

RHIC MULTIPLICITY DISTRIBUTIONS AND SUPERPOSITION MODELS

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The recent PHENIX mid-rapidity measurements of multiplicity distributions for centrality bins are analyzed in the framework of superposition models. A simple superposition of pp events is shown to disagree with the heavy ion data for the scaled variance as a function of centrality. However, it is suggested that a model describing better the pp data and based on the “wounded quark” idea may be compatible with the multiplicity data for heavy ion collisions.

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1. Introduction

There is a long-standing inconsistency in the description of the multiple hadron production in heavy ion collisions at high energy. Many effects are attributed to the collective motion of quark–gluon plasma [1] (ideal fluid? [2]), or the collective production from such a source. Thus the commonly accepted picture adopts the idea of a collective intermediate state. However, surprisingly large amount of data can be described by assuming the superposition of independent nucleon–nucleon collisions as the main mechanism of production. Therefore it is interesting to establish in a possibly precise way the range of applicability of such an assumption.

The multiplicity distributions in selected rapidity bin measured recently by PHENIX Collaboration [3] for different “centrality classes” seem to agree with the simple rules resulting from the superposition hypothesis, if the geometrical fluctuations necessarily present for each centrality class are subtracted from the data. However, the procedure of subtracting the fluctuations relies on the Monte Carlo generator which does not describe properly

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the data. Moreover, this procedure increases significantly the uncertainty of measurements. Therefore the lack of visible discrepancies with the superposition hypothesis does not prove convincingly that the collective effects are irrelevant for the multiplicity distributions (this problem was already addressed in the analyses of the earlier data [4, 5]).

In this note we use the same PHENIX data, but do not perform any “subtractions”. Instead, we formulate a simple model which does not include any assumptions apart from the superposition idea. The heavy ion “production events” we use are simply the final states from the large number N of superimposed pp events obtained from the PYTHIA 8.107 generator [6]. To each centrality class (defined by the range of the number of charged particles observed in the dedicated detector) one may estimate the range and distribution of N to produce a proper sample of heavy ion events.

One should add here that such a construction does not mean that we neglect the obvious effects of the screening, showering or saturation effects summarized in the “wounded nucleon” [7] models. N is neither the assumed number of nucleon–nucleon collisions, nor twice the number of wounded nucleons. In fact, the estimate of the value of N for the most central AuAu collisions exceeds significantly the global number of nucleons in both colliding nuclei. Since we are interested just in testing the validity of the superposition assumption, it is enough to assume that the number n of particles produced by a number N_w of wounded nucleons is proportional to this number (with small fluctuations for large N_w). It is not necessary to assume that the proportionality coefficient is, *e.g.*, half the multiplicity of pp collisions at the same energy.

In the following section we give the details of our generation procedure and of the definitions of quantities to be compared with data. Then we present the results and compare them with the PHENIX data. Short conclusions are contained in the last section.

2. Assumptions and definitions

In this note we are using the recent C++ version of the PYTHIA 8.107 generator [6]. We generate samples of minimum bias events for the pp collisions at RHIC energies. To obtain the “heavy ion” event with a selected value of N we simply count all the particles produced in a series of N pp events.

To compare the model with the PHENIX results we have to find first the relation between N and the number N_d of particles registered in the detector BBC used to define the “centrality class”. Thus we started by generating large numbers N_{ev} of pp events divided into N_{ev}/N “superevents”, each made of N pp events. We register then for each superevent the value

of N_d (counting the number of charged particles falling into the η and Φ bins corresponding to the BBC detector) and produce the histogram of N_d corresponding to the given value of N . *E.g.*, for the AuAu collisions at 200 GeV CM energy we found the following relations:

$$\begin{aligned}\langle N_d \rangle &\approx 3.9 \langle N \rangle, \\ D^2 &\approx \langle N_d^2 \rangle - \langle N_d \rangle^2 \approx 4.7 \langle N \rangle.\end{aligned}$$

In principle, to produce a sample of heavy ion events corresponding to the given range of N_d : $N_{\min} < N_d < N_{\max}$, we have to generate superevents for all values of N in the range for which such values of N_d can occur. Thus we generate the superevents for the range of N corresponding to an extended range $N_{\min} - 3D_{\min} < N_d < N_{\max} + 3D_{\max}$ and remove afterwards the superevents for which N_d falls outside the required range. We have checked that using every second, every fourth or even every eighth value of N gives the same results as using all the values of N in the same range. This allows to shorten the calculations significantly. The number of superevents generated for each value of N should correspond to the known distribution of N_d , which for almost all the considered classes of centrality (except of the most central events) falls down exponentially with a rather small coefficient in the exponent. We assume that the distribution of N has the same shape as the measured distribution of N_d . The number of generated superevents for each N in the required range results from this distribution.

For each of the superevents in the sample corresponding to the given centrality class we count the number of the charged particles n_c in the central bin of η and Φ and p_T corresponding to the PHENIX central detector and produce a histogram of n_c for this class. As expected, the average value of n_c is simply proportional to $\langle N_d \rangle$, and thus to the weighted average of N in the sample. The main non-trivial result of our analysis is the dependence of the scaled variance of n_c on centrality, defined by the range of N_d . This variance contains a contribution from the variation of n_c for given N (which is simply proportional to N in all the superposition models), and a contribution reflecting the spread of N in the sample.

Let us repeat that we do not intend to test any particular model, in which the dependence of average multiplicities on energy and/or the mass of nuclei may be more or less compatible with data. Our modeling of the heavy ion events by superpositions of pp events allows to use the distributions of N as a useful data parametrization tool. Thus for each energy and nucleus we should repeat independently the analysis of the relation between N and N_d and define the proper sample of superevents to be compared with each sample of data.

3. Data and the superposition models

The multiplicity distribution for the central detector (registering the charged particles with pseudorapidity in the range $-0.26 < \eta < 0.26$ and the range in Φ of about two units) is parametrized by the average multiplicity $\langle n \rangle$ and the scaled variance $\omega = D^2 / \langle n \rangle$.

If the distribution is approximated by the negative binomial distribution (NBD) with the parameters m and k , we have $\langle n \rangle = m$, $\omega = 1 + m/k$. It is worth noticing that for the incoherent superposition of K such independent sources we get the multiplicity distribution with the average multiplicity multiplied by K , but the same value of ω .

In [3] the authors argued that defining the centrality bin by the range of N_d one gets the NBD shape with the k parameter rescaled by a “geometrical factor” f_{geo} , in comparison to the distribution at fixed (average) value of the number of nucleon “participants”. The value of f_{geo} estimated on the basis of Monte Carlo simulations by the HIJING generator is about 0.37 for the 200 GeV AuAu data. Thus the “dynamical” value of ω is assumed to be

$$\omega_{\text{dyn}} = 1 + f_{\text{geo}}(\omega - 1)$$

and such “corrected” data are roughly compatible for all centralities with the value measured in the pp collisions. Similar situation is seen for lower energies and for the CuCu collisions. This is regarded as the argument for the absence of collective effects in the multiplicity distributions from the heavy ion collisions.

However, the data show a systematic dependence of ω on centrality. As we shall see, with the increasing number of participants there is first the increase, and then the decrease of ω . The effect is not very strong, and becomes almost insignificant when including the error of the rescaling factor f_{geo} . Nevertheless, it seems to need an explanation.

The procedure outlined in the previous section allows to calculate the parameters of (uncorrected) multiplicity distributions in the central detector for various centrality cuts defined by PHENIX experiment. The results are shown and compared with data in Figs. 1 and 2 for the 200 GeV AuAu collisions for various numbers of participants N_p , as calculated in [1] for the centrality bins. Let us note that in our model this number has no physical meaning: it simply labels the range of N_d for each centrality class. For comparison, the data and PYTHIA results for pp collisions ($N_p = 2$) are also shown.

The average multiplicity, shown in Fig. 1 as stars, slightly overshoots PHENIX data, shown as crosses (we do not show the errors, which are of the order of the size of data points). This may be surprising, since the average multiplicity for pp collisions calculated in PYTHIA (0.198) is much

lower than that measured by PHENIX (0.32) (note that this is not visible in Fig. 1 due to the linear scale). However, one should remember that the centrality bins were defined by the number of particles in BBC detectors, which for the pp collisions is similarly too low in PYTHIA. Thus to match these experimental values one needs to superimpose a very large number of pp events (reaching more than twice the number of nucleons in two colliding nuclei!). In other words, the correlation between the average values of N_d and N is roughly described in the model in which the number of superimposed pp events is chosen in such a range that the proper range of N_d is reproduced.

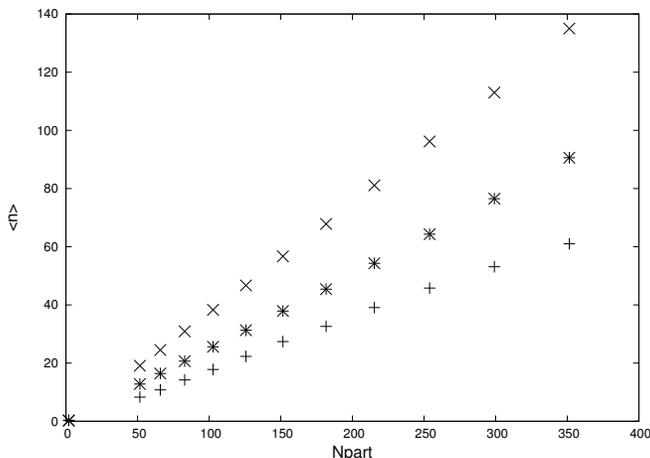


Fig. 1. The average multiplicity in the central detector for the PHENIX pp and AuAu data (crosses), superposition model (stars) and the model with extended range of ϕ (x -s) as a function of the number of participants.

The situation is different for the scaled variance, as shown in Fig. 2. For the peripheral and moderately central events (corresponding to the number of participants below 200) the values of $\omega - 1$ from PYTHIA are by a factor of 1/3 lower than the experimental values. Almost the same factor is found for the model and data for the pp collisions. This discrepancy is not removed if one increases artificially (by extending the range of ϕ) the average multiplicity from PYTHIA to fit the experimental value measured by PHENIX for the pp collisions. The resulting values, shown as x -s, are still much too low. Moreover, the agreement with data for average multiplicity is spoiled (as shown in Fig. 1).

Obviously, there is some serious problem with the variance of the pp multiplicity distribution in PYTHIA or data (or both), and this problem persists in the description of the AuAu data for moderate centralities. How-

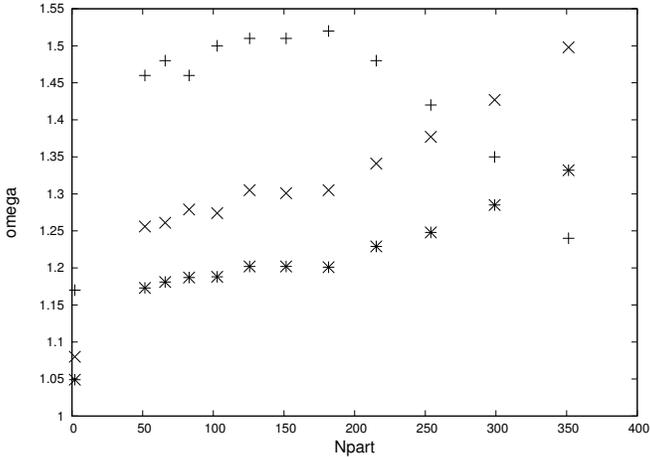


Fig. 2. The scaled variance for the PHENIX pp and AuAu data (crosses), superposition model (stars) and the model with extended range of ϕ (x -s) as a function of the number of participants.

ever, both in the model and in the data $\omega - 1$ is approximately three times higher for the AuAu than for the pp data. The slow increase with centrality is also similar. Thus the observed increase of scaled variance over the values from pp data may result from the fluctuations induced by the fluctuations in the number of participants allowed by the choice of the range of N_d .

Certainly, it would be better to use an event generator reproducing correctly the pp multiplicity distributions measured by PHENIX to test this interpretation, but our results do not exclude the possibility that the superposition of pp events may describe the multiplicity distributions for moderate centralities.

For the most central events the disagreement is much more spectacular. The scaled variance calculated for the superposition of pp PYTHIA events grows approximately linearly with N_p (corresponding to the linear increase with average N_d), whereas the data show the significant decrease and for the most central bin even fall below the model results. Thus the naïve superposition model fails to describe the data for scaled variance in the central collisions. In the next section we discuss the possible interpretation of this effect.

The observed increase of ω with centrality in the superposition model casts some doubts on the correction procedure for the “geometrical” fluctuations applied in [3]. In this procedure it was assumed explicitly that a single rescaling parameter is sufficient to correct the data for all centralities. We see that the fluctuations due to the non-zero range of centralities in each bin do not scale but increase significantly with average centrality in the bin.

4. Discussion of the results

In any superposition model the central collisions, with large number of participants, are expected to yield naturally larger multiplicity fluctuations than the less central ones. Is then the observed decrease of scaled variance a signal of some collective effects, or can one understand it within the superposition models?

To answer this question, let us first remind that the values of N needed to describe the central collisions are much higher than number of nucleons in the Au nucleus. Thus the simple “wounded nucleon” model cannot describe the data: even if all the nucleons in both nuclei are wounded, the average multiplicity from them $\langle n \rangle_{AA}$ should be just $A\langle n \rangle_{NN}$. This fact, known since quite a long time, gave rise to the so-called “wounded quark” model [8].

In such a model the pp collision results in “wounding” just a single quark from each nucleon, but in the multiple nucleon interactions during a heavy ion collision more than one quark of this nucleon is usually wounded, resulting in the enhancement of average multiplicity. In a simple version of wounded quark model considered recently [9] a nucleon consists of a quark and a diquark, both interacting similarly and yielding similar number of hadrons when wounded.

Now let us consider the multiplicity distribution from a single nucleon in such a picture assuming that the distribution of products from one wounded quark (or diquark) may be approximated by NBD with parameters $\langle n \rangle_q$ and k_q , yielding $\omega_q = 1 + \langle n \rangle_q / k_q$. During the heavy ion collision the multiplicity distribution from any nucleon may be thus parametrized as a superposition of two distributions: from one quark (with probability α) and from both constituents (with probability $1 - \alpha$). It is straightforward to prove that the parameters of the resulting distribution are

$$\begin{aligned}\langle n \rangle_1 &= \alpha \langle n \rangle_q + 2(1 - \alpha) \langle n \rangle_q = (2 - \alpha) \langle n \rangle_q, \\ \omega_1 &= 1 + \frac{\langle n \rangle_q}{k_q} + \frac{\langle n \rangle_q \alpha (1 - \alpha)}{(2 - \alpha)}.\end{aligned}$$

We see that both for $\alpha = 1$ (only one quark wounded) and for $\alpha = 0$ (both constituents wounded) the scaled variance is the same, but for the intermediate values of α an additional positive term appears and ω_1 has a maximum at $\alpha = 0.5$.

If we consider a class of events corresponding for some range of centralities (defined, *e.g.*, by the number of participant nucleons N) and assume fixed α in this range, the multiplicity distribution parameters will read

$$\begin{aligned}\langle n \rangle &= \langle n \rangle_1 \langle N \rangle, \\ \omega &= \omega_1 + \langle n \rangle_1 \omega_N,\end{aligned}$$

where $\omega_N = (\langle N^2 \rangle - \langle N \rangle^2) / \langle N \rangle$ for the given range of N . If α increases from 0 to 1 for increasing $\langle N \rangle$, the first term passes through a maximum at $\alpha = 0.5$, and a similar maximum may appear in the dependence of ω on $\langle N \rangle$. Obviously, fixed α for each range of N is not a realistic assumption, but may serve as a first approximation.

This suggests that the wounded quark model may explain, at least qualitatively, the non-monotonic dependence of ω on centrality, as the increase of α with $\langle N \rangle$ is a very natural feature of this model. For the peripheral collisions most of the nucleons interact only once and thus only one quark in each of them is wounded. For the central collisions almost all nucleons interact more than once and both of their constituents are wounded.

5. Conclusions

We have investigated the PHENIX data of the multiplicity distributions in the central rapidity region for changing centrality in heavy ion collisions. We use a simple model, in which each final state for a heavy ion collision is constructed as a superposition of many pp events and the combination of such states is arranged to fit the experimental definition of various centrality classes.

We show that the model which describes roughly the average multiplicity fails to describe the PHENIX data on scaled variance. For moderate centralities the disagreement may result simply from the imperfection of the model for pp collisions. For most central events the discrepancy between the model and data is more severe, but it is qualitatively similar to the effect expected in the wounded quark model. A construction of a superposition model for the multiplicity distributions for different centralities based on this idea would be desirable to check if the superposition mechanism is really excluded by the PHENIX data. Some time ago a “quark participants” model was already shown to describe the centrality dependence of the average multiplicity [10].

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