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ANTISYMMETRIC-TENSOR FIELD MEDIATING IN HIDDEN SECTOR AND REDUCTION OF ITS POLARIZATION DEGREES OF FREEDOM

Wojciech Królikowski

Institute of Theoretical Physics, University of Warsaw
Hoża 69, 00-681 Warszawa, Poland

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In the model of hidden sector of the Universe, proposed and commented recently, a new nongauge mediating field transforming as an antisymmetric tensor (of dimension one) plays a crucial role. If it gets definite parity, say, $-$, it can be split into two three-dimensional fields of spin 1 and parity $-$ and $+$, respectively, much like the electromagnetic field (of dimension two) is split into its electric and magnetic parts. Then, the parity is preserved by a new weak interaction in the hidden sector. *A priori*, the parts of the nongauge mediating field may be either independent or dependent. We discuss a simple natural constraint that may relate them to each other in a relativistically covariant way, reducing their independent polarization degrees of freedom to three. In Appendix, we describe another option, where the mediating field (of dimension one) is gauged by a vector field (of dimension zero).

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1. Introduction

In previous papers [1,2], we have proposed a model of hidden sector of the Universe, consisting of sterile spin-1/2 Dirac fermions ("sterinos"), sterile spin-0 bosons ("sterons"), and sterile nongauge mediating bosons ("A bosons") described by an antisymmetric-tensor field (of dimension one) weakly coupled to steron–photon pairs and, more obviously, to the antisterino–sterino pairs,

$$-\tfrac{1}{2}\sqrt{f}\,\left(\varphi F_{\mu\nu} + \zeta\bar{\psi}\sigma_{\mu\nu}\psi\right) A^{\mu\nu}\,, \tag{1}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the Standard-Model electromagnetic field (of dimension two), while $\sqrt{f}$ and $\sqrt{f}\,\zeta$ denote two dimensionless small coupling constants. Here, it is presumed that $\varphi = \langle\varphi\rangle_{\rm vac} + \varphi_{\rm ph}$ with a spontaneously nonzero vacuum expectation value $\langle\varphi\rangle_{\rm vac} \neq 0$. Such a coupling of photons to the hidden sector has been called "photonic portal" (to hidden sector). It provides a weak coupling between the hidden and Standard-Model sectors of the Universe. The photonic portal is an alternative to the popular "Higgs portal" (to hidden sector) [3].

In the present note, we discuss the polarization degrees of freedom for A bosons, in particular, a simple natural constraint that may reduce these degrees to three in a relativistically covariant way.

The new interaction Lagrangian (1), together with the A-boson kinetic and Standard-Model electromagnetic Lagrangians, leads to the following field equations for $F_{\mu\nu}$ and $A_{\mu\nu}$:

$$\partial^\nu \left[F_{\mu\nu} + \sqrt{f}\,(\langle\varphi\rangle_{\rm vac} + \varphi_{\rm ph})\,A_{\mu\nu}\right] = -j_\mu\,, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad (2)$$

and

$$\left(\Box - M^2\right) A_{\mu\nu} = -\sqrt{f}\,\left[(\langle\varphi\rangle_{\rm vac} + \varphi_{\rm ph})\,F_{\mu\nu} + \zeta\bar{\psi}\sigma_{\mu\nu}\psi\right]\,, \quad (3)$$

where j_μ denotes the Standard-Model electric current and M stands for a mass scale of A bosons, expected typically to be large.

The field equations (2) ("supplemented Maxwell's equations") are modified due to the presence of hidden sector. This modification has a magnetic character, because the hidden-sector contribution to the total electric source-current $j_\mu + \partial^\nu[\sqrt{f}\,(\langle\varphi\rangle_{\rm vac} + \varphi_{\rm ph})A_{\mu\nu}]$ for the electromagnetic field A_μ is a four-divergence giving no contribution to the total electric charge $\int d^3x\{j_0 + \partial^k[\sqrt{f}\,(\langle\varphi\rangle_{\rm vac} + \varphi_{\rm ph})A_{0k}]\} = \int d^3x j_0 = Q$. In particular, it can be seen that the vacuum expectation value $\langle\varphi\rangle_{\rm vac} \neq 0$ generates spontaneously a small sterino magnetic moment

$$\mu_\psi = \frac{f\zeta}{2M^2}\langle\varphi\rangle_{\rm vac}\,, \quad (4)$$

though sterinos are electrically neutral. This is a consequence of an effective sterino magnetic interaction

$$-\mu_\psi\bar{\psi}\sigma_{\mu\nu}\psi F^{\mu\nu} \quad (5)$$

appearing, when the low-momentum-transfer approximation

$$A_{\mu\nu} \simeq \frac{\sqrt{f}\,\zeta}{M^2}\bar{\psi}\sigma_{\mu\nu}\psi \quad (6)$$

effectively implied by Eq. (3) is used in the interaction (1) with $\varphi = \langle\varphi\rangle_{\rm vac} + \varphi_{\rm ph}$.

2. Option of independent field components for *A* bosons

In analogy with the familiar splitting of $F_{\mu\nu}$ into $\vec{E}$ and $\vec{B}$, we can split the field $A_{\mu\nu}$ into the three-dimensional vector and axial fields $\vec{A}^{(E)}$ and $\vec{A}^{(B)}$ of spin 1 and parity $-$ and $+$, respectively (if the field $A_{\mu\nu}$ has a definite parity, say, $-$). Then,

$$(A_{\mu\nu}) = \begin{pmatrix} 0 & A_1^{(E)} & A_2^{(E)} & A_3^{(E)} \\ -A_1^{(E)} & 0 & -A_3^{(B)} & A_2^{(B)} \\ -A_2^{(E)} & A_3^{(B)} & 0 & -A_1^{(B)} \\ -A_3^{(E)} & -A_2^{(B)} & A_1^{(B)} & 0 \end{pmatrix} . \tag{7}$$

Similarly, for the spin tensor $\sigma^{\mu\nu} = (i/2)[\gamma^\mu, \gamma^\nu]$ with $\vec{\alpha} = (\alpha_k) = (\gamma^0\gamma^k) = (i\sigma^{k0})$ and $\vec{\sigma} = (\sigma_k) = \gamma_5\vec{\alpha} = (1/2)\left(\varepsilon_{klm}\sigma^{lm}\right)$ $(k = 1, 2, 3)$, we get

$$(\sigma^{\mu\nu}) = \begin{pmatrix} 0 & i\alpha_1 & i\alpha_2 & i\alpha_3 \\ -i\alpha_1 & 0 & \sigma_3 & -\sigma_2 \\ -i\alpha_2 & -\sigma_3 & 0 & \sigma_1 \\ -i\alpha_3 & \sigma_2 & -\sigma_1 & 0 \end{pmatrix} . \tag{8}$$

Then, the interaction (1) can be rewritten in the form

$$\left(\varphi\vec{E} - i\zeta\bar{\psi}\,\vec{\alpha}\,\psi\right) \cdot \vec{A}^{(E)} - \left(\varphi\vec{B} - \zeta\bar{\psi}\,\vec{\sigma}\psi\right) \cdot \vec{A}^{(B)} , \tag{9}$$

where $\varphi = \langle\varphi\rangle_{\rm vac} + \varphi_{\rm ph}$ with $\langle\varphi\rangle_{\rm vac} \neq 0$. Consequently, the first and second of supplemented Maxwell's equations (2) for photons can be split as follows:

$$\vec{\partial} \times \left(\vec{B} + \sqrt{f}\,\varphi\vec{A}^{(B)}\right) = \partial_0\left(\vec{E} + \sqrt{f}\,\varphi\vec{A}^{(E)}\right) + \vec{j} ,$$
$$\vec{\partial} \cdot \left(\vec{E} + \sqrt{f}\,\varphi\vec{A}^{(E)}\right) = j_0 , \qquad \vec{\partial} \times \vec{E} = -\partial_0\vec{B} , \qquad \vec{\partial} \cdot \vec{B} = 0 \tag{10}$$

and the field equation (3) for A bosons as:

$$\begin{aligned} \left(\Box - M^2\right)\vec{A}^{(E)} &= -\sqrt{f}\left(\varphi\vec{E} - i\zeta\bar{\psi}\,\vec{\alpha}\,\psi\right) , \\ \left(\Box - M^2\right)\vec{A}^{(B)} &= -\sqrt{f}\left(\varphi\vec{B} - \zeta\bar{\psi}\,\vec{\sigma}\,\psi\right) , \end{aligned} \tag{11}$$

where $\varphi = \langle\varphi\rangle_{\rm vac} + \varphi_{\rm ph}$ with $\langle\varphi\rangle_{\rm vac} \neq 0$. Here, $(j_\mu) = (j_0, -\vec{j})$ is the Standard-Model current ($\vec{E} = -\partial_0\vec{A} - \vec{\partial}A_0$ and $\vec{B} = \vec{\partial} \times \vec{A}$ with $(\partial_\mu) = (\partial_0, \vec{\partial})$ and $(A_\mu) = (A_0, -\vec{A})$). Note that the source-free Eqs. (10) are, of course, the ordinary source-free Maxwell's equations.

The sterile A bosons described by the fields $\vec{A}^{(\mathrm{E})}$ and $\vec{A}^{(\mathrm{B})}$, when they propagate freely in space ($\sqrt{f} \to 0$), get the one-particle wave functions

$$\vec{A}^{(E,B)}_{\vec{k}_A}(x) = \frac{1}{(2\pi)^{3/2}} \frac{1}{\sqrt{2\omega_A}} \, \vec{e}^{\,(E,B)} e^{-ik_A \cdot x} \,, \tag{12}$$

where $k_A = (\omega_A, \vec{k}_A)$ with $\omega_A = \sqrt{\vec{k}^2_A + M^2}$, while $\vec{e}^{\,(E,B)}$ are linear polarizations of $A^{(E)}$ and $A^{(B)}$ bosons [2]. If the field $A_{\mu\nu}$ has a definite parity, say, $-$, then due to Eq. (7) the polarizations $\vec{e}^{\,(E)}$ and $\vec{e}^{\,(B)}$ are polar and axial vectors, respectively. Then, the parity is preserved by the new weak interaction (1) or (9) in the hidden sector.

Denoting by $e_{\mu\nu}$ the antisymmetric polarization tensor appearing in the A-boson relativistic free wave function

$$A_{\mu\nu\vec{k}_A}(x) = \frac{1}{(2\pi)^{3/2}} \frac{1}{\sqrt{2\omega_A}} \, e_{\mu\nu} e^{-ik_A \cdot x} \tag{13}$$

split according to Eq. (7) into $\vec{A}^{(E,B)}_{\vec{k}_A}$ given in Eq. (12), we can write

$$(e_{\mu\nu}) = \begin{pmatrix} 0 & e^{(E)}_1 & e^{(E)}_2 & e^{(E)}_3 \\ -e^{(E)}_1 & 0 & -e^{(B)}_3 & e^{(B)}_2 \\ -e^{(E)}_2 & e^{(B)}_3 & 0 & -e^{(B)}_1 \\ -e^{(E)}_3 & -e^{(B)}_2 & e^{(B)}_1 & 0 \end{pmatrix} . \tag{14}$$

(Of course, there is a triplet of antisymmetric polarization tensors $e_{\mu\nu a}$ ($a = 1,2,3$) split into two triplets of linear polarizations $\vec{e}^{\,(E,B)}_a = \left(e^{(E,B)}_{ka}\right)$ ($a = 1,2,3, k = 1,2,3$).)

If the fields $\vec{A}^{(E)}$ and $\vec{A}^{(B)}$ are independent (as can be in Eqs. (11)), then the corresponding polarizations form two independent triplets of orthonormal versors,

$$\vec{e}^{\,(E,B)}_a \cdot \vec{e}^{\,(E,B)}_b = \delta_{ab} \; (a,b=1,2,3) \,, \qquad \sum_{a=1}^{3} e^{(E,B)}_{ka} e^{(E,B)}_{la} = \delta_{kl} \; (k,l=1,2,3) \tag{15}$$

with $\vec{e}^{\,(E,B)}_a = (e^{(E,B)}_{ka})$ ($a = 1,2,3,\ k = 1,2,3$) [2].

In place of the option of independent field components for A bosons, we will discuss in Section 3 an option with a simple natural constraint that may relate the fields $\vec{A}^{(E)}$ and $\vec{A}^{(B)}$ to each other in a relativistically covariant way, reducing their independent polarization degrees of freedom to three.

3. Option of $A_{\mu\nu}A^{\mu\nu} = 0$ for A bosons

Consider the natural option, where the axial polarizations $\vec{e}_a^{\,(B)}$ $(a = 1,2,3)$ are related to the polar polarizations $\vec{e}_a^{\,(E)}$ $(a = 1,2,3)$ through the constraint

$$\vec{e}_{1,2,3}^{\,(B)} = \vec{e}_{2,3,1}^{\,(E)} \times \vec{e}_{3,1,2}^{\,(E)} \tag{16}$$

(both in right- and left-handed frame of reference), where

$$\vec{e}_{2,3,1}^{\,(E)} \times \vec{e}_{3,1,2}^{\,(E)} = (+ \text{ or } -)\vec{e}_{1,2,3}^{\,(E)} \tag{17}$$

in a right- or left-handed frame of reference, respectively. Thus, the constraint (16) can be trivially rewritten as

$$\vec{e}_a^{\,(B)} = (+ \text{ or } -)\vec{e}_a^{\,(E)} \tag{18}$$

$(a = 1,2,3)$, showing that $\vec{e}_a^{\,(B)}$ are parallel or antiparallel to $\vec{e}_a^{\,(E)}$ and have the same magnitudes as $\vec{e}_a^{\,(E)}$,

$$\vec{e}_a^{\,(B)2} = \vec{e}_a^{\,(E)2}\,. \tag{19}$$

Then, from Eqs. (14) and (19) it follows that the products

$$e_{\mu\nu a}e_a^{\mu\nu} = 2\left(\vec{e}_a^{\,(B)2} - \vec{e}_a^{\,(E)2}\right) = 0 \tag{20}$$

$(a = 1,2,3)$ are relativistically covariant in a trivial way. Notice that, when $\vec{e}_a^{\,(E)2} = 1$, the orthonormal conditions (15) are valid in the present option as previously in the option of independent $\vec{e}_a^{\,(E)}$ and $\vec{e}_a^{\,(B)}$ $(a = 1,2,3)$, though now $\vec{e}_a^{\,(B)}$ are dependent on $\vec{e}_a^{\,(E)}$ (through Eqs. (18)).

For the field operators $\vec{A}^{\,(E,B)}(x)$, we can write in the Heisenberg picture that

$$\vec{A}^{\,(E,B)}(x) = \int d^3\vec{k}_A \sum_{a=1}^{3} a_a(\vec{k},t)\frac{1}{(2\pi)^{3/2}}\frac{1}{\sqrt{2\omega_A}}\,\vec{e}_a^{\,(E,B)}\,e^{-ik_A\cdot x} + \text{h.c.}\,, \tag{21}$$

where the annihilation and creation operators, $a_a(\vec{k},t)$ and $a_a^\dagger(\vec{k},t)$, are the same for (E) and (B) components of $A_{\mu\nu}(x)$. In the case of constraint (18), we can infer from Eq. (21) that

$$\vec{A}^{\,(B)}(x) = (+ \text{ or } -)\vec{A}^{\,(E)}(x) \tag{22}$$

and hence,

$$\vec{A}^{\,(B)2}(x) = \vec{A}^{\,(E)2}(x)\,. \tag{23}$$

Then, Eqs. (7) and (23) imply that the product

$$A_{\mu\nu}(x)A^{\mu\nu}(x) = 2\left[\vec{A}^{\,(B)2}(x) - \vec{A}^{\,(E)2}(x)\right] = 0 \tag{24}$$

is relativistically covariant in a trivial manner.

Thus, in conclusion, the new natural option, accepting the constraint (18) for the polarizations $\vec{e}_a^{\,(E)}$ and $\vec{e}_a^{\,(B)}$ or, equivalently, the constraint (22) for the fields $\vec{A}^{\,(B)}(x)$ and $\vec{A}^{\,(E)}(x)$, may be satisfactory in describing polarizations of the mediating A bosons in our model of hidden sector (communicating with the Standard-Model sector through the photonic portal). In this option, the parity is preserved by the new weak coupling (1) or (9) in the hidden sector. It is a scheme practically realizing three axial $\vec{e}_a^{\,(B)}$ in terms of three polar $\vec{e}_a^{\,(E)}$ in a relativistically covariant, trivial way. From the methodological point of view, the constraint (18) (see also its form (16)) has the character of a definition of $\vec{e}_a^{\,(B)}$ in terms of $\vec{e}_a^{\,(E)}$ that, in consequence, is included in the definition of field $A_{\mu\nu}(x)$.

In Sections 4 and 5, we will describe as an illustration two simple particle processes that may be important for the hypothetic phenomenology of A bosons.

4. Illustration 1: annihilation of a pair AA into a pair $\gamma\gamma$

To illustrate the working of our formalism consider in the lowest order the annihilation channel $AA \to \varphi^*_{\rm ph}\gamma\,\varphi^*_{\rm ph}\gamma \to \gamma\gamma$ induced by the coupling

$$-\tfrac{1}{2}\sqrt{f}\,\varphi_{\rm ph}F_{\mu\nu}\,A^{\mu\nu} \tag{25}$$

following from the interaction Lagrangian (1) with $\varphi = \langle\varphi\rangle_{\rm vac} + \varphi_{\rm ph}$.

The corresponding S-matrix element reads (in an obvious notation):

$$\begin{aligned} S(AA\to\gamma\gamma) &= -if\left[\frac{1}{(2\pi)^{12}}\,\frac{1}{16\omega_1\omega_2\omega_{A1}\omega_{A2}}\right]^{1/2}(2\pi)^4\delta^4(k_1+k_2-k_{A1}-k_{A2}) \\ &\times\frac{1}{4}\left[\frac{1}{i}\,e_2^{\mu\nu}(k_{2\mu}e_{2\nu}-k_{2\nu}e_{2\mu})\frac{1}{(k_1-k_{A1})^2-m_\varphi^2}\,\frac{1}{i}\,(k_{1\rho}e_{1\sigma}-k_{1\sigma}e_{1\rho})e_1^{\rho\sigma}+\right. \\ &\left.+\frac{1}{i}\,e_2^{\mu\nu}(k_{1\mu}e_{1\nu}-k_{1\nu}e_{1\mu})\frac{1}{(k_2-k_{A1})^2-m_\varphi^2}\,\frac{1}{i}\,(k_{2\rho}e_{2\sigma}-k_{2\sigma}e_{2\rho})e_1^{\rho\sigma}\right], \end{aligned} \tag{26}$$

where according to the matrix (14) for the antisymmetric polarization tensor $e_{\mu\nu}$ of A bosons we put

$$e^{\mu\nu} = \left\{\begin{array}{ll} e_k^{(E)} & \mu\nu = k\,0\,, \\ -\varepsilon_{klm}e_m^{(B)} & \mu\nu = k\,l\,, \\ -e_l^{(E)} & \mu\nu = 0\,l\,, \end{array}\right. \quad (k,l=1,2,3) \tag{27}$$

and, subsequently, take into account the constraint (18)

$$\vec{e}^{(B)} = \pm \vec{e}^{(E)} \tag{28}$$

in a right- or left-handed frame of reference, respectively. Then, in Eq. (26) there appears four times the expression of the type

$$\frac{1}{i} e^{\mu\nu} \left(k_\mu e_\nu - k_\nu e_\mu \right) = -\frac{1}{i} 2 \vec{e}^{(E)} \cdot \left(\omega \vec{e} \pm \vec{k} \times \vec{e} \right) . \tag{29}$$

Here, $\omega = |\vec{k}|$ and $\omega_A = \sqrt{\vec{k}_A^2 + M^2}$, giving $(k - k_A)^2 - m_\varphi^2 = -2(\omega\omega_A - \vec{k} \cdot \vec{k}_A) + M^2 - m_\varphi^2$ with $\vec{k} \cdot \vec{k}_A = \omega \sqrt{\omega_A^2 - M^2} \cos\theta_{\vec{k}}$.

From Eq. (26) we calculate the differential and total cross-sections for the channel $AA \to \gamma\gamma$:

$$\frac{d^6 \sigma(AA \to \gamma\gamma)}{d^3\vec{k}_1 d^3\vec{k}_2} = \frac{(2\pi)^6}{v_{\rm rel}} \sum_{e_1} \sum_{e_2} \frac{1}{3} \sum_{e_1^{(E)}} \frac{1}{3} \sum_{e_2^{(E)}} \frac{|S(AA \to \gamma\gamma)|^2}{(2\pi)^4 \delta^4(0)} \tag{30}$$

and (two photons are indistinguishable)

$$\sigma(AA \to \gamma\gamma) = \frac{1}{2} \int d^3\vec{k}_1 d^3\vec{k}_2 \frac{d^6\sigma(AA \to \gamma\gamma)}{d^3\vec{k}_1 d^3\vec{k}_2} . \tag{31}$$

The result we obtain in the centre-of-mass frame, where $\vec{k}_{A1} + \vec{k}_{A2} = 0$ and $v_{\rm rel} = 2\sqrt{\omega_{A1}^2 - M^2}/\omega_{A1} = 2|\vec{k}_{A1}|/\omega_{A1} = 2v_{A1}$, is

$$\sigma\left(AA \to \gamma\gamma\right) 2v_{A1} = \frac{1}{144\pi\omega_{A1}^2} \frac{\xi}{v_{A1}^2} \left(\frac{\xi}{1-\xi^2} + \frac{1}{2} \ln \frac{1+\xi}{1-\xi} \right) \tag{32}$$

with

$$\xi \equiv \frac{2\omega_{A1}\sqrt{\omega_{A1}^2 - M^2}}{2\omega_{A1}^2 - M^2 + m_\varphi^2} = \frac{2\omega_{A1}^2}{2\omega_{A1}^2 - M^2 + m_\varphi^2} v_{A1} \tag{33}$$

(here, of course, $\omega_{A1} = \omega_{A2} = \omega_1 = \omega_2$ and $v_{A1} = v_{A2}$).

Note that for nonrelativistic A bosons (when $\omega_{A1} \to M$) we get $\xi \to [2M^2/(M^2 + m_\varphi^2)] v_{A1}$. If it happens that $M^2 \sim m_\varphi^2$ or $\gg m_\varphi^2$, then $\xi \to v_{A1}$ or $2v_{A1}$, respectively. Thus, in the nonrelativistic case, we have from Eq. (32)

$$\sigma\left(AA \to \gamma\gamma\right) 2v_{A1} \to \frac{f^2}{72\pi M^2} \left(\frac{2M^2}{M^2 + m_\varphi^2} \right)^2 \sim \frac{f^2}{72\pi M^2} , \tag{34}$$

the last step working if it happens that $M^2 \sim m_\varphi^2$.

5. Illustration 2: decay of an A boson into a fermion pair $\bar{f}f$

In contrast to sterinos, sterons and A bosons are not stable. For an illustration consider in the lowest order the decay channel $A \rightarrow \gamma^* \rightarrow \bar{f}f$, where f is a charged fermion (*e.g.* $f = e^-, \mu^-, p$). This process is induced by the coupling

$$-\tfrac{1}{2}\sqrt{f}\,\langle\varphi\rangle_{\rm vac} F_{\mu\nu}A^{\mu\nu} - e_f\bar{\psi}_f\,\gamma^\mu\,\psi_f A_\mu\,, \tag{35}$$

where the first term follows from the interaction Lagrangian (1) with $\varphi = \langle\varphi\rangle_{\rm vac} + \varphi_{\rm ph}$, while the second presents the Standard-Model electromagnetic interaction for f fermions (*e.g.* $e_f = -e, -e, e$).

The corresponding S-matrix element reads (in an obvious notation):

$$\begin{aligned} S\left(A \rightarrow \bar{f}f\right) \;=\; & -ie_f\,\sqrt{f}\langle\varphi\rangle_{\rm vac}\left[\frac{1}{(2\pi)^9}\,\frac{m_f^2}{E_1\,E_2\,2\omega_A}\right]^{1/2}(2\pi)^4\delta^4(p_1+p_2-k_A) \\ & \times\frac{1}{2}\bar{u}(p_1)\frac{1}{i}\left(k_A^\mu\gamma^\nu - k_A^\nu\gamma^\mu\right)v(p_2)\frac{1}{k_A^2}\,e_{\mu\nu}\,, \end{aligned} \tag{36}$$

where the A-boson antisymmetric polarization tensor $e_{\mu\nu}$ is given as in Eq. (27) and the constraint (28) is taken into account. Then,

$$\left(k_A^\mu\gamma^\nu - k_A^\nu\gamma^\mu\right)e_{\mu\nu} = 2\left[\omega_A\vec{\gamma} - \vec{k}_A\beta \mp \left(\vec{k}_A \times \vec{\gamma}\right)\right]\cdot\vec{e}^{\,(B)} = 2M\vec{\gamma}\cdot\vec{e}^{\,(E)}\,, \tag{37}$$

the last step being valid for the A boson at rest, where $\vec{k}_A = 0$ and so, $\omega_A = M$. In this case, in consequence of energy-momentum conservation, $\vec{p}_1 + \vec{p}_2 = 0$ and $E_1 = E_2 = \omega_A/2 = M/2$.

From Eq. (36) we calculate the differential and total decay rates in the channel $A \rightarrow \bar{f}f$:

$$\frac{d^6\Gamma\left(A\rightarrow\bar{f}f\right)}{d^3\vec{p}_1 d^3\vec{p}_2} = (2\pi)^3\frac{1}{3}\sum_{e^{(E)}}\sum_u\sum_v\frac{|S\left(A\rightarrow\bar{f}f\right)|^2}{(2\pi)^4\delta^4(0)} \tag{38}$$

and

$$\Gamma\left(A\rightarrow\bar{f}f\right) = \int d^3\vec{p}_1\,d^3\vec{p}_2\frac{d^6\Gamma\left(A\rightarrow\bar{f}f\right)}{d^3\vec{p}_1\,d^3\vec{p}_2}\,. \tag{39}$$

For the A boson at rest, we obtain

$$\Gamma\left(A\,\rightarrow\,\bar{f}f\,\right) \;=\; \frac{e_f^2\,f\langle\varphi\rangle_{\rm vac}^2}{12\pi M}\,\frac{\left(M^2+2m_f^2\right)\sqrt{M^2-4m_f^2}}{M^3}\,. \tag{40}$$

When $M^2 \gg m_f^2$, then Eq. (40) gives

$$\Gamma\left(A \to \bar{f} f\right) \simeq \frac{e_f^2 f \langle\varphi\rangle_{\rm vac}^2}{12\pi M} \sim \frac{e_f^2 f M}{12\pi}\,, \tag{41}$$

the last step applying if it happens that $M^2 \sim \langle\varphi\rangle_{\rm vac}^2$.

Stable sterinos are candidates for thermal cold dark matter. In this case, under the tentative assumption that

$$m_\psi^2 \sim \left(10^{-3} \text{ to } 1\right) \langle\varphi\rangle_{\rm vac}^2 \sim m_\varphi^2\,, \qquad M^2 \sim \langle\varphi\rangle_{\rm vac}^2\,, \tag{42}$$

and putting boldly

$$f \sim e^2 \simeq 0.0917\,, \qquad \zeta \sim 1\,, \tag{43}$$

we estimate (in a similar way as in the third Ref. [1]) that

$$m_\psi \sim (13 \;\text{ to }\; 770)\ {\rm GeV}\,. \tag{44}$$

This gives $M^2 \sim (400 \text{ to } 770)^2$ GeV^2. Here, the experimental value $\Omega_{\rm DM} h^2 \sim 0.11$ is taken for the dark-matter relic abundance [4]. Then, the annihilation cross-section for an antisterino–sterino pair (improved in comparison with the third Ref. [1]) is equal to

$$\sigma_{\rm ann}\left(\bar{\psi}\psi\right) v_{\rm rel} \sim \left[\sigma\left(\bar{\psi}\psi \to \varphi_{\rm ph}\gamma\right) + (20/3 \text{ to } 8)\sigma\left(\bar{\psi}\psi \to e^+e^-\right)\right] 2v_\psi \sim {\rm pb} \tag{45}$$

(pb $= 10^{-12}$b $= 10^{-36}$cm^2), where $3\cdot[1+3\cdot(4/9+1/9)] = 8$ for $m_t < m_\psi \sim 770$ GeV and without top quark $3 + 2\cdot3\cdot(4/9+1/9) + 3\cdot(1/9) = 20/3$ for $m_t > m_\psi \sim 13$ GeV (masses of active leptons and quarks are neglected *versus* $E_\psi \sim m_\psi$).

In the case of these assumptions, we can estimate from Eqs. (34) and (41) that

$$\sigma(AA \to \gamma\gamma)\, 2v_{A1} \sim (2.3 \;\text{ to }\; 0.62) \times \frac{10^{-4}}{{\rm TeV}^2} = (0.088 \;\text{ to }\; 0.024)\ {\rm pb} \tag{46}$$

and

$$\Gamma\left(A \to \bar{f} f\right) \sim (90 \;\text{ to }\; 170)\ {\rm MeV} = \left(\frac{1}{0.72} \;\text{ to }\; \frac{1}{0.38}\right) \times \frac{10^{23}}{\rm s} \tag{47}$$

($\hbar = 1 = c$).

Appendix

Option of a dimensionless vector field gauging our mediating field

The sterile mediating field $A_{\mu\nu}$ of dimension one, discussed in this paper, is not gauged in the conventional manner *i.e.*, it is not the four-dimensional curl of a vector field of *dimension one*, as in the case of electromagnetic field $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ gauged by the vector field A_μ. In this Appendix, we ask the question, what would happen, if the mediating field $A_{\mu\nu}$ (still of dimension one) were the four-dimensional curl of a new *dimensionless* vector field χ_μ,

$$A_{\mu\nu} = \partial_\mu \chi_\nu - \partial_\nu \chi_\mu \,, \tag{A.1}$$

where, for simplicity, we would apply the analogue of electromagnetic Lorentz gauge

$$\partial_\mu \chi^\mu = 0 \tag{A.2}$$

giving $\partial^\nu A_{\mu\nu} = -\partial^\nu \partial_\nu \chi_\mu = \Box \chi_\mu$.

In the case of option (A.1), the hidden-sector interaction Lagrangian (1) in Section 1 takes the form

$$-\tfrac{1}{2}\sqrt{f}\,\left(\varphi F^{\mu\nu} + \zeta \bar{\psi}\,\sigma^{\mu\nu}\psi\right)\left(\partial_\mu \chi_\nu - \partial_\nu \chi_\mu\right) \tag{A.3}$$

that up to a four-divergence can be replaced by the coupling

$$-\sqrt{f}\,\left[\partial_\nu\left(\varphi F^{\mu\nu} + \zeta \bar{\psi}\,\sigma^{\mu\nu}\psi\right)\right]\chi_\mu \,. \tag{A.4}$$

Here, the formal current $\partial_\nu(\varphi F^{\mu\nu} + \zeta\, \bar{\psi}\sigma^{\mu\nu}\psi)$ (of dimension four) is *identically* conserved,

$$\partial_\mu\left[\partial_\nu\left(\varphi F^{\mu\nu} + \zeta \bar{\psi}\,\sigma^{\mu\nu}\psi\right)\right] \equiv 0 \,, \tag{A.5}$$

not providing a new gauge charge (of dimension one), since

$$Q^{(\chi)} \equiv \int d^3\vec{x}\,\partial_\nu\left(\varphi F^{0\nu} + \zeta \bar{\psi}\,\sigma^{0\nu}\psi\right) = 0 \tag{A.6}$$

($F^{00} = 0$ and $\sigma^{00} = 0$) due to the presence of ∂_ν at front of the integrand. However, this guarantees a trivial gauge invariance with respect to χ_μ , when the kinetic Lagrangian (of dimension four) for χ_μ is built up only from $A_{\mu\nu}$ (of dimension one) in the conventional way:

$$-\tfrac{1}{4}\left[\left(\partial_\lambda A_{\mu\nu}\right)\left(\partial^\lambda A^{\mu\nu}\right) - M^2 A_{\mu\nu}A^{\mu\nu}\right] \,. \tag{A.7}$$

Then, the field equations (2) and (3) in Section 1 transit respectively into the forms

$$\partial^\nu\left[F_{\mu\nu} + \sqrt{f}\,\varphi\left(\partial_\mu\chi_\nu - \partial_\nu\chi_\mu\right)\right] = -j_\mu \quad \text{or} \quad \Box A_\mu = -j_\mu - \sqrt{f}\,\partial^\nu\left[\varphi\left(\partial_\mu\chi_\nu - \partial_\nu\chi_\mu\right)\right] \tag{A.8}$$

(with $\partial^\nu A_\nu = 0$) and

$$(\Box - M^2)\Box\chi_\mu = -\sqrt{f}\,\partial^\nu\left(\varphi F_{\mu\nu} + \zeta\bar{\psi}\sigma_{\mu\nu}\psi\right) \tag{A.9}$$

(with $\partial^\nu\chi_\nu = 0$). We can derive Eq. (A.9) either applying the Euler–Lagrange equations to the Lagrangian involving χ_μ, $\partial_\mu\chi_\nu - \partial_\nu\chi_\mu$ and $\partial_\lambda(\partial_\mu\chi_\nu - \partial_\nu\chi_\mu)$ or acting on both sides of the field equation (3) with the operator ∂^ν. Note that in the low-momentum-transfer approximation, when $\Box$ can be neglected *versus* the large mass scale squared M^2, we obtain from Eq. (A.9)

$$\Box\chi_\mu \simeq \frac{\sqrt{f}}{M^2}\partial^\nu\left(\varphi F_{\mu\nu} + \zeta\bar{\psi}\sigma_{\mu\nu}\psi\right)\,. \tag{A.10}$$

In contrast, for processes near the mass shell, $\Box \sim M^2$, we can write $(\Box - M^2)\chi_\mu \simeq (\sqrt{f}/M^2)\partial^\nu\left(\varphi F_{\mu\nu} + \zeta\bar{\psi}\sigma_{\mu\nu}\psi\right)$.

Now, consider for χ_μ the vacuum solution $\chi_\mu^{(\mathrm{vac})}$ to the field equation (A.9) (with $\langle\varphi\rangle_{\mathrm{vac}} \neq 0$ and $\varphi_{\mathrm{ph}} = 0$ as well as $\psi = 0$), satisfying, therefore, the simpler field equation

$$\left(\Box - M^2\right)\Box\chi_\mu^{(\mathrm{vac})} = -\sqrt{f}\,\langle\varphi\rangle_{\mathrm{vac}}\Box A_\mu^{(\mathrm{vac})} = \sqrt{f}\,\langle\varphi\rangle_{\mathrm{vac}}\,j_\mu + f\,\langle\varphi\rangle^2_{\mathrm{vac}}\Box\chi_\mu^{(\mathrm{vac})} \tag{A.11}$$

or

$$\left(\Box - \widetilde{M}^{\,2}\right)\Box\chi_\mu^{(\mathrm{vac})} = \sqrt{f}\,\langle\varphi\rangle_{\mathrm{vac}}\,j_\mu\,, \tag{A.12}$$

where

$$\widetilde{M}^{\,2} = M^2 + f\langle\varphi\rangle^2_{\mathrm{vac}}\,. \tag{A.13}$$

In the particular case of electrically charged particle at rest at the point $\vec{x}_0$, where

$$j^\mu(\vec{x}) = e_0\,g^{\mu 0}\delta^3(\vec{x} - \vec{x}_0)\,, \tag{A.14}$$

we get from Eq. (A.12)

$$\begin{aligned}\chi_\mu^{(\mathrm{vac})}(\vec{x}) &= -\left(\frac{1}{\Delta} - \frac{1}{\Delta - \widetilde{M}^{\,2}}\right)\frac{1}{\widetilde{M}^{\,2}}\sqrt{f}\,\langle\varphi\rangle_{\mathrm{vac}}e_0\,g_{\mu 0}\,\delta^3(\vec{x} - \vec{x}_0)\\ &= \frac{e_0\sqrt{f}\,\langle\varphi\rangle_{\mathrm{vac}}}{\widetilde{M}^{\,2}}\frac{g_{\mu 0}}{4\pi|\vec{x} - \vec{x}_0|}\left(1 - e^{-\widetilde{M}|\vec{x} - \vec{x}_0|}\right)\end{aligned} \tag{A.15}$$

($\Box = \Delta - \partial_0^2$). We can see that here $\chi_\mu^{(\mathrm{vac})}$ is spontaneously generated by $\langle\varphi\rangle_{\mathrm{vac}} \neq 0$ via the field equation (A.12). In the low-momentum-transfer approximation, we can put $\widetilde{M}\,|\vec{x} - \vec{x}_0| \gg 1$ and hence obtain

$$\chi_\mu^{(\mathrm{vac})}(\vec{x}) \simeq \frac{e_0\sqrt{f}\,\langle\varphi\rangle_{\mathrm{vac}}}{\widetilde{M}^{\,2}}\frac{g_{\mu 0}}{4\pi|\vec{x} - \vec{x}_0|}\,. \tag{A.16}$$

Thus, in this approximation, the Coulomb-like dependence on $|\vec{x}-\vec{x}_0|$ dominates in $\chi^{(\mathrm{vac})}_\mu$ given in Eq. (A.15).

The space-integrated interaction energy of two charged particles at rest at the points $\vec{x}_0$ and $\vec{x}'_0$, corresponding to the solution (A.15) for $\chi^{(\mathrm{vac})}_\mu$, is equal to (*cf.* Eq. (A.4)):

$$\begin{aligned} V^{(\mathrm{vac})} &= -\int d^3\vec{x}\sqrt{f}\,\langle\varphi\rangle_{\mathrm{vac}}\partial_\nu\left[F^{0\nu}(\vec{x})\right]\chi'^{(\mathrm{vac})}_0(\vec{x}) \\ &= \int d^3\vec{x}\sqrt{f}\,\langle\varphi\rangle_{\mathrm{vac}} j^0(\vec{x})\chi'^{(\mathrm{vac})}_0(\vec{x}) + O\left(f^2\right) \\ &= \frac{e_0e'_0}{4\pi}\,\frac{f\langle\varphi\rangle^2_{\mathrm{vac}}}{\widetilde{M}^2}\,\frac{1}{|\vec{x}_0-\vec{x}'_0|}\left(1-e^{-\widetilde{M}|\vec{x}_0-\vec{x}'_0|}\right) + O\left(f^2\right) \\ &= V^{(\mathrm{vac})}(|\vec{x}_0-\vec{x}'_0|)\,, \end{aligned} \tag{A.17}$$

where $\chi'^{(\mathrm{vac})}_0(\vec{x}) = (e'_0\sqrt{f}\,\langle\varphi\rangle_{\mathrm{vac}}/\widetilde{M}^2)[1-\exp(-\widetilde{M}\,|\vec{x}-\vec{x}'_0|)]/(4\pi|\vec{x}-\vec{x}'_0|)$ due to Eq. (A.15). For $\widetilde{M}\,|\vec{x}_0-\vec{x}'_0|\to 0$ or ∞, the energy (A.17) tends to $(e_0e'_0/4\pi)f\langle\varphi\rangle^2_{\mathrm{vac}}/\widetilde{M}$ or $(e_0e'_0/4\pi)f\langle\varphi\rangle^2_{\mathrm{vac}}/(\widetilde{M}^2|\vec{x}_0-\vec{x}'_0|)\to 0$, respectively.

This not-observed-yet correction to the Coulomb energy $e_0e'_0/(4\pi\,|\vec{x}_0-\vec{x}'_0|)$ of two charged particles, generated spontaneously by $\langle\varphi\rangle_{\mathrm{vac}}\neq 0$, would require a really small value for the constant $f\langle\varphi\rangle^2_{\mathrm{vac}}/(M^2+f\langle\varphi\rangle^2_{\mathrm{vac}})$ (multiplied by $1-\exp(-\widetilde{M}\,|\vec{x}_0-\vec{x}'_0|)$) in order to be potentially acceptable. In the case of tentative assumption (42) in Section 5, where $M^2\sim\langle\varphi\rangle^2_{\mathrm{vac}}$ and boldly $f\sim e^2\simeq 0.0917$, the value of $f\langle\varphi\rangle^2_{\mathrm{vac}}/\widetilde{M}^2\sim e^2/(1+e^2)\simeq 0.0840$ is dangerously large, so that our correction to the Coulomb energy ought to be seen in experiment. Since it is not yet, this may suggest a smaller value for f. Then, $m_\psi\propto f$ becomes also smaller.

Finally, we would like to point out that our previous option discussed in Section 3, based on the constraint (22), $\vec{A}^{(B)}(x) = (+\text{ or }-)\vec{A}^{(E)}(x)$, is in contradiction with the new option (A.1) which can be considered as an "orthogonal" proposal. In fact, making use of the relations $A^{(E)}_k = A_{0k}$ and $A^{(B)}_k = -(1/2)\varepsilon_{klm}A_{lm}$ $(k=1,2,3)$ (*cf.* Eq. (7)), we obtain from Eq. (A.1) that

$$\vec{A}^{(E)} = -\partial_0\vec{\chi}-\vec{\partial}\chi_0\,,\qquad \vec{A}^{(B)} = \vec{\partial}\times\vec{\chi}\,, \tag{A.18}$$

where $\vec{A}^{(E,B)} = \left(A^{(E,B)}_k\right)$, $(\chi^\mu) = \left(\chi^0,\vec{\chi}\right)$, $(\partial_\mu) = \left(\partial_0,\vec{\partial}\right)$. Hence, applying also Eq. (A.2), we get two pairs of Maxwell-type equations (but of dimension two instead of dimension three):

$$\vec{\partial}\times\vec{A}^{(E)} = -\partial_0\vec{A}^{(B)}\,,\qquad \vec{\partial}\cdot\vec{A}^{(B)} = 0 \tag{A.19}$$

and

$$\vec{\partial} \times \vec{A}^{(B)} = \partial_0 \vec{A}^{(E)} - \Box \vec{\chi} \,, \qquad \vec{\partial} \cdot \vec{A}^{(E)} = -\Box \chi^0 \,, \tag{A.20}$$

where $-(\Box - M^2)\Box\chi^\mu$ is equal to the formal current $\sqrt{f}\,\partial_\nu(\varphi F^{\mu\nu} + \zeta\bar{\psi}\,\sigma^{\mu\nu}\psi)$ (of dimension four) according to Eq. (A.9).

In the case of free wave functions (12) of an A boson, we infer from Eqs. (A.19) that

$$\vec{k}_A \times \vec{e}_a^{\,(E)} = \omega_A \vec{e}_a^{\,(B)} \,, \qquad \vec{k}_A \cdot \vec{e}_a^{\,(B)} = 0 \quad (a = 1, 2) \tag{A.21}$$

(not $a = 1, 2, 3$), where $\left(k_A^\mu\right) = \left(\omega_A \,,\, \vec{k}_A\right)$ with $\omega_A = \sqrt{\vec{k}_A^2 + M^2}$. We choose $\vec{e}_1^{\,(E,B)} \cdot \vec{e}_2^{\,(E,B)} = 0$. Now, the field operators $\vec{A}^{(E,B)}(x)$ in the Heisenberg picture are given as in Eq. (21), but with the summation over $a = 1, 2$. The first relation (A.21) is really "orthogonal" to the constraint (18), $\vec{e}_a^{\,(B)} = (+ \text{ or } -)\vec{e}_a^{\,(E)} (a = 1, 2, 3)$, considered in Section 3. With the ansatz

$$\vec{k}_A \cdot \vec{e}_a^{\,(E)} = 0 \quad (a = 1, 2) \tag{A.22}$$

and the choice $\vec{e}_a^{\,(E)\,2} = 1$, the relations (A.21) define two orthogonal triplets $\vec{e}_a^{\,(E)}$, $\vec{e}_a^{\,(B)}$, $\vec{k}_A/\omega_A$, where $\vec{e}_a^{\,(B)\,2} = 1 - M^2/\omega_A^2 < 1$ and $(\vec{k}_A/\omega_A)^2 = 1 - M^2/\omega_A^2 < 1$, so that the vectors $\vec{e}_a^{\,(B)}$ and $(\vec{k}_A/\omega_A)$ are not versors (they become such in the limit of $\omega_A^2/M^2 \to \infty$). The versors $\vec{e}_a^{\,(E)}$ $(a = 1, 2)$ can be treated as two independent linear polarizations of an A boson.

The first Eq. (A.21)together with Eq. (A.22) implies that

$$\vec{k}_A \times \vec{e}_a^{\,(B)} = -\omega_A \left(1 - M^2/\omega_A^2\right) \vec{e}_a^{\,(E)} \quad (a = 1, 2) \,. \tag{A.23}$$

The last relation and the relation (22) show that the free gauging wave function $\chi^\mu_{a\,\vec{k}_A}(x)$ of an A boson may be presented as

$$\chi^\mu_{a\,\vec{k}_A}(x) = \frac{1}{(2\pi)^{3/2}} \frac{1}{\sqrt{2\omega_A}} \frac{1}{i\,\omega_A} e_a^\mu e^{-ik_A\cdot x} \tag{A.24}$$

with

$$\left(e_a^\mu\right) = \left(0, \vec{e}_a^{\,(E)}\right) \quad (a = 1, 2) \,. \tag{A.25}$$

Then, due to the first Eq. (A.21) and Eq. (A.22), it satisfies together with $A^{\mu\nu}_{a\,\vec{k}_A}(x)$ its defining formulae (A.1) and (A.2). Here, $\chi^0_{a\,\vec{k}_A}(x) = 0$ in consequence of the ansatz (A.22).

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