

ON THE IMPACT OF THE ATMOSPHERIC DRAG
ON THE LARES MISSIONLORENZO IORIO[†]

INFN — Sezione di Pisa

Edificio C — Polo Fibonacci Largo B. Pontecorvo 3, 56127 Pisa, Italy

lorenzo.iorio@libero.it

(Received October 6, 2009; revised version received January 17, 2010)

The goal of the recently approved space-based LARES mission is to measure the general relativistic Lense–Thirring effect in the gravitational field of the spinning Earth at a repeatedly claimed 1% accuracy by combining its node Ω with those of the existing LAGEOS and LAGEOS II laser-ranged satellites. In this paper we show that, in view of the lower altitude of LARES ($h = 1450$ km) with respect to LAGEOS and LAGEOS II ($h \simeq 6000$ km), the cross-coupling between the effect of the atmospheric drag, both neutral and charged, on the inclination of LARES and its classical node precession due to the Earth's oblateness may induce a 3–9% year⁻¹ systematic bias on the total relativistic precession. Since its extraction from the data will take about 5–10 years, such a perturbing effect may degrade the total accuracy of the test, especially in view of the large uncertainties in modeling the drag force.

PACS numbers: 04.80.Cc, 91.10.Sp, 94.05.Hk, 91.10.Qm

1. Introduction

The LARES (LAsER Relativity Satellite) satellite, recently approved¹ by the Italian Space Agency, should have been launched at the end of² 2009 with a VEGA rocket in a circular orbit inclined by 71.5 deg to the Earth's equator at an altitude of³ 1450 km (Barbagallo 2008). Its goal is

[†] Address for correspondence: Viale Unità di Italia 68, 70125, Bari (BA), Italy.

¹ See <http://www.asi.it/SiteEN/MotorSearchFullText.aspx?keyw=LARES>

² See http://www.esa.int/esapub/bulletin/bulletin135/bul135f_bianchi.pdf
Actually, the launch date has been postponed to the end of 2010 or the beginning of 2011. <http://www.spacenews.com/civil/100115-asi-expects-budget-remain-flat-2010.html>

³ In its originally proposed configuration (Ciufolini 1986) the semi-major axis of LARES was equal to that of LAGEOS, *i.e.* $a = 12270$ km.

a measurement of the general relativistic gravitomagnetic Lense–Thirring effect (Lense, Thirring 1918) due to the Earth’s rotation at a repeatedly claimed $\simeq 1\%$ level of accuracy in conjunction with the existing LAGEOS and LAGEOS II laser-ranged satellites which fly at much higher altitudes, *i.e.* $h \simeq 6000$ km. The observable is a suitable linear combination of the longitudes of the ascending nodes Ω of the three satellites, because the gravitomagnetic field of the Earth induces a secular precession on such a Keplerian orbital element

$$\dot{\Omega}_{\text{LT}} = \frac{2GL}{c^2 a^3 (1 - e^2)^{3/2}}, \quad (1)$$

where G is the Newtonian gravitational constant, L is the Earth’s spin angular momentum, c is the speed of light in vacuum, a is the satellite’s semi-major axis and e is its eccentricity. In Table I we quote the Lense–Thirring precessions for LAGEOS, LAGEOS II and LARES: their magnitudes are of the order of 10^1 – 10^2 milliarcseconds per year (mas yr $^{-1}$ in the following).

TABLE I

LAGEOS, LAGEOS II and LARES: orbital parameters and node precessions due to the terrestrial gravitomagnetic field and the first two even zonal harmonics for $L_{\oplus} = 5.86 \times 10^{33}$ kg m 2 s $^{-1}$ (McCarthy, Petit 2004), $J_2 = 0.00108263538$, $J_4 = -1.619989 \times 10^{-6}$ (Tapley *et al.* 2007).

Satellite	a (km)	e	I (deg)	$\dot{\Omega}_{\text{LT}}$ (mas yr $^{-1}$)	$\dot{\Omega}_{J_2}$ (mas yr $^{-1}$)	$\dot{\Omega}_{J_4}$ (mas yr $^{-1}$)
LAGEOS	12270	0.0045	109.9	30.7	4.538082658×10^8	-2.501490×10^5
LAGEOS II	12163	0.014	52.65	31.5	-8.303252509×10^8	9.05051×10^4
LARES	7828	0.0	71.5	118.1	$-2.0298207310 \times 10^9$	2.8925357×10^6

The much larger classical secular precessions induced on Ω by the even zonal harmonic coefficients J_{ℓ} , $\ell = 2, 4, 6, \dots$ of the multipolar expansion of the terrestrial gravitational potential accounting for the centrifugal oblateness of our planet (Kaula 1966) are a major source of systematic uncertainty. They can be written as

$$\dot{\Omega}^{\text{obl}} = \sum_{\ell=2} \dot{\Omega}_{\ell} J_{\ell}, \quad (2)$$

where the coefficients $\dot{\Omega}_{\ell}$ depend on the Earth’s mass M and equatorial radius R , and of the orbital geometry of the satellite through a , e and the inclination I of the orbital plane to the Earth’s equator. Since they have the same temporal signature of the relativistic effect of interest, they cannot

be subtracted from the signal without affecting the recovery of the Lense–Thirring effect itself. Thus, it is of the utmost importance to realistically assess the uncertainty in them in order to evaluate their percent impact on the gravitomagnetic shift. More specifically, the magnitude of the node secular precessions due to the first two even zonals for LAGEOS, LAGEOS II and LARES are listed in Table I. It can be noted that the J_2 -induced rates are of the order of 10^8 – 10^9 mas yr $^{-1}$, *i.e.* seven orders of magnitude larger than the Lense–Thirring precessions. The perturbations by J_4 are of the order of 10^5 – 10^6 mas yr $^{-1}$, *i.e.* four orders of magnitude larger than the gravitomagnetic effects. Such figures immediately demonstrate the difficulty of determining a smallish relativistic effect with respect to a huge classical one, which needs, thus, to be accounted for with the appropriate accuracy (about one part in 10^{10}), or, as combining the three satellite data sets aims at (see Section 3), removed from the signal with the same accuracy.

Up to now major efforts have been devoted to evaluate the bias due to the lingering uncertainty δJ_ℓ in the even zonals according to

$$\delta \dot{\Omega}_{J_\ell}^{\text{obl}} \leq \sum_{\ell=2} |\dot{\Omega}_\ell| \delta J_\ell. \quad (3)$$

A reliable evaluation of such a corrupting effect is made difficult by the fact that the relatively low altitude of LARES brings into play more even zonals than done by LAGEOS and LAGEOS II (Iorio 2009).

Concerning the non-conservative orbital perturbations (Milani *et al.* 1987) like direct solar radiation pressure, Earth’s albedo, direct Earth’s infrared radiation, atmospheric drag, thermal effects like the Yarkovski–Schach and Rubincam ones, they have been so far regarded as a minor concern because their direct impact on the node of the LAGEOS-type satellites is $\lesssim 1\%$ of the Lense–Thirring effect (Lucchesi 2001, 2002).

In this paper we want to investigate their indirect effects through the cross-coupling (Kaula 1966)

$$\delta \dot{\Omega}_I^{\text{obl}} \leq \left| \frac{\partial \dot{\Omega}_\ell}{\partial I} J_\ell \right| \delta I \quad (4)$$

between the zonals-induced node precessions and certain non-gravitational perturbations affecting the LARES inclination. We will show that, in particular, the impact of the atmospheric drag on I_{LR} may play an important role in the evaluation of the error budget of the Lense–Thirring test. Indeed, although the direct secular effect of the atmospheric drag on the node vanishes, it is not so for the indirect one due to the non-vanishing secular decrease of the inclination which maps onto a node effect. Moreover, we will point out that it should not be possible to correct the signal for the measured

value of I arc by arc without likely affecting the gravitomagnetic signal of interest itself. Clearly, since the non-gravitational perturbations do depend on the particular type of satellite considered and since LARES has not yet been launched in orbit, our investigation should not be required to be more accurate than it can be in the sense that it must be viewed as a reasonable sensitivity analysis pointing out a possible source of potential bias and evaluating conservatively the largest possible size of the effect examined.

The paper is organized as follows. In Section 2 we calculate the secular rate of the inclination of a LAGEOS-type satellite induced by a drag force and compute it for LARES. In Section 3 we calculate its indirect effect on the Lense–Thirring shift through the secular precession due to the even zonal harmonics. We also briefly discuss other non-gravitational perturbations which may cause a secular variation of the LARES inclination in Section 4. Section 5 is devoted to the conclusions.

2. The effect of the atmospheric drag on the inclination of LARES

The Gauss equation for the variation of the inclination I is (Milani *et al.* 1987)

$$\frac{dI}{dt} = \frac{r \cos u}{na^2 \sqrt{1-e^2}} A_\nu, \quad (5)$$

where $n \doteq \sqrt{GM/a^3}$ is the unperturbed Keplerian mean motion, $u \doteq g + f$ is the argument of latitude, defined as the sum of the argument of pericentre g , which fixes the position of the pericentre with respect to the line of the nodes, and the true anomaly f which reckons the instantaneous position of the spacecraft from the pericentre, and A_ν is the out-of-plane component of the perturbing acceleration \mathbf{A} .

The drag force per unit mass is (King-Hele 1987)

$$\mathbf{A}_D = -\frac{1}{2} C_D \Sigma \rho \mathbf{V} \mathbf{V}, \quad (6)$$

where

$$\mathbf{V} = \mathbf{v} - \mathbf{V}_A \quad (7)$$

is the satellite velocity with respect to the atmosphere; \mathbf{v} and \mathbf{V}_A are the geocentric satellite and atmosphere velocities, respectively. The other parameters entering Eq. (6) are the drag coefficient C_D , which depends in a complicated way on the interaction between the gas of particles in the surroundings of the satellite and its surface (Afonso *et al.* 1985, Milani *et al.* 1987) $\Sigma \doteq S/m$ is the area-to-mass ratio⁴ of the satellite, and ρ is the density of the atmosphere.

⁴ S denotes the spacecraft cross-sectional area (perpendicular to the velocity).

The velocity of the atmosphere, known as ambient velocity, can be written in terms of geocentric inertial quantities as

$$\mathbf{V}_A = \boldsymbol{\omega}_A \times \mathbf{r}, \quad (8)$$

with

$$\boldsymbol{\omega}_A = (1 + \xi)\boldsymbol{\omega}_\oplus = (1 + \xi)\omega_\oplus \mathbf{k}, \quad (9)$$

where \mathbf{k} is the unit vector of the z -axis in an inertial geocentric frame chosen aligned with the Earth's angular velocity vector. Note that Eq. (9) accounts for the fact that the atmosphere, in general, does not co-rotate exactly with the Earth; maximum observed deviations from the simplifying assumption of exact co-rotation are of the order of 40% (King-Hele 1987). Thus,

$$\mathbf{V}_A = \omega_A (-y \mathbf{i} + x \mathbf{j}), \quad (10)$$

where \mathbf{i} and \mathbf{j} are the unit vectors in the reference $\{xy\}$ plane of the geocentric inertial frame which coincides with the Earth's equator; the angle between \mathbf{v} , which lies in the orbital plane, and \mathbf{V}_A is the inclination I .

In order to have the out-of-plane component A_ν of the drag acceleration evaluated onto the unperturbed Keplerian ellipse, to be inserted into the right-hand-side of Eq. (5), \mathbf{V} must be projected onto the $\hat{\mathbf{n}}$ direction of the frame co-moving with the satellite; since

$$\hat{\mathbf{n}} = \sin I \sin \Omega \mathbf{i} - \sin I \cos \Omega \mathbf{j} + \cos I \mathbf{k}, \quad (11)$$

then, by choosing $\Omega = 0$,

$$\mathbf{V}_A \cdot \hat{\mathbf{n}} = -\omega_A x \sin I. \quad (12)$$

Onto the unperturbed orbit

$$x = r \cos u \cos \Omega - \sin u \cos I \sin \Omega, \quad (13)$$

so that it is possible to obtain

$$\mathbf{V}_A \cdot \hat{\mathbf{n}} = -\omega_A r \sin I \cos u. \quad (14)$$

Since

$$\mathbf{v} = v_r \hat{\mathbf{r}} + v_t \hat{\mathbf{t}}, \quad (15)$$

it appears clear that if the Earth's atmosphere did not rotate there would not be any out-of-plane component of the drag acceleration which, instead, exists because $\mathbf{V}_A \cdot \hat{\mathbf{n}} \neq 0$ for non-equatorial orbits. Thus, the out-of-plane component of Eq. (6) is

$$A_\nu = -\frac{1}{2} C_D \Sigma \rho V \omega_A r \sin I \cos u. \quad (16)$$

It turns out that it can be posed (Abd El-Salam, Sehnal 2004)

$$V = |\mathbf{v} - \mathbf{V}_A| \simeq v\sqrt{k_R}, \quad k_R \simeq 1, \quad (17)$$

so that

$$A_\nu \simeq -\frac{1}{2}C_D \Sigma \rho v \omega_A r \sin I \cos u. \quad (18)$$

Concerning the approximations used in Eq. (17) and Eq. (18), they are justified since

$$k_R \doteq 1 + \left(\frac{V_A}{v}\right)^2 - 2\left(\frac{V_A}{v}\right)\cos I, \quad (19)$$

where typically $V_A \simeq 0.5 \text{ km s}^{-1}$ because of Eq. (9) and Eq. (10) (see also Table II), and $v \simeq \sqrt{GM/a} = 7.1 \text{ km s}^{-1}$ for orbital heights of about 1400 km.

By inserting Eq. (18) into Eq. (5) with the un-perturbed relations

$$r = \frac{a(1-e^2)}{1+e\cos f}, \quad v = na\sqrt{\frac{1+e^2+2e\cos f}{1-e^2}}, \quad (20)$$

and integrating over an orbital period P_b by means of

$$\frac{dt}{P_b} = \frac{(1-e^2)^{3/2}}{2\pi(1+e\cos f)^2} df, \quad (21)$$

one finds that there is a non-vanishing secular rate of the inclination to order zero in the eccentricity

$$\left\langle \frac{dI}{dt} \right\rangle = -\frac{1}{4}C_D \Sigma \rho \omega_A a \sin I; \quad (22)$$

it agrees with (6.17) by⁵ Milani *et al.* (Milani *et al.* 1987). In obtaining Eq. (22) we considered the atmospheric density ρ constant over one orbital revolution; since for LARES $P_b = 3.7 \text{ h}$, this is certainly a reasonable assumption. In general, ρ undergoes many irregular and complex variations both in position and time, being largely affected by solar activity and by the heating and cooling of the atmosphere (King-Hele 1987, Abd El-Salam, Sehnal 2004).

According to Table II, the inclination of LARES will experience a secular decrease of

$$\left\langle \frac{dI}{dt} \right\rangle_{\text{LR}} = -3 \times 10^{-9} \text{ rad yr}^{-1} = -0.6 \text{ mas yr}^{-1}, \quad (23)$$

⁵ ΔI in (Milani *et al.* 1987) is the shift per revolution; in order to be confronted with Eq. (22), (6.17) by Milani *et al.* (1987) must be divided by $P_b = 2\pi/n$. By putting $Z \rightarrow 1$ and $v = na$ one recovers just Eq. (22).

where mas stands for milliarcseconds. Concerning the node, whose Gauss variation equation is identical to Eq. (5) with $\cos u$ replaced by $\sin u/\sin I$, it can be shown that there are no secular effects induced by the atmospheric drag on it; the first non-vanishing term is proportional to $e^2 \sin 2g$.

TABLE II

Relevant physical and orbital parameters of the Earth-LARES system. The quoted value for C_D is usually used in literature, but it refers typically to altitudes of some hundreds km; at 1450 km it may be larger (Milani *et al.* 1987). The value of the area-to-mass ratio Σ has been obtained by using for LARES a diameter of $d = 37.6$ cm and a mass of $m = 400$ kg (<http://esamultimedia.esa.int/docs/LEX-EC/CubeSat%20CFP%20issue%201.pdf>). The value of ρ is that for the Ajisai satellite (Sengoku *et al.* 1996) which has a semimajor axis of 7870 km. Concerning the rotation of the atmosphere, the quoted value has been obtained by assuming it is about 20% faster than the Earth itself.

Parameter	Value	Units	Reference
GM_\oplus	$3.986004418 \times 10^{14}$	$\text{m}^3 \text{s}^{-2}$	(McCarthy, Petit 2004)
R_\oplus	6378136.6	m	(McCarthy, Petit 2004)
J_2	0.00108263538	—	(Tapley <i>et al.</i> 2007)
a_{LR}	7828×10^3	m	(Barbagallo 2008)
e_{LR}	0	—	(Barbagallo 2008)
I_{LR}	71.5	deg	(Barbagallo 2008)
C_D	2.2	—	(Abd El-Salam, Sehnal 2004)
Σ	3×10^{-4}	$\text{m}^2 \text{kg}^{-1}$	(See caption)
ρ	1×10^{-15}	kg m^{-3}	(Sengoku <i>et al.</i> 1996)
ω_A	8.750538×10^{-5}	s^{-1}	—

3. The impact of the secular decrease of the inclination on the node precession due to the oblateness

Such a decrease of I_{LR} affects also the secular precession of the spacecraft node due to the oblateness of the Earth which is a major corrupting effect for the Lense–Thirring signal.

Indeed, since

$$\dot{\Omega}_{J_2} = -\frac{3}{2}n \left(\frac{R}{a}\right)^2 \frac{\cos I}{(1-e^2)^2} \frac{J_2}{I} \quad (24)$$

a bias

$$\delta \dot{\Omega}_i = \frac{3}{2}n \left(\frac{R}{a}\right)^2 \frac{\sin I}{(1-e^2)^2} \frac{J_2}{I} \left\langle \frac{dI}{dt} \right\rangle \Delta t \quad (25)$$

occurs. For LARES Eq. (25) yields a shift of 18.8 mas yr^{-1} over one year; since the Lense–Thirring precession of the node of LARES amounts to 118 mas yr^{-1} , the cross-coupling of the inclination perturbation with the oblateness would yield a systematic error of 16% over just one year.

In fact, the data of LARES will be combined with those of the existing LAGEOS and LAGEOS II spacecraft according to the following linear combination of their nodes (Iorio 2005)

$$\begin{aligned} \dot{\Omega}^{\text{LAGEOS}} + c_1 \dot{\Omega}^{\text{LAGEOS II}} + c_2 \dot{\Omega}^{\text{LARES}}, \\ c_1 = 0.358642219, \quad c_2 = 0.075117522 \end{aligned} \quad (26)$$

in order to cancel out the impact of the mismodelling δJ_2 and δJ_4 of the first two even zonal harmonics; the general relativistic prediction of the total Lense–Thirring shift, according to the linear combination of Eq. (26), is 50.7 mas yr^{-1} . The combination of Eq. (26) is based on a strategy put forth for the first time in (Ciufolini 1996). It turns out that the impact of Eq. (22) on Eq. (26) is $3\% \text{ yr}^{-1}$. Note that since c_1 and c_2 are aimed at removing the classical effects by J_2 and J_4 to the needed extent, we released them with nine decimal digits, *i.e.* with the matching accuracy. Indeed, as shown by Table I, the largest classical effect is about seven orders of magnitude larger than the Lense–Thirring precessions, and the claimed accuracy of the proposed test is 1%. In obtaining such a result we treated the coefficients c_1 and c_2 , which depend on the semi-major axes, the eccentricities and the inclinations of the three satellites, as constant numbers; let us check if a conservative uncertainty of the order of⁶ $\delta I_{\text{LR}} \simeq 1 \text{ mas}$ in I_{LR} can affect the numerical values of c_1 and c_2 at the ninth decimal digit. It turns out that

$$\begin{aligned} |c_1(I_{\text{LR}}) - c_1(I_{\text{LR}} + \delta I_{\text{LR}})| &= 1.7 \times 10^{-9}, \\ |c_2(I_{\text{LR}}) - c_2(I_{\text{LR}} + \delta I_{\text{LR}})| &= 3 \times 10^{-10}. \end{aligned} \quad (27)$$

Thus, we can conclude that the uncertainty in determining the inclination of LARES is not a concern about the accuracy required to compute c_1 and c_2 . By repeating the same analysis for the semimajor axis a_{LR} of LARES, it turns out that an uncertainty $\delta a_{\text{LR}} \simeq 1 \text{ cm}$ yields a similar conclusion because

$$\begin{aligned} |c_1(a_{\text{LR}}) - c_1(a_{\text{LR}} + \delta a_{\text{LR}})| &= 5 \times 10^{-10}, \\ |c_2(a_{\text{LR}}) - c_2(a_{\text{LR}} + \delta a_{\text{LR}})| &= 5 \times 10^{-10}. \end{aligned} \quad (28)$$

Let us see what could be the impact of the uncertainties in parameters like C_{D} and ω_{A} on our estimates. For $2 < C_{\text{D}} < 2.5$ we get a substantially unchanged bias $2.7\text{--}3.4\% \text{ yr}^{-1}$. By assuming $\omega_{\text{A}} = \omega_{\oplus} = 7.292115 \times 10^{-5} \text{ s}^{-1}$,

⁶ In fact, it may likely be about one order of magnitude smaller because $\delta r \simeq 1 \text{ cm}$ yields an uncertainty of approximately 0.3 mas at an height of 1450 km .

i.e. by assuming that the atmosphere co-rotates with the Earth, the bias amounts to $2.5\% \text{ yr}^{-1}$. Concerning the approximation of Eq. (17) used for V , *i.e.* $V = v\sqrt{k_R} \simeq v$, it is fully justified in our case. Indeed, $(V_A/v) \cos I$ appearing in it can be approximated with $(\omega_A/n) \cos I$ for a circular orbit; for LARES it amounts to 0.03 only, thus yielding $\sqrt{k_R} = 0.97$. It must be noted that the effect of Eq. (25) should likely affect the LARES data in full because of the difficulty of realistically modelling the drag force, especially C_D and ρ ; just to give an idea of the uncertainty in their values note that when the solar activity is low a typical atmospheric density at about 1500 km altitude is $2 \times 10^{-16} \text{ kg m}^{-3}$ (Sengoku *et al.* 1996), while for the existing LAGEOS satellite the drag coefficient is $C_D \simeq 4.9$ (Afonso *et al.* 1985, Milani *et al.* 1987). The fact that C_D is larger for LAGEOS than for a lower satellite like LARES is only seemingly contradictory (the higher the orbit, the smaller the drag). Indeed, the drag coefficient depends on the ratio between the average thermal molecular speed of the atmosphere V_T and the orbital speed v of the spacecraft. For relatively low orbits, $V_T \simeq 1 \text{ km s}^{-1}$ is typically smaller than⁷ $v \simeq 7.5 \text{ km s}^{-1}$. On the contrary, at higher altitudes the situation changes because v becomes smaller and smaller, while V_T increases rapidly due to higher temperatures and lower mean molecular weight. In this case, as for LAGEOS, (Afonso *et al.* 1985)

$$C_D = b \left[2 + \frac{4}{3} \left\langle \left(\frac{V_T}{v} \right)^2 \right\rangle - \frac{2}{15} \left\langle \left(\frac{V_T}{v} \right)^4 \right\rangle \right], \quad (29)$$

with $b \simeq 1.4$ and $\langle V_T/v \rangle \simeq 0.8$, so that $C_D \simeq 4$. For more details, see the discussion in (Milani *et al.* 1987), pp. 104–107.

In addition to the neutral particle drag considered so far it should also be taken into account the charged particle drag (Afonso *et al.* 1985) due to the fact that a spacecraft moving in a gas of electrons and ions tends to acquire an electric charge because of the collisions with such particles and also because of the photoelectric effect caused by solar radiation. The effect of the charged particle drag can be obtained by re-scaling the one due to the neutral particle drag by a multiplicative factor b containing, among other things, the satellite's potential V_0 . According to Lucchesi and Paolozzi (Lucchesi, Paolozzi 2001), it may amount to about $V_0 = -0.3 \text{ V}$ for LARES, so that $b = 3.1$ which implies a $9\% \text{ yr}^{-1}$ systematic error in the measurement of the Lense–Thirring effect with Eq. (26). It must be pointed out that the reduction of the impact of the perturbing accelerations of thermal origin should have been reached by the LARES team with two concentric spheres. However, as explained by Andrés (Andrés 2007), this

⁷ For LARES (see Table I for its orbital parameters) it is $v = 7.1 \text{ km s}^{-1}$.

solution will increase the floating potential of LARES because of the much higher electrical resistivity, so that the evaluations presented here may turn out to be optimistic.

Since the extraction of the relativistic effect would require a multi-year analysis, typically $\Delta t = 5\text{--}10$ yr, the action of the overall atmospheric drag on the LARES inclination may be a serious corrupting effect over such timescales.

Ciufolini *et al.* (Ciufolini *et al.* 2009) objected that, in fact, the disturbing effect examined would not appear in the real data analysis procedure because the inclination along with all the other Keplerian orbital elements would be⁸ “measured” arc by arc, so that one should only have to correct the signal for the measured value of the inclination; after all, the same problems, if not even larger, would occur with the semimajor axes of the LAGEOS satellites, which are known to undergo still unexplained secular decrease of 1.1 mm d^{-1} (Rubincam 1982) and their consequent mappings onto the node rates. The problem is that while a perturbation Δa pertains the in-plane, radial \mathcal{R} component (Christodoulidis *et al.* 1988) of the LAGEOSs’ orbits, both the Lense–Thirring node precession and the shifts in the inclination affect the out-of-plane, normal \mathcal{N} component (Christodoulidis *et al.* 1988) of the orbit; thus, even if repeated corrections to the semimajor axis could be applied without affecting the gravitomagnetic signal of interest, the same would not hold for the inclination. This is particularly true in view of the fact that, for still unexplained reasons, the Lense–Thirring effect itself has never been estimated, either as a short-arc or as a global parameter. Moreover, Ciufolini *et al.* (Ciufolini *et al.* 2009) claimed that the recent improvements in atmospheric refraction modelling would allow to “measure” the inclination of the LAGEOSs satellites at a level of accuracy, on average, of $30\text{ }\mu$ as for LAGEOS and $10\text{ }\mu$ as for LAGEOS II. Firstly, the tracking of a relatively low satellite is always more difficult than for higher targets, so that caution would be needed in straightforwardly extrapolating results valid for LAGEOS to the still non-existing LARES. Second, it is difficult to understand the exact sense of such claims because they would imply an accuracy $\delta r \simeq a\delta I$ in reconstructing the orbits of LAGEOS and LAGEOS II, on average, of 0.2 cm and 0.06 cm, respectively.

4. Other effects potentially inducing secular variations of the LARES inclination

Among the other non-conservative forces acting on the LAGEOS-type satellites, also the Rubincam (Rubincam 1987) effect, due to the anisotropic

⁸ Actually, the Keplerian orbital elements are not directly measurable quantities, contrary to, *e.g.*, ranges, range-rates, right ascension, declination.

re-emission of the infrared radiation of the Earth along the satellite's spin axis, is important. Such an effect arises from the fact that the retroreflectors of LAGEOS have a significant thermal inertia of about 3000 s; since such time of thermal response is shorter than the satellite's orbital period $P_b = 13526$ s and larger than the spin period, whose nominal value was about 1 s at the launch epoch, the perturbation induces a temperature asymmetry between the hemisphere facing the Earth and the one opposite to it. The illuminated hemisphere becomes hotter than the dark one only after the Earth has passed its pole, causing a time lag effect accounted for by the thermal lag angle θ . A net recoil acceleration A_{Rub} directed along the satellite spin axis occurs. It induces a secular rate of the inclination according to (Lucchesi 2002)

$$\left\langle \frac{dI}{dt} \right\rangle = -\frac{A_{\text{Rub}}}{8na} \sin \theta \sin 2I (3\sigma_z^2 - 1). \quad (30)$$

In it A_{Rub} is the Rubincam acceleration which depends in a complex way on the physical and thermal properties of the satellite and of its array of retro-reflectors, θ is the thermal lag angle, and σ_z is the component of the satellite's spin along the z axis of a geocentric equatorial inertial frame having the x axis along the vernal equinox direction. For LAGEOS II the secular inclination rate is of the order of 1.5 mas yr^{-1} . By assuming for LARES the same value of A_{Rub} as for LAGEOS II, *i.e.* $A_{\text{Rub}} \simeq -7 \times 10^{-12}$ m s^{-2} , Eq. (30) yields an effect of the order of about 0.7 mas yr^{-1} . In fact, it might be finally smaller because of the currently ongoing manufacturing efforts of the LARES team aimed at reducing the impact of the non-gravitational perturbations of thermal origin on the new spacecraft with respect to the LAGEOS satellites (Bosco *et al.* 2007). Moreover, it will depend on the direction of the satellite's spin at the injection in orbit.

5. Conclusions

In this paper we have shown that certain subtle non-gravitational perturbations acting on the forthcoming LARES satellite may corrupt the claimed goal of performing a $\simeq 1\%$ measurement of the Lense–Thirring effect in the gravitational field of the rotating Earth because of the lower altitude of the new spacecraft with respect to the existing LAGEOS and LAGEOS II spacecraft. In particular, the interplay between the node precessions due to the even zonal harmonics of the geopotential, which are a major source of systematic error, and the LARES inclination has been investigated. The atmospheric drag, both in its neutral and charged components, will induce a non-negligible secular decrease of the inclination of the new spacecraft yielding a correction to the node precession of degree $\ell = 2$ which amounts to

3–9% yr^{-1} of the total gravitomagnetic signal pertaining just the node itself. Such a corrupting bias would be very difficult to be modeled. Since the extraction of the relativistic signature will require a data analysis of about 5–10 yr, the effect examined here may yield a degradation of the achievable total accuracy of the test. In principle, also the Rubincam effect, of thermal origin, should be taken into account because it can induce a non-vanishing secular variation of the inclination. Since both the node and the inclination enter the out-of-plane, normal component of the orbit of a satellite, it would not be possible to correct for the measured values of the inclinations arc by arc without likely affecting also the Lense–Thirring signal itself, especially because it has never been estimated along with the other parameters.

I am grateful to an anonymous referee for his/her detailed and helpful comments.

REFERENCES

- Abd El-Salam, F.A., Sehnal, L. *Celest. Mech. Dyn. Astron.* **90**, 361 (2004).
 Afonso, G., *et al.*, *J. Geophys. Res.* **90**, 9381 (1985).
 Andrés, J.I., *Enhanced Modelling of LAGEOS Non-Gravitational Perturbations. PhD Thesis book*. Ed. Sieca Repro Turbineweg, 20, 2627, BP Delft, The Netherlands. ISBN 978-90-5623-081-4, 2007.
 Barbagallo, D., (ESA-ESRIN), personal communication to the author, September 2008.
 Bosco, A. *et al.*, *Int. J. Mod. Phys.* **D16**, 2271 (2007).
 Christodoulidis, D.C., Smith, D.E., Williamson, R.G., Klosko, S.M., *J. Geophys. Res.* **93**, 6216 (1988).
 Ciufolini, I., *Phys. Rev. Lett.* **56**, 278 (1986).
 Ciufolini, I., *Nuovo Cim.* **A109**, 1709 (1996).
 Ciufolini, I., Paolozzi, A., Pavlis, E.C., Ries, J.C., Koenig, R., Matzner, R.A., Sindoni, G., Neumayer, H., *Space Sci. Rev.* **148**, 71 (2009).
 Iorio, L., *New Astron.* **10**, 616 (2005).
 Iorio, L., *Gen. Relativ. Gravit.* **41**, 1717 (2009).
 Kaula, W.M., *Theory of Satellite Geodesy*, Waltham, Blaisdell 1966.
 King-Hele, D.G., *Satellite Orbits in an Atmosphere*, Blackie and Son Ltd., London 1987.
 Lense, J., Thirring, H., *Phys. Z.* **19**, 156 (1918). Translated and discussed in B. Mashhoon, F.W. Hehl, D.S. Theiss, *Gen. Relativ. Gravit.* **16**, 711 (1984).
 Lucchesi, D.M., *Planet. Space Sci.* **49**, 447 (2001).

Lucchesi, D.M., *Planet. Space Sci.* **50**, 1067 (2002).

Lucchesi, D.M., Paolozzi, A., paper presented at XVI Congresso Nazionale AIDAA 24–28 Settembre 2001, Palermo.

McCarthy, D.D., Petit, G., *IERS Conventions (2003)*, Verlag des Bundesamtes für Kartographie und Geodäsie, Frankfurt am Main 2004.

Milani, A., Nobili, A.M., Farinella, P., *Non-gravitational Perturbations and Satellite Geodesy*, Adam Hilger, Bristol 1987.

Rubincam, D.P., *Celest. Mech. Dyn. Astron.* **26**, 361 (1982).

Rubincam, D.P., *J. Geophys. Res.* **92**, 1287 (1987).

Sengoku, A., Cheng, M.K., Schutz, B.E., Hashimoto, H., *J. Geod. Soc. Japan* **42**, 15 (1996).

Tapley, B.D., Ries, J.C., Bettadpur, S., Chambers, D., Cheng, M., Condi, F., Poole, S., American Geophysical Union, Fall Meeting 2007, abstract No. G42A-03.