Vol. 41 (2010) ACTA PHYSICA POLONICA B No 4

BOUNCING UNIVERSE WITH THE NON-MINIMALLY COUPLED QUINTOM MATTER ON THE WARPED DGP BRANE

Kourosh Nozari[a†], M.R. Setare[b‡], Tahereh Azizi[a§]
Siamak Akhshabi[a¶]

[a]Department of Physics, Faculty of Basic Sciences
University of Mazandaran
P.O. Box 47416-95447, Babolsar, Iran
[b]Department of Physics, Payam-e Nour University, Bijar, Iran

(Received August 14, 2009; revised version received 21 December, 2009; final version received 8 March, 2010)

We construct a quintom dark energy model with two non-minimally coupled scalar fields, one quintessence and the other phantom field, confined on the warped DGP brane. We study some important issues such as phantom divide line crossing, existence of the bouncing solutions and the stability of the solutions in this framework. We show that this model accounts for crossing the phantom divide line and realization of the bouncing solutions. This model allows for stability of the solutions in separate regions of the ω–ω' phase-plane.

PACS numbers: 98.80.–k, 95.36.+x, 98.80.Cq

1. Introduction

Despite all of its successes, the standard model of cosmology suffers from a series of problems. The most serious of these problems is the problem of initial singularity because the laws of physics break down at the singularity point. In order to avoid this lawlessness, there is a huge interest in the solutions that do not display divergences. These solutions could be obtained at a classical level or by quantum modifications. Most of the efforts in quantum gravity is devoted to reveal the nature of the initial singularity and to understand the origin of matter, non-gravitational fields, and the

† knozari@umz.ac.ir
‡ rezakord@ipm.ir
§ t.azizi@umz.ac.ir
¶ s.akhshabi@umz.ac.ir

very nature of the spacetime. In recent analysis based on the loop quantum cosmology, the Big Bang singularity is replaced by a quantum Big Bounce with finite energy density of matter. This scenario has strong quantum effects at the Planck scale too. Another motivation to remove the initial singularity is the initial value problem. A sound gravitational theory needs to have a well posed Cauchy problem. Due to the fact that the gravitational field diverges at the singularity, we could not have a well formulated Cauchy problem as we cannot set the initial values at that point.

On the other hand, one of the most important discoveries over the past few years is that we live in a positively accelerated universe which is almost spatially flat [1]. Another remarkable hint of the cosmological observations is that the equation of state parameter (ω) transits from $\omega > -1$ to $\omega < -1$ [2–4]. These discoveries generated renewed interest in bouncing models of the universe because it can be shown that at a positively accelerated universe a necessary condition for a bounce in general relativity is to violate the null energy condition, *i.e.* to have $\rho+p < 0$. To interpret the cosmic acceleration, a so-called dark energy component has been proposed. On the other hand the nature of dark energy is ambiguous. The simplest candidate of dark energy is a cosmological constant with the equation of state parameter $\omega = -1$. However, this scenario suffers from serious problems like a huge fine tuning and the coincidence problem [5]. Alternative models of dark energy suggest a dynamical form of dark energy, which is often realized by one or two scalar fields. In this respect, dark energy components such as quintessence, k-essence, chaplygin gas, phantom and quintom fields have been studied extensively [6] (see also [7] and [8]). Another alternative approach to explain the universe's late-time acceleration is modifying the General Relativity itself [9]. Also, some braneworld scenarios are other successful models to achieve this goal [10]. In a braneworld scenario, our 3-brane is embedded in a higher dimensional bulk. Matter fields are confined to a four dimensional brane while gravity and possibly non-standard matter fields are free to propagate in entire space time. Among the braneworld models, the Randall–Sundrum II (RSII) model is very popular since it has a new modification of the gravitational potential in the very early stages of the universe evolution [11]. On the other hand, the Dvali–Gabadadze and Porrati (DGP) braneworld scenario is a very interesting model which can describe the origin of the late-time accelerating behavior of the universe without adopting any additional mechanism [12]. In this setup, gravity is modified at large distances because of an induced four-dimensional Ricci scalar on the brane. This term can be obtained by the quantum interaction between the matter confined on the brane and the bulk gravitons. The DGP braneworld scenario explains accelerated expansion of the universe via leakage of gravity to extra dimension without need to introduce a dark energy component. While the RSII model produces ultra-violet modification to the General Relativity, the DGP model leads to infra-red modification. By considering the effect of an induced gravity term as a quantum correction in RSII model, we have a combined model that dubbed *warped DGP braneworld* in the literature [13]. This setup gives also a self-accelerating phase in the brane cosmological evolution.

While DGP-inspired models essentially have the capability to explain late-time acceleration, crossing the cosmological constant line and issues such as realization of bouncing solutions and their stability need additional mechanism to be explained in these models. With this viewpoint, in this paper we construct a quintom dark energy model with two scalar fields non-minimally coupled to induced gravity on the warped DGP brane. We study some currently important cosmological issues such as phantom divide line crossing, avoiding singularities by realization of the bouncing solutions and stability of these solutions. We analyze the parameter space of the model numerically and we show that this model allows for stability of the solutions in the separate regions of the ω–ω' phase-plane.

2. Warped DGP brane

The action of the warped DGP model can be written as follows

$$\mathcal{S} = \mathcal{S}_{\text{bulk}} + \mathcal{S}_{\text{brane}}\,, \tag{1}$$

$$\mathcal{S} = \int\limits_{\text{bulk}} d^5X\sqrt{-{}^{(5)}g}\left[\frac{1}{2\kappa_5^2}{}^{(5)}R + {}^{(5)}\mathcal{L}_m\right] + \int\limits_{\text{brane}} d^4x\sqrt{-g}\left[\frac{1}{\kappa_5^2}K^{\pm} + \mathcal{L}_{\text{brane}}(g_{\alpha\beta},\psi)\right]. \tag{2}$$

Here $\mathcal{S}_{\text{bulk}}$ is the action of the bulk, $\mathcal{S}_{\text{brane}}$ is the action of the brane and $\mathcal{S}$ is the total action. X^A with $A = 0,1,2,3,5$ are coordinates in the bulk while x^μ with $\mu = 0,1,2,3$ are induced coordinates on the brane. κ_5^2 is the 5-dimensional gravitational constant. ${}^{(5)}R$ and ${}^{(5)}\mathcal{L}_m$ are the 5-dimensional Ricci scalar and the matter Lagrangian respectively. $K^{\pm}$ is trace of the extrinsic curvature on either side of the brane. $\mathcal{L}_{\text{brane}}(g_{\alpha\beta},\psi)$ is the effective 4-dimensional Lagrangian on the brane. The action $\mathcal{S}$ is actually a combination of the Randall–Sundrum II and DGP model. In other words, an induced curvature term appears on the brane in the Randall–Sundrum II model, hence the name *Warped* DGP Braneworld [13]. Now consider the brane Lagrangian as follows

$$\mathcal{L}_{\text{brane}}(g_{\alpha\beta},\psi) = \frac{\mu^2}{2}R - \lambda + L_m\,, \tag{3}$$

where μ is a mass parameter, R is the Ricci scalar of the brane, λ is the tension of the brane and L_m is the Lagrangian of the other matters localized on the brane. We assume that bulk contains only a cosmological constant, ${}^{(5)}\Lambda$. With these choices, action (1) gives either a generalized DGP or a generalized RS II model: it gives DGP model if $\lambda = 0$ and ${}^{(5)}\Lambda = 0$, and gives RS II model if $\mu = 0$ [13]. The generalized Friedmann equation on the brane is as follows [13]

$$H^2 + \frac{k}{a^2} = \frac{1}{3\mu^2}\left[\rho + \rho_0\Big(1 + \varepsilon\mathcal{A}(\rho, a)\Big)\right], \tag{4}$$

where $\varepsilon = \pm 1$ corresponds to two possible branches of the solutions (two possible embedding of the brane in the AdS$_5$ bulk) in this warped DGP model and $\mathcal{A} = \left[\mathcal{A}_0^2 + \frac{2\eta}{\rho_0}\Big(\rho - \mu^2\frac{\mathcal{E}_0}{a^4}\Big)\right]^{1/2}$, where $\mathcal{A}_0 \equiv \left[1 - 2\eta\frac{\mu^2\Lambda}{\rho_0}\right]^{1/2}$, $\eta \equiv 6\,m_5^6/\rho_0\mu^2$ with $0 < \eta \leq 1$ and $\rho_0 \equiv m_\lambda^4 + 6\,m_5^6/\mu^2$. By definition, $m_\lambda = \lambda^{1/4}$ and $m_5 = k_5^{-2/3}$. $\mathcal{E}_0$ is an integration constant and corresponding term in the generalized Friedmann equation is called dark radiation term. We neglect dark radiation term in forthcoming arguments. In this case, generalized Friedmann equation (4) attains the following form,

$$H^2 + \frac{k}{a^2} = \frac{1}{3\mu^2}\left[\rho + \rho_0 + \varepsilon\rho_0\Big(\mathcal{A}_0^2 + \frac{2\eta\rho}{\rho_0}\Big)^{1/2}\right], \tag{5}$$

where ρ is the total energy density, including energy densities of the scalar fields and dust matter on the brane:

$$\rho = \rho_\varphi + \rho_\sigma + \rho_{\mathrm{dm}}\,. \tag{6}$$

In what follows, we construct a quintom dark energy model on the warped DGP brane.

3. A quintom dark energy model on the warped DGP brane

As a part of matter fields localized on the brane, we consider a quintom field non-minimally coupled to induced gravity on the warped DGP brane. The action of this non-minimally coupled quintom field is given by

$$\mathcal{S}_{\mathrm{quint}} = \int\limits_{\mathrm{brane}} d^4x\sqrt{-g}\Big[-\tfrac{1}{2}\xi R(\varphi^2 + \sigma^2) - \tfrac{1}{2}\partial_\mu\varphi\partial^\mu\varphi + \tfrac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - V(\varphi, \sigma)\Big], \tag{7}$$

where ξ is a non-minimal coupling and R is induced Ricci scalar on the brane. φ is a normal (canonical or quintessence) component while σ is a phantom field. We have assumed a conformal coupling of the scalar fields and induced gravity. Variation of the action with respect to each scalar field gives the equation of motion of that scalar field

$$\ddot{\varphi}+3H\dot{\varphi}+\xi R\varphi+\frac{\partial V}{\partial\varphi}=0\,, \tag{8}$$

and

$$\ddot{\sigma}+3H\dot{\sigma}-\xi R\sigma-\frac{\partial V}{\partial\sigma}=0\,. \tag{9}$$

The energy density and pressure of the quintom field are given by the following relations

$$\begin{aligned}\rho_{\rm quint} &= \rho_\varphi+\rho_\sigma\\ &= \tfrac{1}{2}\left(\dot{\varphi}^2-\dot{\sigma}^2\right)+V(\varphi,\sigma)+6\xi H(\varphi\dot{\varphi}+\sigma\dot{\sigma})+3\xi H^2\left(\varphi^2+\sigma^2\right)\end{aligned} \tag{10}$$

and

$$\begin{aligned}p_{\rm quint} &= p_\varphi+p_\sigma\\ &= \tfrac{1}{2}\left(\dot{\varphi}^2-\dot{\sigma}^2\right)-V(\varphi,\sigma)-2\xi\left(\varphi\ddot{\varphi}+2\varphi H\dot{\varphi}+\dot{\varphi}^2+\sigma\ddot{\sigma}+2\sigma H\dot{\sigma}+\dot{\sigma}^2\right)\\ &\quad -\xi(2\dot{H}+3H^2)\left(\varphi^2+\sigma^2\right)\,.\end{aligned} \tag{11}$$

In what follows, by comparing the modified Friedmann equation in the warped DGP braneworld with the standard Friedmann equation, we deduce the equation of state of the dark energy component. This is reasonable since all observed features of dark energy are essentially derivable in general relativity (see [14] and references therein). The standard Friedmann equation in four dimensions is written as

$$H^2+\frac{k}{a^2}=\frac{1}{3\mu^2}(\rho_{\rm dm}+\rho_{\rm de})\,, \tag{12}$$

where $\rho_{\rm dm}$ is the dust matter density, while $\rho_{\rm de}$ is dark energy density. Comparing this equation with equation (5), we deduce

$$\rho_{\rm de}=\rho_\varphi+\rho_\sigma+\rho_0+\varepsilon\rho_0\Big(A_0^2+2\eta\frac{\rho}{\rho_0}\Big)^{1/2}. \tag{13}$$

The conservation of the quintom field effective energy density can be stated as

$$\frac{d\rho_{\rm quint}}{dt}+3H(\rho_{\rm quint}+p_{\rm quint})=0\,. \tag{14}$$

Since the dust matter obeys the continuity equation and the Bianchi identity keeps valid, total energy density satisfies the continuity equation. In order to solve the Friedmann equation (5) we choose the following potential

$$V(\varphi,\sigma)=(\zeta\varphi\sigma)^2+\tfrac{1}{2}m^2\left(\varphi^2-\sigma^2\right)\,, \tag{15}$$

where ζ is a dimensionless constant describing the interaction between the scalar fields. With this potential, a possible solution of our basic equations, (5), (9) and (10) with supplemented equations (11) and (12) is as follows (see [15] for a similar argument)

$$\varphi = \sqrt{C_0}\cos(mt)\,, \qquad \sigma = \sqrt{C_0}\sin(mt)\,, \tag{16}$$

where C_0 is a parameter with the dimension of mass squared describing the oscillating amplitude of the fields. For a flat spatial geometry on the brane and setting $\rho_{\rm dm} = 0$, if we consider low-energy limit where by assumption $\rho_{\rm de} \ll \rho_0$, we find

$$\left(\frac{\dot{a}}{a}\right)^2 \approx \frac{1}{3\mu^2}\left[(\rho_\varphi + \rho_\sigma)\left(1 + \frac{\varepsilon\eta}{\mathcal{A}_0}\right) + \rho_0(1 + \varepsilon\mathcal{A}_0)\right]. \tag{17}$$

Using (17) in (11), we find

$$\rho_\varphi + \rho_\sigma = \frac{\zeta^2 C_0^2}{4}\sin^2(2mt) + 3\xi H^2 C_0\,. \tag{18}$$

Therefore, Friedmann equation (17) can be rewritten as follows

$$H = \pm\left(\frac{\frac{\zeta^2 C_0^2}{12\mu^2}\sin^2(2mt)(1 + \frac{\varepsilon\eta}{\mathcal{A}_0}) + \frac{\rho_0}{3\mu^2(1+\varepsilon A_0)}}{1 - \frac{\xi C_0}{\mu^2(1+\frac{\varepsilon\eta}{\mathcal{A}_0})}}\right)^{1/2}. \tag{19}$$

There are four possible combinations of signs in this equation. We use this result in our forthcoming arguments. Before proceeding further, we note that one could choose the quantities in the square root in a way that lead to a imaginary Hubble parameter. We avoid such cases in what follows. Also singularity points of H are treated in forthcoming arguments.

3.1. Bouncing behavior of the model

We start with a detailed examination of the necessary conditions required for a successful bounce. During the contracting phase, the scale factor a(t) is decreasing, *i.e.* $\dot{a}(t) < 0$, and in the expanding phase we have $\dot{a}(t) > 0$. At the bouncing point, $\dot{a}(t) = 0$, and around this point $\ddot{a}(t) > 0$ for a period of time [15,16]. Equivalently in the bouncing cosmology, the Hubble parameter H runs across zero from $H < 0$ to $H > 0$ and $H = 0$ at the bouncing point. A successful bounce requires that around this point the following relation should be satisfied

$$\dot{H} = -4\pi G\rho(1 + \omega) > 0\,. \tag{20}$$

So, at the bouncing point the scale factor reaches a non-zero minimum value while the Hubble parameter reaches zero. By solving the Friedmann equation (19) we plot the behavior of the scale factor *versus* the cosmic time, t, for two branches of the solutions. Figure 1 (a) shows the behavior of $a(t)$ for $\varepsilon = +1$ and Fig. 1 (b) shows the case for $\varepsilon = -1$. As one can see, in both branches of this DGP-inspired model, the scale factor reaches a non-zero minimum and the universe switches between expanding and contracting phases alternatively. As we have emphasized, equation (19) has four alternative representations corresponding to four possible combinations of the signs. If we integrate this equation, we find

$$a(t) = a_0 \exp\left[\pm \int \left(\frac{\frac{\zeta^2 C_0^2}{12\mu^2}\sin^2(2mt)(1+\frac{\varepsilon\eta}{\mathcal{A}_0}) + \frac{\rho_0}{3\mu^2(1+\varepsilon \mathcal{A}_0)}}{1 - \frac{\xi C_0}{\mu^2(1+\frac{\varepsilon\eta}{\mathcal{A}_0})}}\right)^{1/2}\right]. \qquad (21)$$

Other possible combinations of signs lead to only a shift in the corresponding figures.

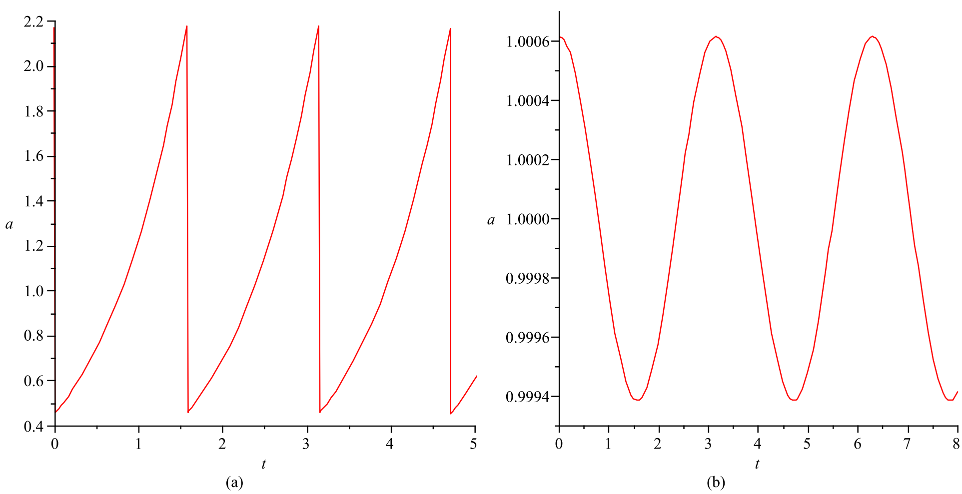

Fig. 1. The evolution of the scale factor for two branches of the warped DGP model with quintom field localized on the brane: (a) Self-accelerating branch of the model (with $\varepsilon = +1$), the universe undergoes an expansion, reaches a maximum radius and then crunches to a finite minimum size and this cycle repeats. There is no bounce at the minimum point since the scale factor has no derivative at that point. (b) Normal branch of the model (with $\varepsilon = -1$). The universe switches alternatively between expanding and contracting phases. The minimum points are the bouncing points.

3.2. Crossing the phantom divide line

In the DGP scenario if we use a single scalar field (ordinary or phantom) on the brane, we can show that the equation of state parameter of dark energy crosses the phantom divide line [17] (see also [14] and [18]). It has been shown that DGP model with a quintom dark energy fluid in the bulk or brane, accounts for the phantom divide line crossing too [19]. Now we try to realize this crossing in the *warped* DGP braneworld with quintom matter localized on the brane and non-minimally coupled to induced gravity. In this warped DGP model, the equation of state parameter, ω of dark energy component has the following form (with $\rho_{\rm dm}=0$)

$$\omega=-1+\frac{(\dot{\varphi}^2-\dot{\sigma}^2)-2\xi\Big[-H(\varphi\dot{\varphi}+\sigma\dot{\sigma})+\dot{H}(\varphi^2+\sigma^2)+\varphi\ddot{\varphi}+\sigma\ddot{\sigma}+\dot{\varphi}^2+\dot{\sigma}^2\Big]}{\rho_{\rm de}}$$
$$\times\left\{1+\varepsilon\eta\left(A_0^2+2\eta\frac{\frac{1}{2}(\dot{\varphi}^2-\dot{\sigma}^2)+V(\varphi,\sigma)+6\xi H(\varphi\dot{\varphi}+\sigma\dot{\sigma})+3\xi H^2(\varphi^2+\sigma^2))}{\rho_0}\right)^{-1/2}\right\}. \tag{22}$$

After substituting corresponding relations for φ, σ, H and V in equation (22), we plot the behavior of ω for two branches of the DGP-inspired model *versus* the cosmic time. Figure 2 shows the variation of ω *versus* cosmic time for two possible branches of the model. In figure 2 (a) which is devoted to self-accelerating branch, the equation of state parameter crosses the cosmological constant line. This behavior is repeated periodically due to oscillating nature of the cosmic expansion. Figure 2 (b) shows the situation for normal (non-self accelerating) branch. In this case crossing the cosmological constant line occurs too, but the behavior of equation of state parameter differs considerably compared to self-accelerating branch. As this figure shows, at the bouncing point ω approaches the negative infinity. Before discussing the stability of solutions in this setup, one important point should be emphasized here: we note that the potential (15) does not make the equations (8)–(11) symmetric with respect to transformations $\varphi\rightarrow-\sigma$ and $\sigma\rightarrow-\varphi$. One may think that this is the case if we take

$$\varphi\rightarrow-i\sigma \tag{23}$$

and

$$\sigma\rightarrow-i\varphi\,, \tag{24}$$

where i is the imaginary unit. However, this is not actually the case: with these transformations, there is a shift in phase of both canonical and phantom fields defined in equation (16). In absence of non-minimal coupling

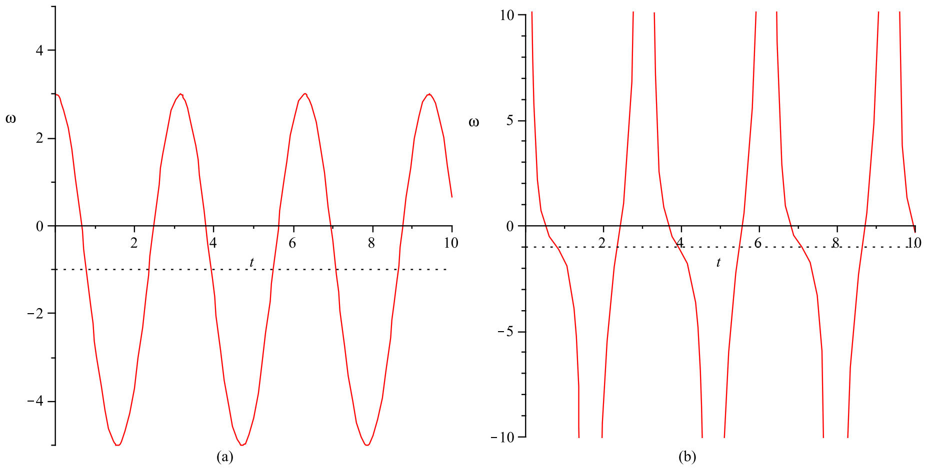

Fig. 2. Time evolution of the equation of state parameter ω. There is a crossing the cosmological constant line in both branches of the scenario: (a) Self-accelerating branch. ω mimics the oscillating nature of the cosmic expansion. (b) Normal branch. As usual, at the bouncing point ω approaches the negative infinity.

parameter ($\xi = 0$), equations (8)–(11) remain unchanged under these transformations and the form of the scale factor and the equation of state parameter are invariant under these transformations. However, the presence of the non-minimal coupling between the scalar fields and induced gravity on the brane leads to a breakdown of this symmetric behavior. In summary, we stress that the presence of the non-minimal coupling breaks the symmetric behavior of the model under transformations (23) and (24).

3.3. Stability of the model

Now we study the stability of our model. The sound speed expresses the phase velocity of the inhomogeneous perturbations of the quintom field. In order to study the classical stability of our model, we analyze the behavior of the model in the $\omega - \omega'$ plane where ω' is the derivative of ω with respect to the logarithm of the scale factor (see [20–23] for a similar analysis for other interesting cases)

$$\omega' \equiv \frac{d\omega}{d\ln a} = \frac{d\omega}{dt}\frac{dt}{d\ln a} = \frac{\dot{\omega}}{H}\,. \tag{25}$$

We define the function c_a as

$$c_a^2 \equiv \frac{\dot{p}}{\dot{\rho}}\,. \tag{26}$$

If the matter is considered as a perfect fluid, this function would be the adiabatic sound speed of this fluid. But, for our model with two scalar fields, this is not actually a sound speed. Nevertheless, we demand that $c_a^2 > 0$ in order to avoid the big rip singularity at the end of the universe evolution. From equation (14) we have

$$\dot{\rho}_{\rm de} = -3H\rho_{\rm de}(1+\omega_{\rm de})\,. \tag{27}$$

Using equation of state $p_{\rm de} = \omega_{\rm de}\rho_{\rm de}$, we get

$$\dot{p}_{\rm de} = \dot{\omega}_{\rm de}\rho_{\rm de} + \omega_{\rm de}\dot{\rho}_{\rm de}\,. \tag{28}$$

So, the function c_a^2 could be rewritten as

$$c_a^2 = \frac{\dot{\omega}_{\rm de}}{-3H(1+\omega_{\rm de})} + \omega_{\rm de}\,. \tag{29}$$

In this situation, the $\omega - \omega'$ plane is divided into four regions defined as follows

$$\begin{cases} \mathrm{I}: & \omega_{\rm de} > -1\,, & \omega' > 3\omega(1+\omega) & \Rightarrow & c_a^2 > 0\,, \\ \mathrm{II}: & \omega_{\rm de} > -1\,, & \omega' < 3\omega(1+\omega) & \Rightarrow & c_a^2 < 0\,, \\ \mathrm{III}: & \omega_{\rm de} < -1\,, & \omega' > 3\omega(1+\omega) & \Rightarrow & c_a^2 < 0\,, \\ \mathrm{IV}: & \omega_{\rm de} < -1\,, & \omega' < 3\omega(1+\omega) & \Rightarrow & c_a^2 > 0\,. \end{cases} \tag{30}$$

As one can see from these relations, the regions I and IV have the classical stability in our model. We plot the behavior of the model in the $\omega - \omega'$ phase plane and identify the regions, mentioned above, in figure 3.

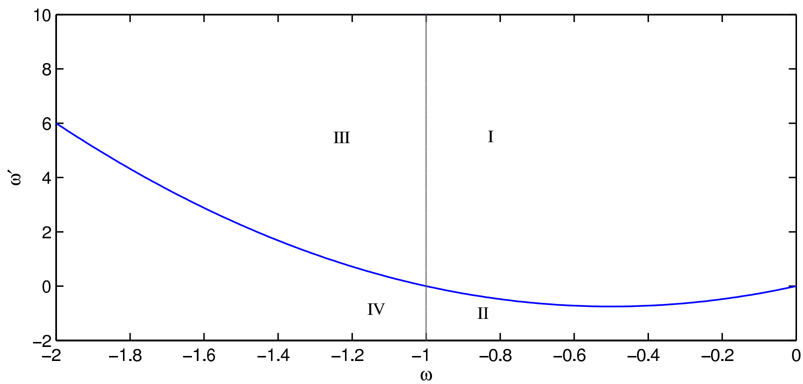

Fig. 3. Bounds on ω' as a function of ω in $\omega - \omega'$ phase plane. The stable regions are I and IV.

4. Summary

One of the most serious drawbacks of the standard model of cosmology is the problem of initial (and possibly final) singularity. In recent analysis done within the loop quantum cosmology, the Big Bang singularity is replaced by a quantum Big Bounce with finite energy density of matter. Also incorporation of the Gauss–Bonnet term in the action of braneworld models with induced gravity provides a phenomenologically rich framework to overcome initial singularity with possible realization of bouncing solutions [24]. On the other hand, a sound gravitational theory needs also to have a well posed Cauchy problem. A Model universe which realizes bouncing solution is a good candidate to overcome these singularities.

An alternative approach to explain late-time positively accelerated expansion of the universe is a multi-component dark energy with at least one non-canonical phantom field. The analysis of the properties of dark energy from recent observations favors models where $\omega = \frac{p}{\rho}$ crosses the phantom divide line, $\omega = -1$ in the near past. In this respect, construction of theoretical frameworks with potential to describe this positively accelerated expansion and crossing the phantom divide line by the equation of state parameter is an interesting challenge. In this paper, we have considered a quintom field non-minimally coupled to induced gravity on the warped DGP braneworld as a dark energy component. We have studied the bouncing behavior of the solutions in both branches of this DGP-inspired scenario. In the self-accelerating branch of the model (with $\varepsilon = +1$), the universe undergoes an expansion, reaches to a maximum radius and then crunches to a finite minimum size and this cycle repeats. In this case, there is no bounce at the minimum point since the scale factor has no derivative at that point. In the normal (non-self accelerating) branch of the model (with $\varepsilon = -1$), the universe switches alternatively between expanding and contracting phases. The minimum points of the scale factor *versus* cosmic time are the bouncing points. In fact, there is a sequence of phases as: Expansion $\rightarrow$ Turn-around $\rightarrow$ Contraction $\rightarrow$ Bounce and this cycle repeats regularly. This model can be regarded as an oscillating universe: this oscillation can be regarded as the result of the existence of positive pressure of the standard scalar field competing with the negative pressure of the phantom field [25].

Next, we study the dynamics of the equation of state parameter. One can see that there is a crossing the phantom divide line in both branches of this DGP-inspired model although the evolution of the equation of state parameter is different in these two branches. We have studied the stability of this model. As a result, there are appropriate regions of $\omega - \omega'$ phase plane that solutions are stable.

Finally, we should stress on the ghost instabilities present in the self-accelerating branch of this DGP-inspired model. The self-accelerating branch of the DGP model contains a ghost at the linearized level [26]. Since the ghost carries negative energy density, it leads to the instability of the space-time. The presence of the ghost can be attributed to the infinite volume of the extra-dimension in DGP setup. When there are ghosts instabilities in self-accelerating branch, it is natural to ask what are the results of solutions decay. As a possible answer we can state that since the normal branch solutions are ghost-free, one can think that the self-accelerating solutions may decay into the normal branch solutions. In fact, for a given brane tension, the Hubble parameter in the self-accelerating universe is larger than that of the normal branch solutions. Then it is possible to have nucleation of bubbles of the normal branch in the environment of the self-accelerating branch solution. This is similar to the false vacuum decay in de Sitter space. However, there are arguments against this kind of reasoning which suggest that the self-accelerating branch does not decay into the normal branch by forming normal branch bubbles (see [26] for more details). It was also shown that the introduction of Gauss–Bonnet term in the bulk does not help to overcome this problem [27]. Also recently it has been argued that the presence of higher derivatives in the field equations on the brane introduce very massive ghost excitations with mass of the order of Planck mass which are generated in the ordinary branch of the modified DGP model [28]. In fact, it is still unclear what is the end state of the ghost instability in self-accelerated branch of DGP inspired setups. On the other hand, non-minimal coupling of scalar field and induced gravity in our setup provides a new degree of freedom which requires special fine tuning and this may provide a suitable basis to treat ghost instability. It seems that in our model this additional degree of freedom has the capability to provide the background for a more reliable solution to ghost instability due to a wider parameter space.

REFERENCES

[1] S. Perlmutter *et al.*, *Astrophys. J.* **517**, 565 (1999); A.G. Riess *et al.*, *Astron. J.* **116**, 1006 (1998); A.D. Miller *et al.*, *Astrophys. J. Lett.* **524**, L1 (1999); P. de Bernardis *et al.*, *Nature* **404**, 955 (2000); S. Hanany *et al.*, *Astrophys. J. Lett.* **545**, L5 (2000); D.N. Spergel *et al.*, *Astrophys. J. Suppl.* **148**, 175 (2003); L. Page *et al.*, *Astrophys. J. Suppl.* **148**, 233 (2003); G. Hinshaw *et al.* [WMAP Collaboration], *Astrophys. J. Suppl.* **180**, 225 (2009) [`arXiv:0803.0732[astro-ph]`]; D.N. Spergel *et al.*, *Astrophys. J. Suppl.* **170**, 377 (2007); G. Hinshaw *et al.*, *Astrophys. J. Suppl.* **170**, 288 (2007); L. Page *et al.*, *Astrophys. J. Suppl.* **170**, 335 (2007); A.G. Reiss *et al.*, *Astrophys. J.* **607**, 665 (2004); S.W. Allen *et al.*, *Mon. Not. R. Astron. Soc.* **353**, 457 (2004); E. Komatsu *et al.* [WMAP Collaboration], *Astrophys. J. Suppl.* **180**, 330 (2009) [`arXiv:0803.0547[astro-ph]`].

[2] U. Alam, V. Sahni, A. Starobinsky, *JCAP* **0406** (2004) 008; Y.G. Gong, *Class. Quantum Grav.* **22**, 2121 (2005); Y. Wang, M. Tegmark, *Phys. Rev.* **D71**, 103513 (2005); Y. Wang, P. Mukherjee, *Astrophys. J.* **606**, 654 (2004); R. Daly, S. Djorgovski, *Astrophys. J.* **612**, 652 (2004); U. Alam, V. Sahni, T. Saini, A. Starobinsky, *Mon. Not. R. Astron. Soc.* **354**, 275 (2004); T. Choudhury, T. Padmanabhan, *Astron. Astrophys.* **429**, 807 (2005).

[3] S. Nesseris, L. Perivolaropoulos, *JCAP* **0701**, 018 (2007) [`arXiv:astro-ph/0610092`].

[4] S. Yin, B. Wang, E. Abdalla, C.-Y. Lin, *Phys. Rev.* **D76**, 124026 (2007) [`arXiv:0708.0992[hep-th]`].

[5] T. Padmanabhan, *Phys. Rep.* **380**, 253 (2003); V. Shani, A.A. Starobinsky, *Int. J. Mod. Phys.* **D9**, 373 (2000); S.M. Carroll, *Living Rev. Rel.* **4**, 1 (2001).

[6] E.J. Copeland, M. Sami, S. Tsujikawa, *Int. J. Mod. Phys.* **D15**, 1753 (2006) [`arXiv:hep-th/0603057`].

[7] A. Vikman, *Phys. Rev.* **D71**, 023515 (2005) [`arXiv:astro-ph/0407107`]; Y.H. Wei, Y.Z. Zhang, *Grav. Cosmol.* **9**, 307 (2003); M.P. Dabrowski, T. Stachowiak, M. Szydlowski, *Phys. Rev.* **D68**, 103519 (2003); V. Sahni, Y. Shtanov, *JCAP* **0311**, 014 (2003); Y.H. Wei, Y. Tian, *Class. Quantum Grav.* **21**, 5347 (2004); F.C. Carvalho, A. Saa, *Phys. Rev.* **D70**, 087302 (2004); F. Piazza, S. Tsujikawa, *JCAP* **0407**, 004 (2004); R.-G. Cai, H.S. Zhang, A. Wang, *Commun. Theor. Phys.* **44**, 948 (2005); I.Y. Arefeva, A.S. Koshelev, S.Y. Vernov, *Phys. Rev.* **D72**, 064017 (2005); A. Anisimov, E. Babichev, A. Vikman, *JCAP* **0506**, 006 (2005); B. Wang, Y.G. Gong, E. Abdalla, *Phys. Lett.* **B624**, 141 (2005); S. Nojiri, S.D. Odintsov, *Gen. Relativ. Gravitation* **38**, 1285 (2006) [`arXiv:hep-th/0506212`]; S. Nojiri, S.D. Odintsov, S. Tsujikawa, *Phys. Rev.* **D71**, 063004 (2005) [`arXiv:hep-th/0501025`]; E. Elizalde, S. Nojiri, S.D. Odintsov, P. Wang, *Phys. Rev.* **D71**, 103504 (2005); W. Zhao, Y. Zhang, *Phys. Rev.* **D73**, 123509 (2006); P.S. Apostolopoulos, N. Tetradis, *Phys. Rev.* **D74**, 064021 (2006) [`arXiv:hep-th/0604014`]; I.Ya. Arefeva, A.S. Koshelev, *J. High Energy Phys.* **0702**, 041 (2007) [`arXiv:hep-th/0605085`].

[8] Z.-K. Guo *et al.*, *Phys. Lett.* **B608**, 177 (2005); W. Zhao, Y. Zhang, *Phys. Rev.* **D73**, 123509 (2006); Y.-F. Cai, H. Li, Y.-S. Piao, X. Zhang, *Phys. Lett.* **B646**, 141 (2007); X. Zhang, *Phys. Rev.* **D74**, 103505 (2006); Y.-F. Cai *et al.*, *Phys. Lett.* **B651**, 1 (2007); Y.-F. Cai *et al.*, *J. High Energy Phys.* **0710**, 071 (2007); M.R. Setare, *Phys. Lett.* **B641**, 130 (2006); M.R. Setare, E.N. Saridakis, *Phys. Lett.* **B668**, 177 (2008); M.R. Setare, E.N. Saridakis, *JCAP* **0809**, 026 (2008) [`arXiv:0809.0114[hep-th]`]; H.-H. Xiong *et al.*, *Phys. Lett.* **B666**, 212 (2008); M.R. Setare, E.N. Saridakis, `arXiv:0807.3807[hep-th]` to appear in *Int. J. Mod. Phys.* **D** (2009); M.R. Setare, E.N. Saridakis, `arXiv:0810.4775[astro-ph]`.

[9] S. Nojiri, S.D. Odintsov, *Int. J. Geom. Meth. Mod. Phys.* **4**, 115 (2007); `arXiv:hep-th/0601213`; S. Nojiri, S.D. Odintsov, `arXiv:0807.0685`; T.P. Sotiriou, V. Faraoni, `arXiv:0805.1726[gr-qc]`; S. Capozziello, M. Francaviglia, *Gen. Relativ. Gravitation* **40**, 357 (2008).

[10] G. Dvali, G. Gabadadze, M. Porrati, *Phys. Lett.* **B485**, 208 (2000) [`arXiv:hep-th/0005016`].

[11] L. Randall, R. Sundrum, *Phys. Rev. Lett.* **83**, 4690 (1999).

[12] C. Deffayet, *Phys. Lett.* **B502**, 199 (2001); C. Deffayet, G. Dvali, G. Gabadadze, *Phys. Rev.* **D65**, 044023 (2002); C. Deffayet, S.J. Landau, J. Raux, M. Zaldarriaga, P. Astier, *Phys. Rev.* **D66**, 024019 (2002).

[13] Kei-ichi Maeda, S. Mizuno, T. Torii, *Phys. Rev.* **D68**, 024033 (2003) [arXiv:gr-qc/0303039].

[14] H. Zhang, Z.-H. Zhu, *Phys. Rev.* **D75**, 023510 (2007); S.-F. Wu *et al.*, *Phys. Lett.* **B659**, 45 (2008) [arXiv:0708.1038[astro-ph]].

[15] H.-H. Xiong, Y.-F. Cai, T. Qiu, Y.-S. Piao, X. Zhang, *Phys. Lett.* **B666**, 212 (2008) [arXiv:0805.0413[astro-ph]].

[16] F. Finelli, *JCAP* **0310**, 011 (2003); M.R. Setare, *Phys. Lett.* **B602**, 1 (2004); M.R. Setare, *Eur. Phys. J.* **C47**, 851 (2006); A. Biswas, S. Mukherji, *JCAP* **0602**, 002 (2006); M. Bojowald, *Phys. Rev.* **D74**, 081301(R) (2007) [arXiv:gr-qc/0608100]; T. Stachowiak, M. Szydlowski, *Phys. Lett.* **B646**, 209 (2007) [arXiv:gr-qc/0610121]; Y.-F. Cai, T. Qiu, Y.-S. Piao, M. Li, X. Zhang, *J. High Energy Phys.* **0710**, 071 (2007) [arXiv:0704.1090[gr-qc]]; A. Cardoso, D. Wands, *Phys. Rev.* **D77**, 123538 (2008) [arXiv:0801.1667[hep-th]]; M. Novello, S.E. Perez Bergliaffa, *Phys. Rep.* **463**, 127 (2008) [arXiv:0802.1634[astro-ph]]; M.P. Dabrowski, T. Stachowiak, *Ann. Phys.* **248**, 199 (2006); M.P. Dabrowski, T. Stachowiak, *Ann. Phys.* **321**, 771 (2006) [arXiv:hep-th/0411199].

[17] K. Nozari, N. Behrouz, T. Azizi, B. Fazlpour, *Prog. Theor. Phys.* **122**, 735 (2009) [arXiv:0808.0318[gr-qc]].

[18] L.P. Chimento, R. Lazkoz, R. Maartens, I. Quiros, *JCAP* **0609**, 004 (2006) [arXiv:astro-ph/0605450].

[19] K. Nozari, M.R. Setare, T. Azizi, N. Behrouz, *Phys. Scri.* **80**, 025901 (2009) [arXiv:0810.1427[hep-th]]; M.R. Setare, P. Moyassari, *Phys. Lett.* **B674**, 237 (2009) [arXiv:0806.2418[gr-qc]]; M.R. Setare, E.N. Saridakis, arXiv:0810.0645[hep-th]; M.R. Setare, E.N. Saridakis, arXiv:0811.4253[hep-th].

[20] R.R. Caldwell, E.V. Linder, *Phys. Rev. Lett.* **95**, 141301 (2005) [arXiv:astro-ph/0505494].

[21] R.J. Scherrer, *Phys. Rev.* **D73**, 043502 (2006) [arXiv:astro-ph/0509890].

[22] T. Chiba, *Phys. Rev.* **D73**, 063501 (2006) [arXiv:astro-ph/0510598].

[23] W. Zhao, Y. Zhang, *Phys. Rev.* **D73**, 123509 (2006) [arXiv:astro-ph/0604460].

[24] R.A. Brown, arXiv:gr-qc/0701083; R.A. Brown *et al.*, *JCAP* **0511**, 008 (2005) [arXiv:gr-qc/0508116]; K. Nozari, B. Fazlpour, *JCAP* **06**, 032 (2008) [arXiv:0805.1537[hep-th]].

[25] M.P. Dabrowski, *Ann. Phys.* **248**, 199 (1996) [arXiv:gr-qc/9503017].

[26] K. Koyama, *Class. Quantum Grav.* **24**, R231 (2007) [arXiv:0709.2399[hep-th]].

[27] C. de Rham, A.J. Tolley, *JCAP* **0607**, 004 (2006) [arXiv:hep-th/0605122].

[28] M. Cadoni, P. Pani, arXiv:0812.3010[hep-th].