EXTRACTION OF THE NEUTRINO OSCILLATION PARAMETERS FROM DIRECT OBSERVABLES*

Jakub Żmuda

Institute of Theoretical Physics, University of Wrocław, Poland
pl. M. Borna 9, 50-204 Wrocław

(Received May 17, 2010)

An alternative method of the neutrino oscillation parameter extraction is discussed. It is based on the directly observed quantities in opposition to the traditional neutrino energy reconstruction. The Monte Carlo oscillation parameter extraction algorithm is tested on the example of the predicted T2K beam profile and event statistics for the Super-Kamiokande detector. A set of MC data samples is generated using the NuWro neutrino generator in order to estimate the statistical error of the proposed method.

PACS numbers: 14.60.Pq

1. Introduction

The past decade has brought a big development in the field of neutrino long baseline experiments. The K2K experiment has discarded the no-oscillation hypothesis at the level of $4.3\,\sigma$. It has also made an attempt to measure the Θ_{23} and Δm^2_{23} parameters, although with a large uncertainty [1]. New experiments, like the currently operating T2K, aim to measure these parameters with significantly higher precision, mainly due to the huge improvement in the neutrino flux intensity. One can compare the 58 measured one-ring muon events and 122 in total in K2K with expected thousands of ν_μ CC events in T2K [2, 3]. This fact opens new possibilities in the field of data analysis methodology. The methodology, which will be described below, has already been introduced in [4].

1.1. Motivation

The main motivation for seeking a method alternative to the traditional one is the requirement of neutrino energy reconstruction for each recorded event. This procedure is based on a few assumptions, which are in many

* Presented at the Cracow Epiphany Conference on Physics in Underground Laboratories and Its Connection with LHC, Cracow, Poland, January 5–8, 2010.

cases not justified. First of all the interaction, in which the detected particles were produced, is assumed to be of the charged current quasi-elastic nature (CCQE). Second assumption is the approximation of all of the complicated nuclear dynamics by a simple potential well with the mean binding energy ϵ_{b}. The Fermi motion is neglected and the target nucleon is assumed to be at rest, having free on-shall mass M. The neutrino energy is reconstructed from the detected charged lepton kinematics (here the muon). There are two variables: the lepton production angle Θ_μ with respect to the neutrino beam direction and the energy ε_μ (or, alternatively, its momentum p_μ). The resulting formula is:

$$E_{\mathrm{rec}} = \frac{\varepsilon_\mu (M - \epsilon_{\mathrm{b}}) + \frac{1}{2}\left(2M\epsilon_{\mathrm{b}} - \epsilon_{\mathrm{b}}^2 - m_\mu^2\right)}{(M - \epsilon_{\mathrm{b}}) - \varepsilon_\mu + p_\mu \cos(\Theta_\mu)} \,. \tag{1}$$

The denominator in the above formula may lead to occurrence of a singularity and thus one has to introduce lower limits on its value *e.g.* 200 MeV. Considering the mean neutrino energies in the T2K beam (peaked around 700 MeV) one concludes, that:

- The Fermi motion of nucleons inside the nucleus may be a large fraction of the reconstructed energy. For example, the Fermi momentum of oxygen is approximately 225 MeV. The oxygen nuclei are the main targets in the water Cherenkov detectors, like the SK. If one uses the nuclear spectral function formalism, the influence of nucleon motion is even bigger.

- The contribution of other dynamics, like the production of pions, becomes significant. One needs less, then 140 MeV of the energy transfer to produce an on-shell π^+. There are also important contributions coming from other processes. If the products of such interactions are stable particles, they may remain undetected for low-energy parent neutrinos, leading to the confusion of non-QE processes with the CCQE processes and an error in the neutrino energy reconstruction.

It should be recalled here, that the K2K experiment has used in the analysis the likelihood functions dependent on the reconstructed neutrino energy and its estimated uncertainty [1–3]. This has worked in a satisfactory way for the beam with the neutrino energies above 1 GeV. However, for the neutrino energies below 1 GeV the above pointed flaws in the neutrino energy reconstruction procedure may introduce a significant and hard to control error [5]. This is the main motivation to seek for alternative methods of the oscillation parameters extraction. This problem has been addressed both by the author and by F. Di Lodovico from Queen Mary University of London, and suggested by F. Sanchez from IFAE.

2. Direct observable-based neutrino oscillation parameters measurement

The main idea for a solution to the issue of oscillation parameters extraction without the knowledge of the neutrino energy has come from the specifications of the modern neutrino appearance/disappearance experiments. The T2K experiment, which is a core example in these considerations, will measure the muon neutrino disappearance due to the oscillations. It will operate at the baseline of the length of 295 km in order to measure the Θ_{23} and Δm_{23}^2, which are the leading order parameters responsible for the ν_μ oscillations. It is also supposed to answer the question, whether Θ_{13} is nonzero by searching for the ν_e appearance. The expected errors of $\sin^2(2\Theta_{23})$ and Δm_{23}^2 are less than 1% and 4%, respectively. Thus the proposed method should be at least as accurate as these predictions.

The most important fact about this experiment is that in the far Super-Kamiokande detector one would collect about 1600 ν_μCC events/year if there were no oscillations [2, 3]. The size of these statistics suggests, that one could try to extract the leading oscillation parameters from the distributions of directly measurable variables describing the muons in the far detector (Super-Kamiokande). These variables are the muon energy ϵ_μ (or momentum p_μ) and the cosine of its production angle $\cos(\Theta_\mu)$. One must know the following:

- The beam profile (monitored at ND280).
- The expected number of events in Super-K.
- The detector parameters, such as the angular and energetic resolutions as well as the particle detection thresholds.
- The best available way of describing the nuclear dynamics for a Monte Carlo simulation.

One must produce the distributions of events in $(\epsilon_\mu, \cos(\Theta_\mu))$ or $(p_\mu, \cos(\Theta_\mu))$ for the different values of $\sin^2(2\Theta_{23})$ and Δm_{23}^2 and then compare them with the experimental data using the appropriate statistical estimator.

F. di Lodovico assumed that it is possible to predict the $(p_\mu, \cos(\Theta_\mu))$ muon event distribution in the Super-Kamiokande having given the distributions in the near ND280 detector and using the knowledge about their response to the different types of events.

Both of these approaches assume, that it is enough to look for the muon distributions in $(\epsilon_\mu, \cos(\Theta_\mu))$ and omit the neutrino energy reconstruction step and thus the error related to it.

2.1. The algorithm

The idea of measuring the oscillation parameters, presented in the previous subsection, is realised by the following algorithm. All the primary (without oscillations) sets of events were produced by the `NuWro` Monte Carlo generator [6]:

1. Generate a large number of the CC neutrino events with a Monte Carlo generator for the specified neutrino beam profile (author used the sample of about 1 000 000 events). The size must provide "continuous" distribution of the events.

2. Impose the detector conditions like:
   - The approximate limit for the detection of the charged particle is its Cherenkov threshold.
   - The neutral pions are always visible.

3. Discard all the events with the visible pions.

4. Create the reference oscillation samples for a set of different parameters $(\Delta m^2_{23},\ \sin^2(2\Theta_{23}))$ using the muon neutrino survival probability $P(\nu_\mu \to \nu_\mu)$.

5. Create the $(\epsilon_\mu, \cos(\Theta_\mu))$ distributions for each data set corresponding to the given values of $(\Delta m^2_{23},\ \sin^2(2\Theta_{23}))$. Re-weight them according to the expected number of events in experiment with the scaling weight $W_{\rm s}$.

   Each of these sample MC distributions represents a *typical* set of events as they are expected to be seen in SK.

6. Create similar $(\epsilon_\mu, \cos(\Theta_\mu))$ maps for the experimental data.

7. Compare the experimentally measured muon distribution with each of the reference samples. Find the best fit using an appropriate statistical estimator and thus identify the data with the pair of oscillation parameters $(\sin^2(2\Theta_{23}), \Delta m^2_{23})$. Because the number of events in each bin is being measured here, there are two basic tests, which can be performed:

   (a) If the typical number of events in a bin exceeds 10, one can use the χ^2 estimator:

$$\chi^2 = \frac{1}{N_{\rm b}-2}\sum_i \frac{(N_i - n_i)^2}{n_i}\,. \tag{2}$$

(b) If one wants to consider the bins with the typical number of events below ten, then the Poisson statistical estimator [7] is used (it is applicable even for the bins with only three events expected):

$$F = 2\sum_i \left[n_i - N_i + N_i \ln \frac{N_i}{n_i} \right] . \tag{3}$$

In both cases the index i labels the bins with data. The number of measured events in each bin is N_i, whereas n_i is the expected number of events predicted by the Monte Carlo simulation. The normalisation factor of χ^2 is $N_\mathrm{b} - 2$, *i.e.* the number of bins minus two unknown parameters, which are measured: $\sin^2(2\Theta_{23})$ and Δm^2_{23}.

2.2. Performance of the method

The tests of this method have been performed for the Monte Carlo data. In order to estimate the statistical error the following steps have been performed:

1. An additional independent large sample of events for the T2K beam has been created.

2. The same detector conditions and cuts have been imposed as for the typical sets of events.

3. The oscillation samples for a few chosen parameters $(\Delta m^2_{23}, \sin^2(2\Theta_{23}))$ have been created, 200 for each. One has sampled with the muon neutrino survival probability $P(\nu_\mu \to \nu_\mu)$ multiplied by the scaling weight W_s. In this way the statistical fluctuations of the expected muon maps have been taken into account.

4. Use the samples to create the muon distribution maps, which will mimic the experimental data. Notice, that these maps have the same meaning, as the corresponding reference histograms from the previous section, but they were produced with approximately two orders of magnitude smaller number of events. This corresponds to the expected number of events in a real experiment.

5. Compare each one of them with the reference distributions and check how many of them have been identified with each pair of the $(\Delta m^2_{23}, \sin^2(2\Theta_{23}))$ parameters. This will give the approximate statistical error of the method, in the same manner as performing the same experiment 200 times.

The lattice of reference oscillation samples has been created within the constrains of the physical expectations for the oscillation parameters, *i.e.* $\sin^2(2\Theta_{23}) \leq 1$, $21 \times 10^{-4} \leq \Delta m^2_{23} \leq 29 \times 10^{-4}$ eV^2. The size of lattice interval was 5×10^{-5} eV^2 in Δm^2_{23} and 5×10^{-3} in $\sin^2(2\Theta_{23})$.

There were a few problems, which had to be solved in order to optimise the performance of this method. First of all, one had to find the area in ϵ_μ, which was most sensitive to the changes in oscillation parameters. In the final tests the considered muon energies were situated between 200 and 1200 MeV.

Secondly, one had to find an optimal histogram bin shape and statistical cut, *i.e.* how many events should be in the reference bin in order to accept it for the statistical test.

For the χ^2 statistics 100, 80, 75 and 50 MeV bins have been used. There were two tested types of the angular binning: uniform bins in the $\cos(\Theta_\mu)$ varying in number from 4 to 20 as well as the uniform bins in θ_μ, 10 degree each from 0 to 90 degree. The smallest bins were at the verge of Super-K resolution [7]. It is worthy to notice here, that the uniform binning in θ_μ has been used to extract the data from the near 1 Kt detector in the Super-Kamiokande experiment [8].

The χ^2 method has been tested with two statistical cuts: at least 10 and at least 20 events in the bin. Unfortunately, the results have been unsatisfying. Too few of the 200 MC data samples were identified with the true $(\Delta m^2_{23}, \sin^2(2\Theta_{23}))$ oscillation parameters.

For the Poisson method tested bins were 100, 80, 75, 50 MeV in the muon energy and 5, 10, 15, 20 uniform bins in $\cos(\Theta_\mu)$. The best result has been found for 50 MeV $\times$ 0.4 $\cos(\Theta_\mu)$ (5 bins) binning. Distribution of the results on the reference lattice is shown in the Fig. 1. The true values of the oscillation parameters of MC data samples were chosen to be $(\Delta m^2_{23} = 24 \times 10^{-4}$ eV^2, $\sin^2(2\Theta_{23}) = 0.92)$ and $(\Delta m^2_{23} = 26 \times 10^{-4}$ eV^2, $\sin^2(2\Theta_{23}) = 0.96)$. Each bin of the plot gives the number of data samples identified with one point of the lattice.

3. Conclusions

After applying the Poisson statistical test the method has given quite good concentration of the results around the expected value. Rough estimation of the statistical error from the plots gives the $1\,\sigma$ areas of about $\pm 0.5 \times 10^{-4}$ eV^2 in Δm^2_{23} and ± 0.02 in $\sin^2(2\Theta_{23})$. The error in Δm^2_{23} is close to the expected uncertainties of T2K [2, 3], but the uncertainty of $\sin^2(2\Theta_{23})$ is larger. The values of these uncertainties vary strongly with the region of the $(\Delta m^2_{23}, \sin^2(2\Theta_{23}))$ plane, in which the true values of oscillation parameters are located. In general, the bigger are the values of

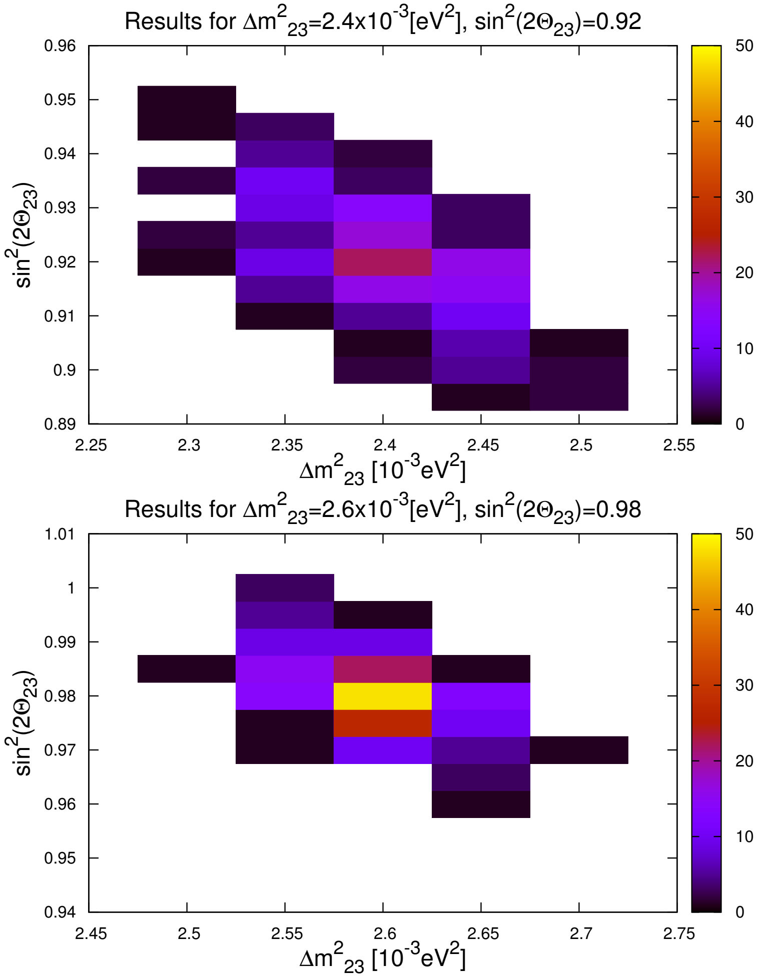

Fig. 1. The figure shows how many of the MC data samples have been identified with each point of the reference lattice.

both $\sin^2(2\Theta_{23})$ and Δm^2_{23} the better is the concentration of the results around the true value. For the true oscillation parameter values located at the present lower bound of the physical expectations the concentration is rather poor. Judging from the plots it seems, that this method is better at extracting the squared mass difference then the mixing angle. The most probable explanation is related to the error of the overall normalisation. The position of the oscillation probability maximum depends on Δm^2, whereas its depth depends on $\sin^2(2\Theta)$. Thus the total number of recorded events should depend more on $\sin^2(2\Theta)$ than on Δm^2.

The optimal method of binning still remains an open problem. It is very probable, that the improvement of the oscillation parameter extraction algorithm may be obtained by choosing non regular bin shapes on the $(\epsilon_\mu, \Theta_\mu)$ maps. The general rule is to have both many bins in the region sensitive to the oscillation signal and high statistics in each bin.

The systematic errors will add more uncertainty but their evaluation is a separate complicated problem.

The author was supported by the grant 35/N-T2K/2007/0 (the project number DWM/57/T2K/2007). I would like to thank Federico Sanchez and Francesca Di Lodovico for discussions. Special thanks to Jan Sobczyk.

REFERENCES

[1] M.H. Ahn *et al.* [K2K Collaboration], *Phys. Rev.* **D74**, 072003 (2006).
[2] P.F. Loverre [K2K and T2K Collaborations], *J. Phys. Conf. Ser.* **39**, 323 (2006).
[3] Y. Obayashi [T2K Collaboration], `arXiv:0807.4012 [hep-ex]`.
[4] J. Sobczyk, J. Żmuda, `arXiv:0912.0021v1 [hep-ex]`.
[5] A.V. Butkevich, *Phys. Rev.* **C78**, 015501 (2008).
[6] C. Juszczak, *Acta Phys. Pol. B* **40**, 2507 (2009).
[7] Y. Ashie *et al.* [Super-Kamiokande Collaboration], *Phys. Rev.* **D71**, 112005 (2005).
[8] A. Rodriguez *et al.* [K2K Collaboration], *Phys. Rev.* **D78**, 032003 (2008).