

PARTICLE–PARTICLE AND HOLE–HOLE RANDOM PHASE APPROXIMATION CALCULATIONS FOR ^{42}Ca AND ^{38}Ca NUCLEI

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The nuclear structure of some closed shell ± 2 -nucleons nuclei ^{42}Ca and ^{38}Ca is studied in the framework of the particle–particle and hole–hole Random Phase Approximation (pp RPA and hh RPA) with the space of wave functions being extended to include orbits up to the $2d_{5/2}$. This model assumes that the low-lying states of ^{42}Ca are correlated two-particle operators acting on a correlated ^{40}Ca core, and those of ^{38}Ca are correlated two-hole operators acting on the same correlated ^{40}Ca ground state. The Hamiltonian is to be diagonalized in this extended space in the presence of the Modified Surface Delta Interaction (MSDI). The spectra of nuclear excitation energy levels for both ^{42}Ca and ^{38}Ca are compared with the experimental data. The pp RPA checked by using the resultant eigenvalues and eigenvectors to calculate the longitudinal form factors of the inelastic electron scattering, and then compared with the available experimental data. More correlations to the ground state are assumed by the inclusion of admixture from higher orbits, which leads to enhance the calculations of the longitudinal form factors. Effective charges are also used to account for the core polarization effect.

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1. Introduction

The problem of a nuclear structure is a many body problem. Various approaches exist to deal with the nuclear many-body problem. A comparison of these approaches has been given in Ref. [1]. The Hartree–Fock (HF) self

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consistent field method is approximation for reducing the problem of many interacting particles to one of non-interacting particles in a field. Clearly this affects an enormous simplification of the problem, but ignores correlations in the ground state and the residual interaction (part of the inter-nucleon force). HF theory can also be made time-dependent (TDHF) to describe excited states and to take into account, in particular, the long range or field-producing part of the residual interaction. This theory is also expressed in other languages as the Random phase approximation (RPA). In the case of closed shell nuclei, the simplest correlation beyond the Hartree–Fock can only be taken into account by breaking the HF core and raising a nucleon from below to above the Fermi level; then the resulting states must have a particle–hole pair. The excited collective oscillation can be described as a linear combination of particle–hole states. Such an approximation is called the particle–hole Tamm–Dancoff Approximation (*ph* TDA). A system of state more general than that considered in the *ph* TDA appears when treating the ground states and the excited states more symmetrically. In that case, the ground states and the excited states are treated on the same footing; both the ground states and the excited states can be described as a linear combination of particle–hole states. Such an approximation is referred as the particle–hole Random Phase Approximation *ph* RPA [2,3].

In the calculations of the simple shell model, the spectra of two valence particle nuclei formed by the addition of two particles beyond an inert core. Correlation among the valence nucleons alone seem to be responsible for a variety of experimental facts known about these nuclei. A whole theory has been developed in treating open shell nuclei, known most widely under the heading of the shell model configuration mixing calculation [4]. The theory of collective motion in nuclei can be treated through the theory of random phase approximation (RPA) where the particle–particle Random Phase Approximation (*pp* RPA) assumes that the low-lying states of $(A + 2)$ nuclei are correlated two-particle operators acting on a correlated core and the hole–hole Random Phase Approximation (*hh* RPA) assumes that the low-lying states of $(A - 2)$ nuclei are correlated two-hole operators acting on the same core [5,6].

In this study, the structure of ^{42}Ca and ^{38}Ca is to be studied in the framework of the *pp* and *hh* RPA with the wave functions is to be extended to include $1d_{5/2}$, $2s_{1/2}$, $1d_{3/2}$, $1f_{7/2}$, $2p_{3/2}$, $1f_{5/2}$, $2p_{1/2}$, $1g_{9/2}$, and $2d_{5/2}$ orbits. This model assumes that the low-lying states of ^{42}Ca are correlated two-particle operators acting on a correlated ^{40}Ca core, and those of ^{38}Ca are correlated two-hole operators acting on the same ^{40}Ca ground state. The Hamiltonian is diagonalized in this extended space in the presence of the modified surface delta interaction (MSDI). The low-lying spectra of nuclear excitation energy levels for both ^{42}Ca and ^{38}Ca are compared with the

experimental data. The collective low-lying isovector ($T = 1$) transitions in ^{42}Ca nuclei are studied, these transitions include states. The radial wave functions for the single-particle matrix elements were calculated with the harmonic oscillator (HO) potential. More correlations to the ground state are assumed by the inclusion of admixture from higher orbits, which leads to enhance the calculations of the longitudinal form factors. Effective charges are also used to account for the core polarization effects.

2. Formulations

The collective excited states of the $A + 2$ and $A - 2$ systems of multipolarity J and isospin T are generated by operating on the ground state $|0\rangle$ of A nucleons system with operators $Q_{\tau, JT}^\dagger$ and $P_{\lambda, JT}^\dagger$, respectively [5,7]:

$$Q_{\tau, JT}^\dagger |0\rangle = |A+2, \tau, JT\rangle = \left(\sum_{m \leq n} X_{mn}^{\tau, JT} a_m^\dagger a_n^\dagger - \sum_{i \leq j} Y_{ij}^{\tau, JT} a_j^\dagger a_i^\dagger \right) |A, 0\rangle, \quad (1)$$

$$P_{\lambda, JT}^\dagger |0\rangle = |A-2, \lambda, JT\rangle = \left(\sum_{i \leq j} X_{ij}^{\lambda, JT} a_i a_j - \sum_{m \leq n} Y_{mn}^{\lambda, JT} a_m a_n \right) |A, 0\rangle, \quad (2)$$

where

$$Q_{\tau, JT}|0\rangle = P_{\lambda, JT}|0\rangle = 0. \quad (3)$$

The operator $Q_{\tau, JT}^\dagger$ creates two particle to give states of the nucleus with atomic mass $A + 2$, while the operator $P_{\lambda, JT}^\dagger$ destroys two particles (creates two holes) to give states of the nucleus with atomic mass $A - 2$. The label τ and λ distinguishes states with the same angular momentum and parity. $a^\dagger$ and a are creation and annihilation operators. Here and below $m \equiv n_m, \ell_m, j_m$, both indices (mn) represent the quantum numbers of orbits above the Fermi sea, and we shall refer to these states as particle states. Indices (ij) represent orbits below the Fermi sea and we call such states hole states. $X_{mn}^{\tau, JT}$ and $Y_{mn}^{\lambda, JT}$ are particle-particle pp eigenvectors (amplitudes), $Y_{ij}^{\tau, JT}$ and $X_{ij}^{\lambda, JT}$ are hole-hole hh eigenvectors.

Using the ansatz of Eqs. (1) and (2) for the wave function and linearizing the equation of motion to the familiar pp RPA and hh RPA eigenvalue problem

$$\begin{pmatrix} A^{\tau, \lambda} & B^{\tau, \lambda} \\ B^{\dagger \tau, \lambda} & C^{\tau, \lambda} \end{pmatrix} \begin{pmatrix} X^{\tau, \lambda} \\ Y^{\tau, \lambda} \end{pmatrix} = \hbar \Omega_{\tau, \lambda} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} X^{\tau, \lambda} \\ Y^{\tau, \lambda} \end{pmatrix}, \quad (4)$$

with

$$\begin{aligned} X^\tau &= X_{pp}^{\tau, JT} = X_{mn}^{\tau, JT}, & A^\tau &= A_{mnm'n'}^{\tau, JT} = \delta_{mm'}\delta_{nn'}(\epsilon_m + \epsilon_n) + V_{mnm'n'}^{JT}, \\ Y^\tau &= Y_{hh}^{\tau, JT} = Y_{ij}^{\tau, JT}, & C^\tau &= C_{iji'j'}^{\tau, JT} = -\delta_{ii'}\delta_{jj'}(\epsilon_i + \epsilon_j) + V_{iji'j'}^{JT}, \\ B^\tau &= B_{mni}^{\tau, JT} = -V_{mni}^{JT}, \end{aligned} \quad (5)$$

and

$$\begin{aligned} X^\lambda &= X_{hh}^{\lambda, JT} = X_{ij}^{\lambda, JT}, & A^\lambda &= A_{iji'j'}^{\lambda, JT} = -\delta_{ii'}\delta_{jj'}(\epsilon_i + \epsilon_j) + V_{iji'j'}^{JT}, \\ Y^\lambda &= Y_{pp}^{\lambda, JT} = Y_{mn}^{\lambda, JT}, & C^\lambda &= C_{mnm'n'}^{\lambda, JT} = \delta_{mm'}\delta_{nn'}(\epsilon_m + \epsilon_n) + V_{mnm'n'}^{JT}, \\ B^\lambda &= B_{ijmn}^{\lambda, JT} = -V_{ijmn}^{JT}. \end{aligned} \quad (6)$$

ϵ is the single-particle energy which can be obtained from [8]:

$$\epsilon_{n\ell j} = (2n + \ell - 1/2)\hbar\omega + \begin{cases} -\frac{1}{2}(\ell + 1)\langle f(r) \rangle_{n\ell} & \text{for } j = \ell - 1/2, \\ -\frac{1}{2}\ell\langle f(r) \rangle_{n\ell} & \text{for } j = \ell + 1/2, \end{cases} \quad (7)$$

with $\langle f(r) \rangle_{n\ell} \approx -20A^{-2/3}$ MeV and $\hbar\omega = -45A^{-1/3} - 25A^{-2/3}$ MeV [9].

By diagonalizing the matrix of Eq. (4), one gets the eigenvalues (excited state) $\hbar\Omega_\tau$ for the $A + 2$ system (nuclides of type: core+2 proton, +2 neutron, +proton+neutron) with eigenvectors (amplitudes) $X_{mn}^{\tau, JT} = Y_{ij}^{\tau, JT}$, and the eigenvalues $\hbar\Omega_\lambda$ for the $A - 2$ system (nuclides of type: core-2 proton, -2 neutron, -proton-neutron) with eigenvectors $X_{ij}^{\lambda, JT} = Y_{mn}^{\lambda, JT}$.

The matrix element V can be defined in terms of the quantum number associated with the single-particle states that participate in the particle-particle and hole-hole configurations, (mn) and (ij) , and for an isospin dependent modified surface delta interaction MSDI can be write them as [8]:

$$\begin{aligned} V_{ab,cd}^{JT} &= (-1)^{n_a+n_b+n_c+n_d} \frac{A_T}{2(2J+1)} \sqrt{\frac{2(2j_a+1)(2j_b+1)(2j_c+1)(2j_d+1)}{(1+\delta_{ab})(1+\delta_{cd})}} \\ &\times \left\{ h_J(j_a j_b) h_J(j_c j_d) \left[1 - (-1)^{J+T+\ell_a+\ell_b} \right] \right. \\ &\quad \left. - K_J(j_a j_b) K_J(j_c j_d) \left[1 + (-1)^T \right] \right\} \\ &+ \left\{ [2T(2T+1) - 3]B + C \right\} \delta_{ac} \delta_{bd}, \end{aligned} \quad (8)$$

where

$$h_J(j_a j_b) = (-1)^{j_b+\ell_b} \langle j_b - 1/2 \ j_a \ 1/2 \ | J0 \rangle, \quad (9)$$

$$K_J(j_a j_b) = \langle j_b \ 1/2 \ j_a \ 1/2 \ | J1 \rangle, \quad (10)$$

$$A_T = \begin{cases} A_0 & T = 0, \\ A_1 & T = 1, \end{cases}$$

and A_0 , A_1 , B and C are the strength parameters of the MSDI obtained from fits to experimental spectra in various mass region.

The reduced single particle matrix elements reduced in both spin and isospin, are written in terms of the single-particle matrix elements reduced in spin only [8]

$$\langle j_a | \left\| \hat{T}_{JT} \right\| | j_b \rangle = \sqrt{\frac{2T+1}{2}} \sum_{t_z} I_T(t_z) \langle j_a | \left\| \hat{T}_{Jt_z} \right\| | j_b \rangle, \quad (11)$$

with

$$I_T(t_z) = \begin{cases} 1 & \text{for } T = 0, \\ (-1)^{1/2-t_z} & \text{for } T = 1, \end{cases} \quad (12)$$

where $t_z = 1/2$ for proton and $-1/2$ for neutron.

The reduced matrix element of a one-particle operator between multi-particle states can be expressed as a sum of the product of the elements of multi-particle transition amplitudes times the single-particle matrix elements. Can be written as:

$$\begin{aligned} \langle J_f T_f | \left\| \hat{T}_{JT}^\eta \right\| | J_i T_i \rangle &= \sum_{a \leq b} X_{JT}^{\tau, \lambda}(ab) \langle j_a | \left\| \hat{T}_{JT}^\eta \right\| | j_b \rangle \\ &+ \sum_{c \leq d} Y_{JT}^{\tau, \lambda}(cd) \langle j_c | \left\| \hat{T}_{JT}^\eta \right\| | j_d \rangle, \end{aligned} \quad (13)$$

where

$$X_{JT}^{\tau, \lambda}(ab) = \begin{cases} X_{JT}^\tau(mn) & \text{for } pp, \\ X_{JT}^\lambda(ij) & \text{for } hh, \end{cases} \quad \text{and} \quad Y_{JT}^{\tau, \lambda}(cd) = \begin{cases} Y_{JT}^\tau(ij) & \text{for } pp, \\ Y_{JT}^\lambda(mn) & \text{for } hh, \end{cases} \quad (14)$$

are the eigenvectors (amplitudes), which are obtained from the diagonalization of Eq. (4). η selects the longitudinal (L), transverse electric (E) and transverse magnetic (M) operators, respectively.

Electron scattering form factor involving the angular momentum J and the momentum transfer q , between the initial and final nuclear shell model

states of spin $J_{i,f}$ and isospin $T_{i,f}$ are [10]:

$$|F_J^L(q)|^2 = \frac{4\pi}{Z^2(2J_i + 1)} \left| \sum_{T=0,1} \begin{pmatrix} T_f & T & T_i \\ -T_z & 0 & T_z \end{pmatrix} \langle J_f T_f | \hat{T}_{JT}^\eta | J_i T_i \rangle \right|^2 \\ \times F_{\text{cm}}^2(q) F_{fs}^2(q), \quad (15)$$

where T_z is given by $T_z = (Z - N)/2$. The nucleon finite size (fs) form factor is $F_{fs}(q) = e^{-0.43q^2/4}$ and $F_{\text{cm}}(q) = e^{-q^2b^2/4A}$ is the correction for the lack of translational invariance in the shell model (center of mass correction), where A is the mass number and b is the harmonic oscillator size parameter.

The reduced single-particle matrix element of the Coulomb operator is given by [2]

$$\langle j_a | \hat{T}_{Jt_z}^L | j_b \rangle = \int_0^\infty dr r^2 j_J(qr) \langle j_a || Y_J || j_b \rangle R_{n_a \ell_a}(r) R_{n_b \ell_b}(r), \quad (16)$$

where $j_J(qr)$ is the spherical Bessel function and $R_{n\ell}(r)$ is the single-particle radial wave function.

3. Results and calculations

The structure of ^{42}Ca and ^{38}Ca is to be studied in the framework of the pp and hh RPA with the wave function is to be extended to include $1d_{5/2}$, $2s_{1/2}$, $1d_{3/2}$, $1f_{7/2}$, $2p_{3/2}$, $1f_{5/2}$, $2p_{1/2}$, $1g_{9/2}$, and $2d_{5/2}$ orbits. This model assumes that the low-lying states of ^{42}Ca are correlated two-particle operators acting on a correlated ^{40}Ca core, and those of ^{38}Ca are correlated two-hole operators acting on the same ^{40}Ca ground state. The Hamiltonian is diagonalized in this extended space in the presence of the modified surface delta interaction (MSDI).

The pp RPA checked by using the resultant eigenvalues and eigenvectors to calculate the longitudinal form factors of the inelastic electron scattering, and then compared with the available experimental data. Results are interpreted in terms of the harmonic oscillator wave functions. Effective charges are also used to account for the core polarization effects. In the calculation of the form factors, the ground state wave function is modified to include the admixture from higher configuration parameters that mixes the state $|n\ell j\rangle$ with amplitudes γ with the states $|n+1\ell j\rangle$ with amplitudes δ , such that

$$|^{42}\text{Ca g.s.}\rangle = \gamma |(1d_{5/2})(2s_{1/2})(1d_{3/2})\rangle + \delta |(2d_{5/2})(3s_{1/2})(2d_{3/2})\rangle, \quad (17)$$

where $\gamma^2 + \delta^2 = 1$.

3.1. The ^{42}Ca nucleus

According to the simple shell-model picture ^{42}Ca can be described in terms of two neutrons placed in the lower orbits of f - p shell and coupled to an inert ^{40}Ca core. However, this picture fails to explain the experimental spectrum of ^{42}Ca . According to pp RPA the ground states as well as the excited states are constructed by removing a particle from the closed shells and promoting it to a higher shell leaving a hole state within the closed shells.

In this work, using the modified surface delta interaction MSDI, we have solved the pp RPA Eqs. (4) and (5) within a model space with particles in orbits $1f_{7/2}$, $2p_{3/2}$, $1f_{5/2}$, $2p_{1/2}$, $1g_{9/2}$, and $2d_{5/2}$ and holes in $1d_{5/2}$, $2s_{1/2}$ and $1d_{3/2}$. These orbits will be denoted as orbits (mn) and (ij) , respectively. The (J^+, T) combinations $(0^+, 1)$, $(2^+, 1)$, $(4^+, 1)$, $(6^+, 1)$ and $(8^+, 1)$ are allowed for the two extra particles to occupy the same particle orbit. The unperturbed energies for these positive parity states are calculated using Eq. (7). The scope of this study is restricted including nine 0^+ excitations, the sixteen 2^+ excitations, the eleven 4^+ excitations, the four 6^+ excitations and one 8^+ excitation.

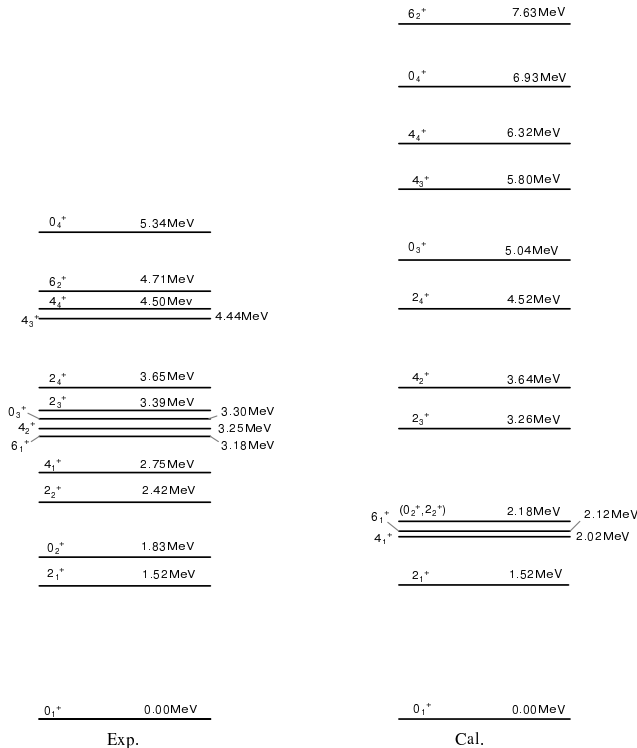


Fig. 1. Comparison of the experimental observed energy levels of ^{42}Ca with our pp RPA calculations. Experimental data are from Ref. [11].

The low-lying positive parity, $T = 1$ spectra of ^{42}Ca are presented in Fig. 1. Our results are reported and plotted in second column and compared with experimental spectrum [11] (first column). In agreement with experiment, the first 2^+ state occurs at 1.52 MeV, this state is missing from the spectrum of the simple-shell model calculations [12], but the results of two-particle RPA calculation within a model space smaller than those used in this work (presented by Bouyssy and Vinh Mau [13]) predicts that the first 2^+ state is high in energy by nearly 1 MeV. Our first 4^+ and 6^+ states are below the corresponding experimental states, this was already obtained from the deformed state calculations presented by Flowers and Skouras [14]. The overall agreement with the experimental systematic is modest.

Our calculated longitudinal $C2$ form factor for the 2_1^+ state is shown in Fig. 2. The radial wave functions for the single-particle matrix elements were calculated with the harmonic oscillator (HO) potential. The oscillator length parameter $b = 2.10$ fm was chosen to reproduce the measured root mean square charge radius [15]. The dashed curve represents the longitudinal form factor calculation without using the core-polarization effects (effective charges) and without introducing the admixture of higher orbits

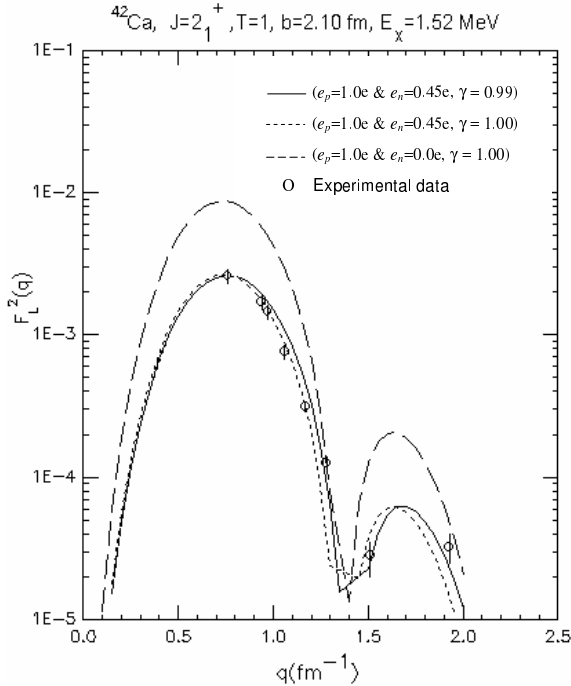


Fig. 2. Calculated and experimental longitudinal form factor of ($T = 1$) state in ^{42}Ca . Experimental data are from Ref. [16].

in the ground states, this result over predicts the experimental data [16] by a factor of five. The dotted curve is calculated using effective charges $e_p = 1.0e$ and $e_n = 0.45e$ to fit the data. A good agreement with the experimental is shown by the solid curve with effective charge $e_p = 1.0e$ and $e_n = 0.45e$ and by introducing the admixture of higher orbits in the ground states with $\gamma = 0.99$ difficult to reproduce by the simple $f_{7/2}^2$ shell model [16,17]. The admixture of α -cluster configuration in the 2_1^+ state is presented by Toshimi Sakuda and Shigeo Ohkubo [18], this model disagreed in high q values more than 1.5 fm^{-1} .

The 2_2^+ (2.42 MeV), C2 form factor is shown in Fig. 3 compared with the experimental data from Ref. [16]. The dashed curve represents the calculations without effective charges and without introducing the admixture of higher orbits in the ground states. The calculations are based on the single particle wave functions of the HO potential with size parameter $b = 2.1 \text{ fm}$. The calculation does not yield an adequate description of the measured longitudinal form factor, even with the inclusion of the core-polarization effects (dotted curve). The deviations of the longitudinal form factors for this state

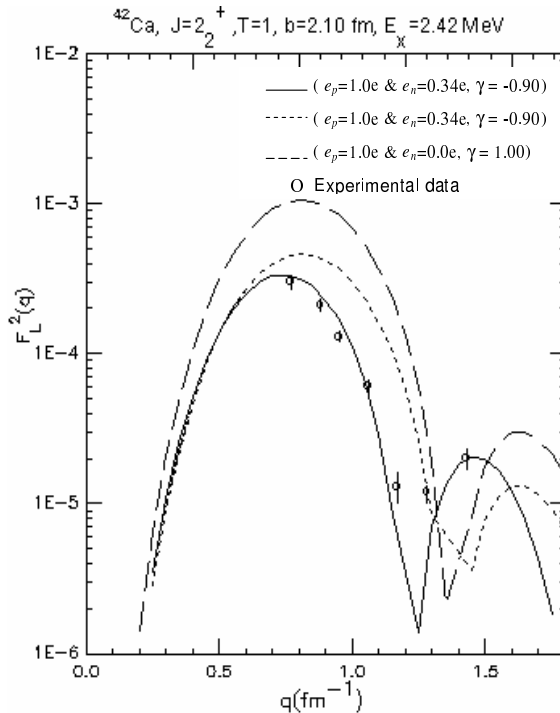


Fig. 3. Calculated and experimental longitudinal form factor of ($T = 1$) state in ^{42}Ca . Experimental data are from Ref. [16].

reflect the fact that enormous degrees of collectivity are required for proper treatment of this state. In this work, we removed this deviation by introducing the admixture of higher orbits in the ground states. Therefore, a good agreement with the experimental is shown by the solid curve with effective charge $e_p = 1.0e$ and $e_n = 0.34e$ and with $\gamma = -0.90$. Our prediction to diffraction minima is an excellently consistent with that of experimental data at 1.2 fm^{-1} , while α -cluster configuration [18] predict it at 1.5 fm^{-1} .

3.2. The ^{38}Ca nucleus

In this study, the structure of ^{38}Ca is to be studied in the framework of the hole-hole Random Phase Approximation. The system of hh RPA Eqs. (4) and (6) are solved in a model space which involves the $1d_{5/2}$, $2s_{1/2}$ and $1d_{3/2}$ orbits for hole states and $1f_{7/2}$, $2p_{3/2}$, $1f_{5/2}$, $2p_{1/2}$, $1g_{9/2}$ orbits for particle states. The unperturbed energies are calculated using Eq. (7), then the configurations mixing is allowed and the Hamiltonian is diagonalized in the presence of a modified surface delta interaction (MSDI).

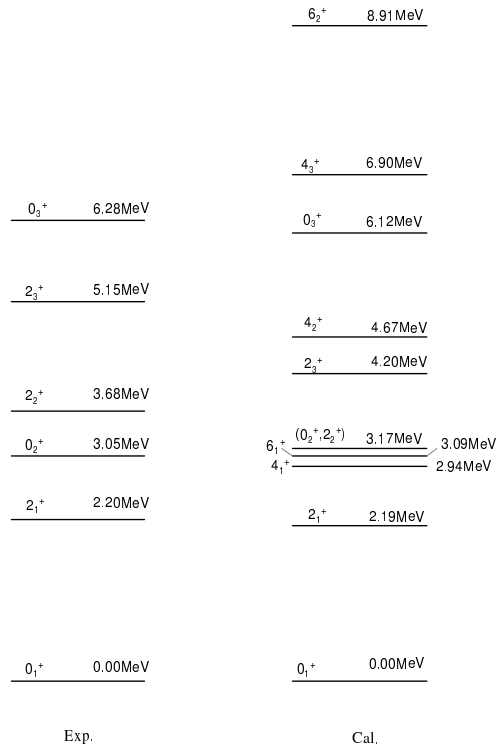


Fig. 4. Comparison of the experimental observed energy levels of ^{38}Ca with our hh RPA calculations. Experimental data are from Refs. [11,19].

In Fig. 4 we have plotted the low-lying positive parity, $T = 1$ spectra of ^{38}Ca . Our calculations are compared with the few available experimental values (experimental values are taken from Refs. [11,19]). Our prediction for the first 2_1^+ state is at 2.19 MeV in excellent agreement with the experimental value 2.20 MeV, Bouyssy and Vinh Mau [13] predicts this state is low in energy by nearly 0.5 MeV. The 2_2^+ and 2_3^+ well positioned and shifted by 0.6 MeV and 0.68 MeV, respectively. Our third 0_3^+ state is obtained at 6.49 MeV in very good agreement with experiment an error about 2.5%. The overall agreement with the available experimental data is good. In general this nucleus needs more experimental studies.

4. Conclusions

The structure of closed shell-nucleons nuclei ^{42}Ca and ^{38}Ca was studied in the framework of the pp RPA and hh RPA, respectively. According to the results, a number of conclusions can be extracted; pp RPA and hh RPA had success in prediction the excited energies (eigenvalues) spectrum of certain states in the calcium isotopes ^{42}Ca and ^{38}Ca . When the model space is extended to include orbits up to $2d_{5/2}$, the results give reasonable description of the experimental data. The core polarization effects are essential in obtaining a reasonable description of the electron scattering data. The longitudinal form factors are enhanced when the effective charges are included. Better agreements with the available data are found by introducing the admixture of higher orbits in the ground states.

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