB})^2}} \left( -\Gamma_L^{BB} b_3 + \Gamma_L^{AB} b_0 \right) \quad (73)$$

decouples from  $\phi_3$ .

### 6.2. Decoupling of the unphysical modes in the neutral sector at $p^2 = 0$

At  $p^2 = 0$  there are some unphysical modes in the neutral sector. They show up in the propagator of the  $Z_\mu$  in the Landau gauge and in the propagator of the  $\phi_3$ . There is a further  $p^2 = 0$  unphysical pole in the photon propagator.

We have Eq. (64) which follows from the WTI (59) and (63). In the limit  $p^2 = 0$  only the pole parts survive. By using the relations in Eq. (68) the WTI (64) yields

$$\left[ ip^\mu \mathfrak{M}_{\widehat{B^\mu \dots}} - \frac{v' g'}{2} \mathfrak{M}_{\widehat{\phi_3 \dots}} \right]_{p^2=0} = 0. \quad (74)$$

Now we use Eq. (65). The multiplication by the square of the external momentum and its limits to zero selects only the pole parts. From Eqs. (C.2), (C.4) and (C.7) in Appendix C and Eq. (72)

$$\lim_{p^2=0} p^2 \mathfrak{M}_{b_3 \dots} = \lim_{p^2=0} \left( ip^\mu \mathfrak{M}_{\widehat{A_3^\mu \dots}} - \frac{v' g'}{2} \frac{\Gamma_L^{AB}}{\Gamma_L^{BB}} \mathfrak{M}_{\widehat{\phi_3 \dots}} \right) = 0. \quad (75)$$

By removing the contribution of  $\phi_3$  between Eqs. (74) and (75) we get

$$\lim_{p^2=0} p^\mu \left( \mathfrak{M}_{\widehat{A_3^\mu \dots}} - \frac{\Gamma_L^{AB}}{\Gamma_L^{BB}} \mathfrak{M}_{\widehat{B^\mu \dots}} \right) = 0 \quad (76)$$

which guarantees that longitudinally polarized photons decouple from physical states. Now we consider the massless modes present in the  $Z_\mu$  sector. The combination of Eqs. (74) and (75) orthogonal to the one in Eq. (76) is

$$\lim_{p^2=0} \left( ip^\mu \Gamma_L^{AB} \mathfrak{M}_{\widehat{A_3^\mu \dots}} + ip^\mu \Gamma_L^{BB} \mathfrak{M}_{\widehat{B^\mu \dots}} - \frac{v' g'}{2} \frac{(\Gamma_L^{AB})^2 + (\Gamma_L^{BB})^2}{\Gamma_L^{BB}} \mathfrak{M}_{\widehat{\phi_3 \dots}} \right) = 0. \quad (77)$$

The  $Z$ – $Z$  propagator (68) written for the linear combination (71) is

$$W_{Z^\mu Z^\nu} = \frac{\Pi^{\mu\nu}}{\Delta_T \left[ (\Gamma_L^{AB})^2 + (\Gamma_L^{BB})^2 \right]} \left( -\Gamma_T^{BB} \Gamma_L^{AB} \Gamma_L^{AB} + 2\Gamma_T^{AB} \Gamma_L^{BB} \Gamma_L^{AB} - \Gamma_T^{AA} \Gamma_L^{BB} \Gamma_L^{BB} \right). \quad (78)$$

Now we require that the two-point functions  $\Gamma$  be non singular at  $p^2 = 0$ , *i.e.* [5]

$$\lim_{p^2=0} \left( \Gamma_T^{XY} - \Gamma_L^{XY} \right) = 0, \quad (79)$$

$$W_{Z^\mu Z^\nu}|_{p^2 \sim 0} = \frac{\Pi^{\mu\nu}}{\left[ (\Gamma_L^{AB})^2 + (\Gamma_L^{BB})^2 \right]} \left( -\Gamma_L^{BB} \right). \quad (80)$$

Eqs. (77) and (80) imply

$$\begin{aligned} \lim_{p^2=0} p^2 \mathfrak{M}_{\widehat{Z}^\mu \dots}^* W_{Z^\mu Z^\nu} \mathfrak{M}_{\widehat{Z}^\nu \dots} &= \lim_{p^2=0} \mathfrak{M}_{\widehat{\phi_3 \dots}}^* \frac{(g'v')^2}{4\Gamma_L^{BB}} \mathfrak{M}_{\widehat{\phi_3 \dots}} \\ &= - \lim_{p^2=0} p^2 \mathfrak{M}_{\widehat{\phi_3 \dots}}^* W_{\phi_3 \phi_3} \mathfrak{M}_{\widehat{\phi_3 \dots}}. \end{aligned} \quad (81)$$

The last term cancels the  $\phi_3$  contribution coming from the full propagator (68).

## 7. Conclusions

The electroweak model based on the nonlinearly realized  $SU(2) \otimes U(1)$  gauge group can be consistently defined in the perturbative loop-wise expansion. In this formulation there is no Higgs in the perturbative series.

The present approach is based on the LFE and the WPC. The LFE encodes the invariance of the path-integral Haar measure under local  $SU(2)_L$  transformations and provides a hierarchy among 1-PI Green functions by fixing all amplitudes involving at least one Goldstone leg. The ancestor amplitudes (*i.e.* those with no Goldstone legs) obey the WPC theorem.

There is a unique classical action giving rise to Feynman rules compatible with the WPC formula in Eq. (34). In particular the anomalous couplings, which would be otherwise allowed on symmetry grounds, are excluded by the WPC. Two gauge boson mass invariants are compatible with the WPC and the symmetries. Thus the tree-level Weinberg relation is not working in the nonlinear framework.

The discovery of the LFE suggests a unique *Ansatz* for the subtraction procedure which is symmetric, *i.e.* it respects the LFE itself, the STI (necessary for the fulfillment of the Physical Unitarity) and the LGE (controlling the stability of the gauge-fixing under radiative corrections). A linear Ward identity exists for the electric charge (despite the nonlinear realization of the gauge group). The strategy does not alter the number of tree-level parameters apart from a common mass scale of the radiative corrections. The

algorithm is strictly connected with dimensional regularization and the symmetric subtraction of the pole parts in the Laurent expansion of the 1-PI amplitudes.

The theoretical and phenomenological consequences of this scenario are rather intriguing. A Higgs boson could emerge as a non-perturbative mechanism, but then its physical parameters are not constrained by the radiative corrections of the low energy electroweak processes. Otherwise the energy scale for the radiative corrections  $\Lambda$  is a manifestation of some other high-energy physics.

Many aspects remain to be further studied. We only mention some of them here. The issue of unitarity at large energy (violation of Froissart bound) [14] at fixed order in perturbation theory when the Higgs field is removed (as in [15–17]) can provide additional insight in the role of the mass scale  $\Lambda$ . The electroweak model based on the nonlinearly realized gauge group satisfies Physical Unitarity as a consequence of the validity of the Slavnov–Taylor identity. Therefore violation of the Froissart bound can only occur in evaluating cross sections at finite order in perturbation theory. This requires the evaluation of a scale at each order where unitarity at large energy is substantially violated.

The phenomenological implications of the nonlinear theory in the electroweak precision fit have to be investigated.

Finally the extension of the present approach to larger gauge groups (as in Grand-Unified models) could help in understanding the nonlinearly realized spontaneous symmetry breaking mechanism (selection of the identity as the preferred direction in the  $SU(2)$  manifold) and the associated appearance of two independent gauge boson mass invariants.

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## Appendix A

### *Propagators in the Landau gauge*

We summarize here the propagators in the Landau gauge evaluated in the symmetric formalism. It is convenient to rescale the Goldstone fields according to



$$\phi_{1,2} \rightarrow \frac{v}{2M} \phi_{1,2}, \quad \phi_3 \rightarrow \frac{v}{2M(1+\kappa)^{1/2}}. \quad (\text{A.1})$$

This ensures the common normalization of the Goldstone propagators. We define the Weinberg angle via the relation

$$\tan \theta_W = \frac{g'}{g}. \quad (\text{A.2})$$

The sine and cosine of the Weinberg angle are denoted by

$$c = \cos \theta_W, \quad s = \sin \theta_W. \quad (\text{A.3})$$

We also define the masses of the charged and neutral gauge boson mass eigenstates:

$$M_W^2 = (gM)^2, \quad M_Z^2 = \frac{(gM)^2}{c}(1+\kappa). \quad (\text{A.4})$$

By inverting the two-point functions in  $\Gamma^{(0)}$  in Eq. (18) one finds (the common pre-factor  $\Lambda^{-(D-4)}$  is always left understood)

$$\begin{aligned} \Delta_{A_{1\mu}A_{1\nu}} &= \Delta_{A_{2\mu}A_{2\nu}} = \frac{i}{-p^2 + M_W^2} T_{\mu\nu}, & \Delta_{A_{1\mu}A_{2\nu}} &= \Delta_{A_{1\mu}A_{3\nu}} = \Delta_{A_{2\mu}A_{3\nu}} = 0, \\ \Delta_{A_{3\mu}A_{3\nu}} &= \frac{i}{-p^2 + M_Z^2} T_{\mu\nu}, & \Delta_{A_{3\mu}B_\nu} &= cs \left( \frac{i}{-p^2} - \frac{i}{-p^2 + M_Z^2} \right) T_{\mu\nu}, \\ \Delta_{A_{1\mu}B_\nu} &= \Delta_{A_{2\mu}B_\nu} = 0, & \Delta_{B_\mu B_\nu} &= \left( c^2 \frac{i}{-p^2} + s^2 \frac{i}{-p^2 + M_Z^2} \right) T_{\mu\nu}, \\ \Delta_{\phi_a \phi_b} &= \delta_{ab} \frac{i}{p^2}, \\ \Delta_{b_i A_{j\mu}} &= -\frac{p_\mu}{p^2} \delta_{ij}, & \Delta_{b_i b_j} &= 0, & \Delta_{b_i \phi_j} &= -i \delta_{ij} \frac{M_W}{p^2}, & i, j &= 1, 2, \\ \Delta_{B_3 A_{3\mu}} &= \frac{-p^\mu}{p^2}, & \Delta_{B_3 B_\mu} &= 0, & \Delta_{B_3 \phi_3} &= -ic \frac{M_Z}{p^2}, & \Delta_{B_3 B_3} &= 0, \\ \Delta_{b_0 A_{3\mu}} &= 0, & \Delta_{b_0 B_\mu} &= -\frac{p^\mu}{p^2}, & \Delta_{b_0 \phi_3} &= is \frac{M_Z}{p^2}, & \Delta_{b_0 b_0} &= 0, \\ \Delta_{\bar{c}_a c_b} &= \delta_{ab} \frac{i}{p^2}, & \Delta_{\bar{c}_0 c_0} &= \frac{i}{p^2}. \end{aligned} \quad (\text{A.5})$$

The mixed  $A - \phi$  propagators are zero.

The relation with the mass eigenstates is given by

$$A_\mu = cB_\mu + sA_{3\mu}, \quad Z_\mu = -sB_\mu + cA_{3\mu}. \quad (\text{A.6})$$

In the fermion sector the propagators are

$$\Delta_{\bar{f}f} = \frac{i}{\not{p} - m_f}, \quad (\text{A.7})$$

where  $m_f$  is the mass of the fermionic species  $f$ .

## Appendix B

### *Proof of the weak-power counting formula*

In this Appendix we prove the weak power-counting formula in Eq. (34) by extending the analysis carried out for massive SU(2) Yang–Mills theory [4] to the electroweak model based on the nonlinearly realized  $\text{SU}(2)_L \otimes \text{U}(1)_R$  gauge group.

Let  $\mathcal{G}$  be an arbitrary  $n$ -loop 1-PI ancestor graph with  $I$  internal lines,  $V$  vertexes and a given set  $\{N_A, N_B, N_F, N_{\bar{F}}, N_c, N_V, N_\Omega, N_{\phi_0^*}, N_{K_0}, N_{\phi_a^*}, N_{A^*}, N_{c^*}, N_{L^*}, N_{\bar{L}^*}\}$  of external legs.  $F, \bar{F}$  are a collective notation for the fermion and anti-fermion matter fields, which can be treated in a unified manner.

We do not need to consider  $\Delta_{b_0B}$  since there are no vertexes involving  $b_0$ . By Eq. (A.5) all the remaining propagators behave as  $p^{-2}$  as  $p$  goes to infinity, with the exception of  $\Delta_{bA} \sim p^{-1}$ .

Let us denote by  $\hat{I}$  the number of internal lines associated with propagators behaving as  $p^{-2}$ , by  $I_b$  the number of internal lines with propagators  $\Delta_{bA}$  and by  $I_F$  the number of internal fermionic lines. One has

$$I = \hat{I} + I_b + I_F. \quad (\text{B.1})$$

According to the Feynman rules generated by the tree-level vertex functional in Eq. (18) the superficial degree of divergence of  $\mathcal{G}$  is

$$\begin{aligned} d(\mathcal{G}) = & nD - 2\hat{I} - I_b - I_F + V_{AAA} \\ & + \sum_k V_{A\phi^k} + \sum_k V_{B\phi^k} + 2 \sum_k V_{\phi^k} + V_{\bar{c}cA} + V_{\bar{c}cV}. \end{aligned} \quad (\text{B.2})$$

In the above equation we have denoted by  $V_{AAA}$  the number of vertexes in  $\mathcal{G}$  with three  $A$ -fields, with  $V_{A\phi^k}$  the number of vertexes with one  $A$  and  $k$   $\phi$ s and so on.

By using Eq. (B.1) we can rewrite Eq. (B.2) as

$$d(\mathcal{G}) = nD - 2I + I_b + I_F + V_{AAA} + \sum_k V_{A\phi^k} + \sum_k V_{B\phi^k} + 2 \sum_k V_{\phi^k} + V_{\bar{c}cA} + V_{\bar{c}cV}. \quad (\text{B.3})$$

The total number of vertexes  $V$  is given by

$$\begin{aligned} V = & V_{AAA} + V_{AAAA} + \sum_k V_{A\phi^k} + \sum_k V_{B\phi^k} + \sum_k V_{\phi^k} \\ & + V_{bVA} + V_{\bar{c}cA} + V_{\bar{c}cV} + V_{\bar{c}cVA} + V_{\bar{c}A\Theta} + V_{\phi_0^* \phi c} \\ & + \sum_k V_{\phi_a^* \phi^k c} + V_{A^*Ac} + V_{c^*cc} + \sum_k V_{K_0 \phi^k} \\ & + V_{\bar{F}FA} + \sum_k V_{\bar{F}F\phi^k} + V_{\bar{F}FB} + V_{\bar{L}^* \bar{L}c} + V_{L^* Lc}. \end{aligned} \quad (\text{B.4})$$

Euler's formula yields

$$I = n + V - 1. \quad (\text{B.5})$$

Moreover, since  $b$  only enters into the trilinear vertex  $\Gamma_{b_a V_{b\mu} A_{c\nu}}^{(0)}$ , the number of  $bVA$  vertexes must be greater than or equal to the number of propagators  $\Delta_{bA}$

$$I_b \leq V_{bVA}. \quad (\text{B.6})$$

On the other hand, the number of internal fermion lines fulfills the following bound

$$I_F \leq V_{\bar{F}FA} + V_{\bar{F}FB} + \sum_k V_{\bar{F}F\phi^k}. \quad (\text{B.7})$$

By using Eqs. (B.4), (B.5), (B.6) and (B.7) into Eq. (B.2) one gets

$$\begin{aligned} d(\mathcal{G}) = & (D - 2)n + 2 + I_b + I_F - V_{AAA} - \sum_k V_{A\phi^k} - \sum_k V_{B\phi^k} - V_{\bar{c}cA} - V_{\bar{c}cV} \\ & - 2 \left[ V_{AAAA} + V_{bVA} + V_{\bar{c}cVA} + V_{\bar{c}A\Theta} + V_{\phi_0^* \phi c} + \sum_k V_{\phi_a^* \phi^k c} + V_{A^*Ac} + V_{c^*cc} \right. \\ & \left. + \sum_k V_{K_0 \phi^k} + V_{\bar{F}FA} + V_{\bar{F}FB} + \sum_k V_{\bar{F}F\phi^k} + V_{\bar{L}^* \bar{L}c} + V_{L^* Lc} \right] \end{aligned}$$

$$\begin{aligned}
&\leq (D-2)n + 2 \\
&\quad -V_{AAA} - \sum_k V_{A\phi^k} - \sum_k V_{B\phi^k} - V_{\bar{c}cA} - V_{\bar{c}cV} - V_{bVA} - V_{\bar{F}FA} - V_{\bar{F}FB} \\
&\quad - \sum_k V_{\bar{F}F\phi^k} - 2 \left[ V_{AAAA} + V_{\bar{c}cVA} + V_{\bar{c}A\Theta} + V_{\phi_0^*\phi c} + \sum_k V_{\phi_a^*\phi^k c} \right. \\
&\quad \left. + V_{A^*Ac} + V_{c^*cc} + \sum_k V_{K_0\phi^k} + V_{\bar{L}^*\bar{L}c} + V_{L^*Lc} \right]. \tag{B.8}
\end{aligned}$$

Clearly, one has

$$\begin{aligned}
V_{\bar{c}A\Theta} &= N_{\Theta}, & V_{\phi_0^*\phi c} &= N_{\phi_0^*}, \\
V_{A^*Ac} &= N_{A^*}, & V_{c^*cc} &= N_{c^*}, \\
\sum_k V_{\phi_a^*\phi^k c} &= N_{\phi_a^*}, & \sum_k V_{K_0\phi^k} &= N_{K_0}, \\
V_{\bar{c}cV} + V_{bVA} + V_{\bar{c}cVA} &= N_V, \\
V_{\bar{L}^*\bar{L}c} &= N_{\bar{L}^*}, & V_{L^*Lc} &= N_{L^*}. \tag{B.9}
\end{aligned}$$

Moreover,

$$\begin{aligned}
&V_{AAA} + \sum_k V_{A\phi^k} + 2V_{AAAA} + V_{\bar{F}FA} + \sum_k V_{B\phi^k} + V_{\bar{F}FB} + \sum_k V_{\bar{F}F\phi^k} \\
&+ V_{\bar{c}cA} + V_{\bar{c}cVA} + \sum_k V_{\phi_a^*\phi^k c} \geq N_A + N_B + N_c + N_F + N_{\bar{F}}. \tag{B.10}
\end{aligned}$$

In fact, the quadrilinear vertex  $V_{AAAA}$  can give one or two external  $A$  lines and the vertexes  $V_{\bar{F}FB}$ ,  $V_{\bar{F}FA}$  can give rise to at most one external  $B$  and  $A$  line, respectively.

By using Eqs. (B.9) and (B.10) into Eq. (B.8) we obtain in a straightforward way the following bound:

$$\begin{aligned}
d(\mathcal{G}) &\leq (D-2)n + 2 - N_A - N_B - N_c - N_F - N_{\bar{F}} - N_V - N_{\phi_a^*} \\
&\quad - 2(N_{\Theta} + N_{A^*} + N_{\phi_0^*} + N_{L^*} + N_{\bar{L}^*} + N_{c^*} + N_{K_0}). \tag{B.11}
\end{aligned}$$

This establishes the validity of the weak power-counting formula.

## Appendix C

### *Two-point functions results*

The results of this Appendix are valid for a generic value of  $p^2$ .

From the  $U(1)$  LGE

$$-J_{b_0} = \Lambda^{(D-4)} \partial^\mu W_{B^\mu} \quad (C.1)$$

we get

$$\begin{aligned} W_{B^\mu b_0} &= -i \frac{p_\mu}{\Lambda^{(D-4)} p^2}, & W_{B^\mu b_3} &= 0, & W_{B^\mu \phi_3} &= 0, \\ p^\mu W_{B^\mu A_3^\nu} &= 0, & p^\mu W_{B^\mu B^\nu} &= 0. \end{aligned} \quad (C.2)$$

From the  $SU(2)$  LGE (25)

$$-J_{b_3} = \Lambda^{(D-4)} \partial^\mu (W_{A_3^\mu} - V_{3\mu}) \quad (C.3)$$

we get

$$\begin{aligned} W_{A_3^\mu b_0} &= 0, & W_{A_3^\mu b_3} &= -i \frac{p_\mu}{\Lambda^{(D-4)} p^2}, & W_{A_3^\mu \phi_3} &= 0, \\ p^\mu W_{A_3^\mu A_3^\nu} &= 0, & p^\mu W_{A_3^\mu B^\nu} &= 0. \end{aligned} \quad (C.4)$$

From the  $U(1)$  STI (21)

$$-\frac{\Lambda^{(D-4)}}{g'} \square W_{b_0} + \frac{1}{g'} \partial^\mu J_{B^\mu} + \frac{v'}{2} J_{\phi_3} = 0, \quad (C.5)$$

where

$$v' \equiv \frac{1}{\Lambda^{(D-4)}} \Gamma_{K_0}, \quad (C.6)$$

we get

$$\begin{aligned} W_{b_0 A_3^\mu} &= 0, & W_{b_0 B^\mu} &= i \frac{p_\mu}{\Lambda^{(D-4)} p^2}, & W_{b_0 \phi_3} &= -\frac{v' g'}{2 \Lambda^{(D-4)} p^2}, \\ W_{b_0 b_0} &= 0, & W_{b_0 b_3} &= 0. \end{aligned} \quad (C.7)$$

From the  $SU(2)$  STI (62)

$$\int d^D x \left( -W_{A_{a\mu}^*} J_{a\mu} - W_{\phi_a^*} K_a + \Lambda^{(D-4)} \eta_a W_{b_a} \right) = 0 \quad (C.8)$$

we get

$$\begin{aligned} W_{b_3 b_0} &= 0, & W_{b_3 B^\mu} &= 0, & W_{b_3 A_3^\mu} &= \frac{1}{\Lambda^{(D-4)}} W_{\bar{c}_3 A_{a\mu}^*}, \\ W_{b_3 \phi_3} &= \frac{1}{\Lambda^{(D-4)}} W_{\bar{c}_3 \phi_3^*}, & W_{b_3 b_3} &= 0. \end{aligned} \quad (C.9)$$

Eqs. (C.4) and (C.9) imply the interesting result

$$W_{\bar{c}_3 A_{a\mu}^*} = i \frac{p^\mu}{p^2}. \quad (\text{C.10})$$

We now consider the 1PI two-point functions. From the U(1) LGE

$$\Gamma_{b_0} = \Lambda^{(D-4)} \partial^\mu B_\mu \quad (\text{C.11})$$

we get

$$\begin{aligned} \Gamma_{b_0 B^\mu} &= -i \Lambda^{(D-4)} p_\mu, & \Gamma_{b_0 A_3^\mu} &= 0, & \Gamma_{b_0 b_3} &= 0, \\ \Gamma_{b_0 \phi_3} &= 0, & \Gamma_{b_0 b_0} &= 0. \end{aligned} \quad (\text{C.12})$$

From the SU(2) LGE (25)

$$\Gamma_{b_a} = \Lambda^{(D-4)} \left( D^\mu [V] (A_\mu - V_\mu) \right)_a \quad (\text{C.13})$$

we get

$$\begin{aligned} \Gamma_{b_3 A_3^\mu} &= -i \Lambda^{(D-4)} p_\mu, & \Gamma_{b_3 B^\mu} &= 0, & \Gamma_{b_3 b_3} &= 0, \\ \Gamma_{b_3 \phi_3} &= 0, & \Gamma_{b_0 b_3} &= 0. \end{aligned} \quad (\text{C.14})$$

From the U(1) STI (21)

$$-\frac{2}{g'} \Lambda^{(D-4)} \square b_0 - \frac{2}{g'} \partial^\mu \Gamma_{B^\mu} - \frac{1}{\Lambda^{(D-4)}} \Gamma_{K_0} \Gamma_{\phi_3} = 0 \quad (\text{C.15})$$

we get

$$\begin{aligned} p^\mu \Gamma_{B^\mu \phi_3} &= -i \frac{v' g'}{2} \Gamma_{\phi_3 \phi_3}, & p^\mu \Gamma_{B^\mu A_3^\nu} &= -i \frac{v' g'}{2} \Gamma_{\phi_3 A_3^\nu}, \\ p^\mu \Gamma_{B^\mu B^\nu} &= -i \frac{v' g'}{2} \Gamma_{\phi_3 B^\nu}, & \implies p^2 \Gamma_L^{BB} &= \left( \frac{v' g'}{2} \right)^2 \Gamma_{\phi_3 \phi_3}. \end{aligned} \quad (\text{C.16})$$

From the SU(2) STI (20)

$$\int d^D x \left[ \Lambda^{-(D-4)} \left( \Gamma_{A_{a\mu}^*} \Gamma_{A_a^\mu} + \Gamma_{\phi_a^*} \Gamma_{\phi_a} \right) + b_a \Gamma_{\bar{c}_a} \right] = 0 \quad (\text{C.17})$$

we get

$$\begin{aligned} p_\mu \Gamma_{c(p) A_{3\mu}^*} &= i \Gamma_{c(p) \bar{c}}, \\ \Gamma_{c A_{3\mu}^*} \Gamma_{A_3^\mu \phi_3} + \Gamma_{c \phi_3^*} \Gamma_{\phi_3 \phi_3} &= 0, \\ \Gamma_{c A_{3\mu}^*} \Gamma_{A_3^\mu B^\nu} + \Gamma_{c \phi_3^*} \Gamma_{\phi_3 B^\nu} &= 0, \\ \Gamma_{c A_{3\mu}^*} \Gamma_{A_3^\mu A_3^\nu} + \Gamma_{c \phi_3^*} \Gamma_{\phi_3 A_3^\nu} &= 0. \end{aligned} \quad (\text{C.18})$$

From Eqs. (C.16) and (C.18) we get

$$\begin{aligned}
 \Gamma_{cA_{3\mu}^*} &= i \frac{p^\mu}{p^2} \Gamma_{c\bar{c}}, \\
 \Gamma_{A_3^\mu \phi_3} &= i \frac{p_\mu \Gamma_{c\phi_3^*}}{\Gamma_{c\bar{c}}} \Gamma_{\phi_3 \phi_3}, \\
 \Gamma_L^{AB} &= ip^\nu \frac{\Gamma_{c\phi_3^*}}{\Gamma_{c\bar{c}}} \Gamma_{\phi_3 B^\nu} = -p^2 \frac{2}{v'g'} \frac{\Gamma_{c\phi_3^*}}{\Gamma_{c\bar{c}}} \Gamma_L^{BB} = -\frac{v'g'}{2} \frac{\Gamma_{c\phi_3^*}}{\Gamma_{c\bar{c}}} \Gamma_{\phi_3 \phi_3}, \\
 \Gamma_L^{AA} &= ip^\nu \frac{\Gamma_{c\phi_3^*}}{\Gamma_{c\bar{c}}} \Gamma_{\phi_3 A_3^\nu} = p^2 \left( \frac{\Gamma_{c\phi_3^*}}{\Gamma_{c\bar{c}}} \right)^2 \Gamma_{\phi_3 \phi_3}.
 \end{aligned} \tag{C.19}$$

From the condition in Eq. (66) we get the following constraints

$$(\Gamma W)_{A^\mu \phi} = 0, \quad \Rightarrow \quad \frac{2}{v'g'} \Gamma_L^{BA} W_{\phi\phi} = \Lambda^{D-4} W_{b_3\phi}, \tag{C.20}$$

$$(\Gamma W)_{B^\mu \phi} = 0, \quad \Rightarrow \quad \Gamma_{\phi\phi} W_{\phi\phi} = -1, \tag{C.21}$$

$$(\Gamma W)_{Ab_0} = 0, \quad \Rightarrow \quad \Gamma_L^{AB} - \Gamma_L^{BA} = 0, \tag{C.22}$$

$$(\Gamma W)_{Ab_3} = 0, \quad \Rightarrow \quad \frac{1}{p^2 \Lambda^{D-4}} \Gamma_L^{AA} + \frac{2}{v'g'} \Gamma_L^{AB} W_{\phi b_3} = 0, \tag{C.23}$$

$$(\Gamma W)_{BA} = 0, \quad \Rightarrow \quad \Gamma_T^{BA} W_T^{AA} + \Gamma_T^{BB} W_T^{BA} = 0, \tag{C.24}$$

$$(\Gamma W)_{AB} = 0, \quad \Rightarrow \quad \Gamma_T^{AA} W_T^{AB} + \Gamma_T^{AB} W_T^{BB} = 0, \tag{C.25}$$

$$(\Gamma W)_{AA} = -\mathbb{I}, \quad \Rightarrow \quad \Gamma_T^{AA} W_T^{AA} + \Gamma_T^{AB} W_T^{BA} = -1, \tag{C.26}$$

$$(\Gamma W)_{BB} = -\mathbb{I}, \quad \Rightarrow \quad \Gamma_T^{BA} W_T^{AB} + \Gamma_T^{BB} W_T^{BB} = -1. \tag{C.27}$$

From Eqs. (C.20) and (C.23) we can deduce the following identity

$$\frac{\Gamma_L^{AA}}{p^2} = \left( \frac{2 |\Gamma_L^{AB}|}{v'g'} \right)^2 \frac{1}{\Gamma_{\phi_3 \phi_3}}. \tag{C.28}$$

Subsequently we use Eq. (C.19)

$$\Gamma_L^{AA} = |\Gamma_L^{AB}|^2 \frac{1}{\Gamma_L^{BB}}, \tag{C.29}$$

*i.e.* the  $2 \times 2$  determinant

$$\Delta_L \equiv \Gamma_L^{AA} \Gamma_L^{BB} - |\Gamma_L^{AB}|^2 = 0 \tag{C.30}$$

and moreover, again from Eq. (C.19)

$$\frac{\Gamma_L^{AA}}{\Gamma_L^{AB}} = \frac{\Gamma_L^{BA}}{\Gamma_L^{BB}} = -\frac{2p^2}{v'g'} \frac{\Gamma_{c_3\phi_3^*}}{\Gamma_{c_3\bar{c}_3}}. \tag{C.31}$$

## Appendix D

\*

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