INFORMATION FLOW WITH TIME LAG IN COMMUNICATION NETWORKS

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The information flow with time lag is investigated to fulfill the efficient functioning of communication in networks. As one crucial factor to determine the processing ability of nodes, the information flow with potential time lag is modeled by co-processing diffusion which couples the continuous time processing and the discrete diffusing dynamics. Exact results on master equation and stationary state are achieved to disclose the formation and control of congestion which results from the time lag. Besides, some statistical properties are obtained to provide well understanding on the co-processing diffusion, *e.g.*, memorylessness of the information flow. Moreover, numerical simulations suggest that many scale-free network systems undergo a special convention from free diffusion state to congestion due to the variety of processing ability.

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1. Introduction

As statistical mechanics widens into the technological, biological and social realms, various topological and dynamical properties of networks resulting from real systems have attracted many researchers in diverse fields [1–5]. Most attention have been paid to complex diffusion process. The epidemic spreading in networks has been deeply discussed and widely applied to control the disease diffusion in social networks [6–8]. The oscillator theory [9,10] and chaotic dynamics [11,12] have been made best use of to propose the synchronization in networks [13–15]. Besides, researchers have gone deep into many other diffusion processes, *e.g.*, information transmission on Internet [16,17], efficient routing for transportation [17] and navigability

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of complex networks [19,20]. The inherent complexity of networks results in rich behaviors in the diffusion processing of the complex systems, which depends on the topological structure of networks.

As a basic model of the diffusion process, random walk [21–23], especially biased random walk [24] has been deeply investigated to understand the essential dynamical properties of physical systems on networks [25–27] and also has many practical applications to real networks, such as information searching in the Internet [28–31] and local navigation in networks. However, congestion often occurs in diffusion processing, which is mainly determined by two factors, capability of nodes and time lag on nodes. Zero-range process (ZRP), as a powerful technique to control the congestion by modulating the capability of nodes, has been investigated deeply. While in the traffic flow [32–35], the car may be jammed at a crossing for a long time before it travels to another crossing. For the information flow in the networks, the information packet may mass at one processor so that it cannot be transferred to the destination in time. If the information packet is delivered following the discrete transition rule, it will be processed without hindrance and the congestion can not be formed. The traffic flow or the information packet is jammed at nodes only when there is time consumed on nodes. Thus, the continuous stochastic process with time consumed on nodes is much more corresponding to the practical situation in our usual life.

Examples with time lag on nodes can be found in many domains [36–39]. In the queueing theory [40–42], there is time consumed for every customer in the queue, and the waiting time follows the exponential distribution. In information security [43,44], we can take full advantage of the time lag on nodes to defend the hackers capturing our secret information when they survey the structure of network. Time consumed on nodes has a determinant role in the transport properties of complex systems. Thus, all these situations make it of great importance to investigate the time lag information flow in networks.

In this paper, a co-processing model is proposed to investigate the information flow with time lag in communication networks. The master equation of this diffusion process is derived, which can reflect the relationship between the transition rate matrix and the transition probability matrix. Based on the master equation, the stationary distribution and the mean first passage time between any two nodes are obtained, which can measure the centrality of the nodes in the process. When the ability of nodes is not powerful enough to deal with the arrived packets in this diffusion process, the congestion occurs so that the information packet cannot be transferred to its destination efficiently. At the end, simulations on the co-processing model are carried out on scale-free networks, which suggest that congestion takes place facilely in the classical diffusion process and can be avoided by adjusting the system locally in our model for scale-free networks.

2. Co-processing model in communication networks

Regarding the information packet as a particle, we consider a co-processing model in a connected network with the nodes $S = \{1, 2, \cdots, N\}$: after staying at the initial state i_0 for time t_0, the particle hops to a neighbor node i_1 at time t_0 with probability $t_{i_0 i_1}$; it will hops to node i_2 with probability $t_{i_1 i_2}$ at time t_1, with the sojourn time $t_1 - t_0$ at node i_1, and so on.

Denote $P(t) = (p_{ij}(t))_{N\times N}$ as the probability transition matrix for this time lag process $\{X(t) \in S, t \geq 0\}$, where $p_{ij}(t)$ is the probability that the particle reaches node j at time t starting from node i. We consider that $p_{ij}(t)$ only relies on the time interval t, but not the initial time. Neglecting the time lag at every node, the jumps can be recorded by a random walk with discrete probability transition matrix $T = (t_{ij})_{N\times N}$, where t_{ij} is the probability of the jump from node i to node j, then this random walk is the discrete representation of this process and T is the represent matrix.

By the knowledge of stochastic process, the transition matrix $P(t) = (p_{ij}(t))_{N\times N}$ of $\{X(t)\}$ must satisfy the following properties,

(1) $\quad 0 \leq p_{ij}(t) \leq 1\,, \quad p_{ij}(0) = \delta_{ij}\,,$
(2) $\quad p_{ij}(t+s) = \sum_k p_{ik}(t) p_{kj}(s)\,,$
(3) $\quad \lim_{t\to 0+} p_{ij}(t) = p_{ij}(0) = \delta_{ij}\,.$

For the properties above, we can derive that

$$\lim_{t\to 0+} \frac{p_{ij}(t) - \delta_{ij}}{t} = r_{ij}\,,$$

in which the limit r_{ij} does exist and $r_{ij} < \infty$ for all $i, j \in S$.

For

$$\sum_{i\neq j} \frac{p_{ij}(t)}{t} = \frac{1 - p_{ii}(t)}{t}\,,$$

then

$$\sum_{i\neq j} r_{ij} = -r_{ii} \doteq r_i\,.$$

$\{r_{ij},\ i \neq j\}$ reflects the rate of the transition probability from node i to j, and $R = (r_{ij})_{N\times N}$ is the transition rate matrix of the process $\{X(t)\}$.

By property (2)

$$p_{ij}(t + \triangle t) = \sum_{k\in S} p_{ik}(t) p_{kj}(\triangle t)\,,$$

the *master equation* for this time lag process $\{X(t)\}$ can be derived

$$p'_{ij}(t) = \sum_{k\in S} p_{ik}(t)\, r_{kj}\,. \tag{1}$$

In practice, it is difficult to determine the transition matrix $P(t) = (p_{ij}(t))_{N\times N}$. However, the rate matrix $R = (r_{ij})_{N\times N}$ for this process consists of the differential coefficient of $\{p_{ij}(t)\}$ at $t = 0$, and it is easy to measure $\{p_{ij}(t)\}$ nearby $t = 0$. Usually, we get $R = (r_{ij})_{N\times N}$ at first, and then deduce the probability transition matrix $P(t)$ according to equation (1).

In the following, an interpretation of r_i will be given to have a good understanding of the transition rate matrix R. Denote τ as the time the particle firstly departs from the initial node i, then

$$\begin{aligned} P\{\tau > t|X(0) = i\} &= P\{X(u) = i, 0 < u < t|X(0) = i\} \\ &= \lim_{n\to\infty} P\{X(kt/2^n) = i, k = 1, 2, \cdots, 2^n|X(0) = i\} \\ &= \lim_{n\to\infty} [p_{ii}(t/2^n)]^{2^n} \\ &= \lim_{n\to\infty} \exp\left(\frac{\ln p_{ii}(t/2^n)}{t/2^n}\,\frac{t}{2^n}\,2^n\right) \\ &= \exp(-r_i t)\,. \end{aligned}$$

This formula illustrates that the sojourn time at node i follows the exponential distribution with parameter r_i and r_i, which is determined by the handling ability of node i, also determines the transition rate that the particle departs from node i.

Supposing that j and k are neighbors of node i, the relation between t_{ij} and t_{ik} is

$$\frac{t_{ij}}{t_{ik}} = \lim_{t\to 0+} \frac{p_{ij}(t)}{p_{ik}(t)} = \frac{r_{ij}}{r_{ik}}$$

and $r_i = -r_{ii} = \sum_{i\neq j} r_{ij}$, then

$$t_{ij} = \frac{r_{ij}}{r_i}\,.$$

At every jump, the particle hops to node j from node i with probability $t_{ij} = r_{ij}/r_i$.

Therefore, this process $X(t)$ is that the particle hops to node j from node i with probability $t_{ij} = r_{ij}/r_i$ in discrete time series, and the time it stays at the node i before hopping to node j follows the exponential distribution with parameter r_i.

2.1. The stationary distribution and mean first passage time in the process

As the probability in the stationary state reflects the centrality for nodes, the stationary distribution is explored in the following. For the connected network, the stationary distribution $\{\mu_j = \lim_{t\to\infty} p_{ij}(t), \forall\, j \in \boldsymbol{S}\}$ uniquely exists (not relying on the initial state i [21]) and satisfies

$$(1) \qquad \mu P = \mu, \forall\, t \geq 0\,,$$

$$(2) \qquad \sum_i \mu_i = 1\,. \tag{2}$$

For

$$\lim_{t\to 0+} \frac{p_{ij}(t) - \delta_{ij}}{t} = r_{ij}\,,$$

μ satisfies

$$\mu R = 0\,,$$

with the transition rate matrix R, and the stationary probability can be derived.

For the probability in the stationary distribution is the probability that we can find the particle in the steady state regime, it equals the number of times that the particle passes the node multiplies the mean sojourn time at the node. Then, the relation of the stationary distribution between the process $\{\mu_i\}_{i=1}^N$ and the represent random walk with transition matrix T of this process $\{P_i^\infty\}_{i=1}^N$ [21] is

$$\mu_i \propto P_i^\infty \times \frac{1}{r_i} = \frac{P_i^\infty}{r_i}\,, \tag{3}$$

which reflects that the probability in the stationary state relies on not only the times that the particle arrives at the node, but also the sojourn time the particle stays at this node. This can be used against the attack of the hacker, that is, we could just adjust the sojourn time at some node to modify the stationary distribution to against the hacker capturing the secrete information.

To reveal the impact of the time lag on the transition efficiency of the process, we investigate the relationship between the mean first passage time and the stationary distribution.

Denote σ_{ij} as the first passage time from node i to node j and $E\sigma_{ij}$ as the mean first passage time (MFPT) from node i to node j.

For $i = j$, $1/E\sigma_{ii}$ is the mean times of the particle returning to node i itself in unit time. For each arrival, the average time lag at node i is $1/r_i$, thus there exists

$$\mu_i = \frac{1}{r_i E\sigma_{ii}}\,,$$

that is,

$$E\,\sigma_{ii} = \frac{1}{r_i \mu_i}\,.$$

For $i \neq j$, the MFPT from i to j is the sum of the mean time lag at node i and the mean MFPT from the neighbors of i to j, so

$$E\,\sigma_{ij} = \frac{1}{r_i} + \sum_{k\neq i, k\neq j} t_{ik} E\sigma_{kj}\,, \qquad i \neq j\,. \tag{4}$$

With the expression of $E\sigma_{ii}$, for each pair (i, j), $E\sigma_{ij}$ can be calculated accurately.

As an application in information security, to protect the information against the hacker, an information process with time lag can be introduced as follows: the particle at node i hops to its neighbors with equal probability $1/k_i$ and the time it stays at node i follows the exponential distribution with parameter k_i^α, in which k_i is the degree of node i and α is a parameter reflecting the handling ability of the nodes. The discrete representation corresponds to the classical unbiased random walk [21] and the transition rate matrix of this time lag process is

$$R = \begin{pmatrix} -k_1^\alpha & A_{12}k_1^{\alpha-1} & \cdots & A_{1N}k_1^{\alpha-1} \\ A_{21}k_2^{\alpha-1} & -k_2^\alpha & \cdots & A_{2N}k_2^{\alpha-1} \\ \cdots & \cdots & \cdots & \cdots \\ A_{N1}k_N^{\alpha-1} & A_{N2}k_N^{\alpha-1} & \cdots & -k_N^\alpha \end{pmatrix}.$$

The stationary distribution μ and the MFPT from node i to itself $E\sigma_{ii}$ are separately

$$\begin{aligned} \mu &= \left(\frac{k_1^{1-\alpha}}{\sum_i k_i^{1-\alpha}}, \frac{k_2^{1-\alpha}}{\sum_i k_i^{1-\alpha}}, \dots, \frac{k_N^{1-\alpha}}{\sum_i k_i^{1-\alpha}} \right), \\ E\sigma_{ii} &= \frac{1}{\mu_i r_i} = \frac{\sum_l k_l^{1-\alpha}}{k_i}\,. \end{aligned} \tag{5}$$

For $i \neq j$, by equation (3)

$$E\sigma_{ij} = \frac{1}{k_i^\alpha} + \sum_{k\neq i, k\neq j} \frac{A_{ik}}{k_i}\, E\sigma_{kj}\,.$$

For $\alpha > 1$, nodes with larger degree have weaker centrality, and *vice versa*, which is reflected by the stationary distribution. To act against the attack of hacker, we can adjust the handling ability of the node locally to strengthen the robustness and optimize the function of the network.

2.2. Congestion in the co-processing model

Congestion [45,46] is of utmost importance on the communication in networks. In this section, we will discuss how the time lag determine the emergence of congestion in the co-processing model.

Consider m particles in the co-processing model without interaction between each other and the tolerance time of each particle for node i is l_i. If the waiting time of a new particle arrived at node i exceeds the tolerance time l_i, the transmission of this particle is noneffective, and then many particles mass together at node i, what results in the jam at node i. Furthermore, congestion will occur in this system when lots of nodes are in the state of jamming.

If there is no congestion in the network, the mean occupation number of particles at node i is $m\mu_i$ in the stationary state. Then the jam happens at node i when

$$m\mu_i \times \frac{1}{r_i} > l_i \,, \tag{6}$$

and $W = \sum_i \Theta(m\mu_i - r_i l_i)$ is the number of jammed nodes, where

$$\Theta(x) = \begin{cases} 1\,, & x > 0\,, \\ 0\,, & \text{otherwise}\,. \end{cases}$$

In the application above,

$$W = \sum_i \Theta\left(\frac{mk(i)^{1-\alpha}}{\sum_i k(i)^{1-\alpha}} - k_i^{\alpha} l_i\right) .$$

For simplicity, we take $l_i = C$ for every node. When $\alpha > 1$, the mean occupation number of particles at nodes with smaller degree is larger and the handling ability of such node is weaker, thus the jam can easily occur at such smaller degree nodes; while for $\alpha < 0$, the mean occupation number of particles at nodes with larger degree is larger and the handling ability of such node is weaker, thus the jam can easily occur at such larger degree nodes. By the calculation of W, the congestion takes place when W is proportional to the network scale N. Thus, adjusting the handling ability α of nodes can avoid the congestion phenomenon in communication networks. Numerical simulations in specific cases are given in the next section.

3. Numerical simulations on scale-free networks

We generate scale-free networks with $N = 1\,000$ nodes and degree distribution $p(k) \sim k^{-\gamma}$, the rule of which is preferential attachment: at each time step, a new node is added creating 3 links with other nodes which are selected with probability proportional to their degree [47].

3.1. Numerical simulations on stationary distribution and mean first passage time

Consider the complete unbiased time lag process in this network, and every node has the same handling ability. That is, the particle at node i hops to its neighbors with equal probability $1/k_i$, and the time it stays at node i follows the exponential distribution with parameter 1, which is just the case $\alpha = 0$ of the process in Sec. 2.1. Thus the transition rate matrix of this process is

$$R = \begin{pmatrix} -1 & \frac{A_{12}}{k_1} & \cdots & \frac{A_{1N}}{k_1} \\ \frac{A_{21}}{k_2} & -1 & \cdots & \frac{A_{2N}}{k_2} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{A_{N1}}{k_N} & \frac{A_{N2}}{k_N} & \cdots & -1 \end{pmatrix},$$

and the stationary distribution and the MFPT are separately

$$\mu_i = \frac{k_i}{\sum k_i}, \qquad E\sigma_{ii} = \frac{\sum_i k_i}{k_i},$$

which are the same as that of the unbiased random walk on the network [21]. Therefore, the sojourn time of random walk on this network is defaulted as unit time for every node. For $i \neq j$, $\{E\sigma_{ij}\}$ satisfies

$$E\sigma_{ij} = 1 + \sum_{k\neq i, k\neq j} \frac{A_{ik}}{k_i} E\sigma_{kj}.$$

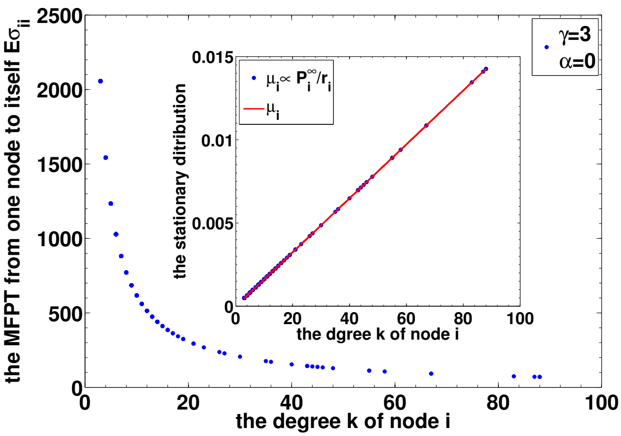

Fig. 1. MFPT from node i to itself, $E\sigma_{ii}$ as a function of degree k_i is shown in the main graph, and the subgraph is the theoretical result for μ_i which is shown in the red line, and the regularity result for P_i^∞/r_i is shown in the (blue) dot, changing with the degree k_i in the case of $\alpha = 0$.

The MFPT $E\sigma_{ii}$ from node i to itself and the stationary distribution μ_i as functions of the degree of node i are shown in Fig. 1. Then, it is verified that the proportional relationship in Eq. (2) is correct in this model. Although every node has the same handling ability and the particle hops to its neighbors with equal probability, the nodes with larger degree possess higher centrality and shorter MFPT for their better connectivity, and *vice versa*.

The average MFPT from other nodes to node i

$$\langle E\,\sigma_{ji}\rangle_i = \frac{\sum_j E\sigma_{ji}}{N}$$

and from node i to other nodes

$$\langle\, E\sigma_{ij}\rangle_i = \frac{\sum_j E\sigma_{ij}}{N}$$

as functions of k_i are shown in Fig. 2. For the better connectivity of nodes with larger degree, the average MFPT from other nodes to these nodes is shorter than those with smaller degree. However, it is worth noting that the MFPT $E\sigma_{ji}$ does not just rely on the degree [21] and nodes with the same degree can have different average MFPT $\langle E\sigma_{ji}\rangle_i$. As every node has the same handling ability and the particle hops to every neighbor with the same probability, the average MFPT $\langle E\,\sigma_{ij}\rangle_i$ from one node to its neighbors is in disorder with the degree.

As another case, the unbiased time lag process with different handling ability for different nodes is studied in the following. That is, the particle at node i hops to its neighbors with equal probability $1/k_i$ and the time

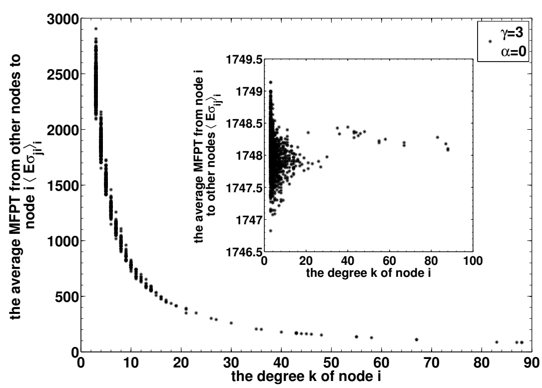

Fig. 2. The main graph is the average mean first passage time from other nodes to node i $\langle E\sigma_{ji}\rangle_i$ and the subgraph is the average mean first passage time from node i to other nodes $\langle E\sigma_{ij}\rangle_i$, both changing with the degree k_i in the case of $\alpha = 0$.

it stays at node i follows the exponential distribution with parameter k_i, which is just the case $\alpha = 1$ of the process in Sec. 2.1. This is exactly the situation that the large degree node has strong handling ability, which can avoid congestion in this process. Then

$$R = \begin{pmatrix} -k_1 & A_{12} & \cdots & A_{1N} \\ A_{21} & -k_2 & \cdots & A_{2N} \\ \cdots & \cdots & \cdots & \cdots \\ A_{N1} & A_{N2} & \cdots & -k_N \end{pmatrix}.$$

The stationary distribution and MFPT are

$$\mu_i = \frac{1}{N}, \qquad E\sigma_{ii} = \frac{N}{k_i}.$$

For $i \neq j$, $\{E\sigma_{ij}\}$ satisfies

$$E\sigma_{ij} = \frac{1}{k_i} + \sum_{k\neq i, k\neq j} \frac{A_{ik}}{k_i} E\sigma_{kj}.$$

The MFPT $E\sigma_{ii}$ from node i to itself and the stationary distribution μ_i as functions of k_i are shown in Fig. 3. For the handling ability is proportional to the degree of the nodes, although particle passes through large degree nodes with high probability for their better connectivity, the time the particle staying at the node is inverse to the degree which counteracts the attraction

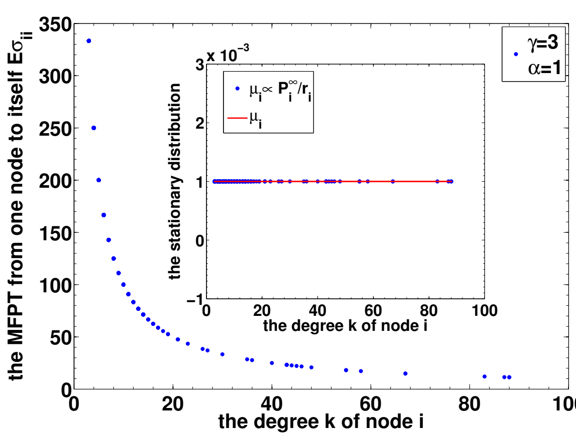

Fig. 3. MFPT from node i to itself $E\sigma_{ii}$ as a function of degree k_i is shown in the main graph, and the subgraph is the theoretical result for μ_i which is shown in the continuous (red) line, and the regularity result for P_i^∞/r_i shown in the (blue) dot, changing with the degree k_i in the case of $\alpha = 1$.

of the large degree. By the above calculation of the stationary distribution, it is obtained that the probability to find the particle in the stationary state is equal for every node.

The average MFPT from other nodes to node i $\langle E\sigma_{ji}\rangle_i$ and from node i to other nodes $\langle E\sigma_{ij}\rangle_i$ as functions of the degree of node i are shown in Fig. 4. The $\langle E\sigma_{ji}\rangle_i$ has the same regularity as that in Fig. 2, which is shown in the main graph of Fig. 4. As the handling ability of the nodes with larger degree is stronger and the rate that the particle departs from them is larger, the average MFPT $\langle E\sigma_{ij}\rangle_i$ from these nodes to other nodes are shorter than those of smaller nodes, which is shown in the subgraph of Fig. 4. Furthermore, the average MFPT $\langle E\sigma_{ji}\rangle_i$ from other nodes to one node in this case is shorter than those in the complete unbiased information process with time lag in Fig. 2, for both the transition rate and the handling ability in this case are stronger than the unbiased one.

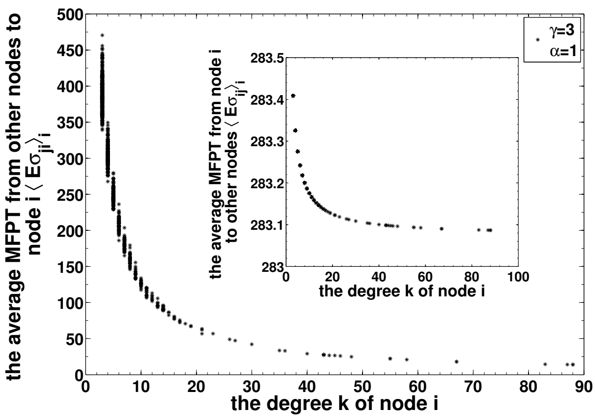

Fig. 4. The main graph is the average mean first passage time from other nodes to node i $\langle E\sigma_{ji}\rangle_i$ and the subgraph is the average mean first passage time from node i to other nodes $\langle E\sigma_{ij}\rangle_i$, both changing with the degree k_i in the case of $\alpha = 1$.

3.2. Simulation results of congestion in communication networks

Consider the time lag process in this generated scale-free networks. For the mean degree of this network is $\langle k\rangle \approx 6$, and the network can deal with about N$\langle k\rangle^\alpha$ particles at the same time, we can take $m \in [1,\ 4\times 10^4]$ particles without interactions among each other in this process. The tolerance time for every node is taken as $l_i = C = 1$ for simplicity. Take $z = 10^7$ times of jumps for every particle, and then record the number of particles $n(i)$ at node i in the stationary state. According to equation (4), if $n(i) > k(i)^\alpha$, the jam happens at node i.

The number of jammed nodes as a function of the number of particles with different parameters $\alpha = -1, 0, 1, 2, 3$ is shown in Fig. 5. For every α, this system undergoes a sharp convention from free diffusion state to congestion with the increase number of particles. For $\alpha > 0$, the handling ability for every node increases along with α increasing, then the congestion occurs only when the particle number is sufficiently large. While for $\alpha < 0$, almost all the particles concentrate at nodes with larger degree which occupy a smaller proportion in scale-free network, and then the congestion takes place requiring more particles than the case $\alpha = 0$. Anyway, with the particle number increasing in the system, the congestion takes place firstly in the complete unbiased time lag process with $\alpha = 0$.

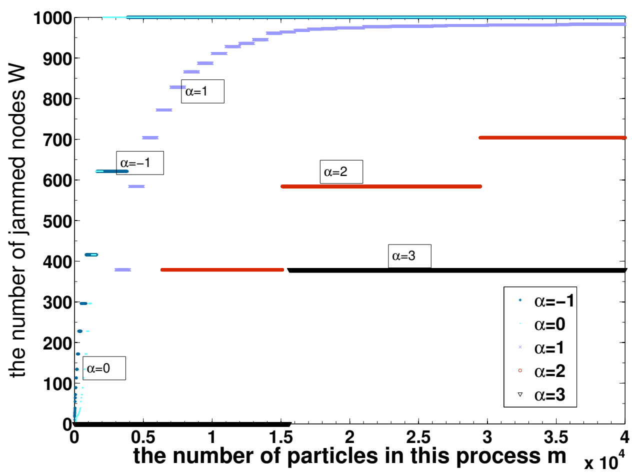

Fig. 5. The number of jammed nodes as a function of the number of particles in this time lag information process.

The number of jammed nodes as a function of the parameter α with different particle numbers $m = 10^3, 10^4, 10^5, 10^6$ is shown in Fig. 6. It is suggested that the most easy-congestion case in this process is at $\alpha = 0$, that is every node has the same handling ability. For $\alpha < 0$, more particles concentrate on nodes with larger degree in the stationary state, while the handling ability for these nodes is worse, then congestion takes place at such nodes. However, nodes with larger degree occupy a smaller proportion in scale-free network, then the number of jammed nodes decreases along with the decreasing of α for $\alpha < 0$. It can be used to control the disease diffusion, that is, we can weaken the communication (handling ability) for active persons (with more neighbors) so that the congestion only occurs on the active persons and the disease cannot diffuse widely. While for $\alpha > 0$, especially for $\alpha > 1$, the handling ability for every node is stronger with

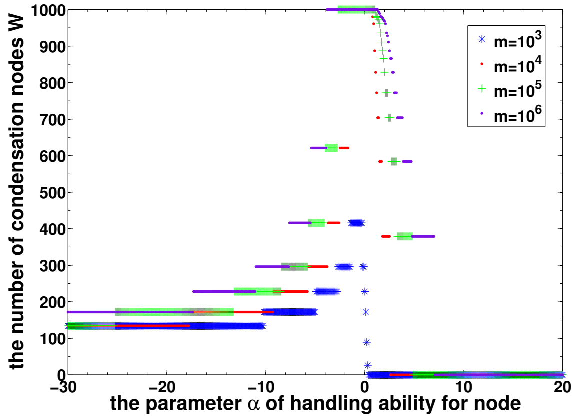

Fig. 6. The number of jammed nodes as a function of the parameter of the handling ability.

the increase of α, and particles stay at nodes with smaller degree. However, such nodes are in majority in scale-free networks and they can share the handling of the particles, so the number of jammed nodes decreases along with the increasing of α for $\alpha > 0$. In summary, the most easy-congestion case in this process for scale-free network is the case $\alpha = 0$, which is just the classical diffusion process, so we would be better to design protocols for information transmission with $\alpha \neq 0$.

By adjusting the parameter α and according to the actual demand, we can formulate the optimal time lag process to optimize the transmission function of the network at the lease cost.

4. Conclusion

In summary, we study the information flow with time lag in communication network, which is a dynamical process with continuous time and discrete states. In this process, the particle hops on the nodes in discrete time series, and it stays at the current node for some time before it hops to the next one. In this diffusion process, the master function of this process, stationary distribution and the mean first passage time are derived. Then, the congestion is well studied to optimize the transmission function of the network at the least cost by adjusting the handling ability of the nodes. In this paper, we just consider the handling ability relying on the degree of the nodes. In practice, the handling ability is not always a constant, and determined by actual requirement, such as the clustering coefficient [47] and the betweenness centrality [48], which will be proceeded in our future work.

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