THE MASS AND LEPTONIC DECAY CONSTANT OF $D_{s0}(2317)$ MESON IN THE FRAMEWORK OF THERMAL QCD SUM RULES

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In the present work, we assume $D_{s0}(2317)$ meson as the $c\bar{s}$ state and study its parameters at finite temperature using QCD sum rules. It is calculated the annihilation and scattering parts of spectral function in the lowest order of perturbation theory. Taking into account perturbative two-loop order α_s corrections and nonperturbative corrections up to the dimension six condensates it is investigated the temperature dependences of mass and leptonic decay constant of $D_{s0}(2317)$ meson.

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1. Introduction

In 2003 BaBar Collaboration discovered a positive-parity scalar charm strange meson $D_{s0}(2317)$ with a very narrow width [1], which was confirmed by CLEO Collaboration [2] and BELLE Collaboration [3] later. This observed state has attracted much attention because its measured mass and width do not match the predictions from potential-based quark models [4]. To resolve the difficulties, many theoretical models have appeared in the literature. Various theoretical models, based on the $c\bar{s}$ quark structure, are suggested to explain the low mass and the narrow width for the $D_{s0}(2317)$ meson [5–10]. QCD sum rule analysis in [11,12] supports the $c\bar{s}$ postulation of nature $D_{s0}(2317)$. Apart from the quark–antiquark interpretation, this state has been interpreted as a DK molecule [13], a $D_s\pi$ molecule [14], a $c\bar{s}q\bar{q}$ four-quark state [15], and a mixing of the conventional state and the four-quark state [16]. Also this state was investigated in the framework of chiral symmetry considerations [17].

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The discussion of D mesons properties in literature has a rather long history and in our understanding of nonperturbative dynamics of QCD these mesons play an important role. The first determinations of leptonic decay constants of these mesons were made twenty years ago [18–21] and due to further theoretical and experimental progress, this problem was reconsidered, taking into account the running quark masses and perturbative three-loop α_s^2 corrections to the correlation function [22, 23]. Recently, first attempts have been made to calculate the leptonic decay constants of some D mesons at finite temperature in the framework of thermal QCD sum rules [24, 25].

At high temperature the restoration of chiral symmetry and deconfinement are expected. In this connection the study of scalar mesons at finite temperature receives special attention. The study of temperature dependence of physical parameters of these mesons can give us hints of a possible restoration of symmetries [17]. Although the scalar mesons have been studied for several decades, many properties of them are not so clear yet and identifying the scalar mesons is experimentally difficult. Hence, the theoretical works can play a crucial role in this point of view. The investigation of $D_{s0}(2317)$ is of a great importance to explore the structure charmed mesons and may provide some information to help understanding the nature of scalar mesons. Can we get any new information about the nature of the scalar mesons from the thermal QCD sum rule analysis? Present work is addressed to the investigation of this problem.

In this work, we assume $D_{s0}(2317)$ meson as the $c\bar{s}$ state and study its parameters at finite temperature using QCD sum rules [26]. The extending of QCD sum rules method to finite temperature has been made in the paper [27]. This extension based on two basic assumptions, that the Operator Product Expansion (OPE) and notion of quark–hadron duality remain valid at finite temperature, but the vacuum condensates must be replaced by their thermal expectation values. The thermal QCD sum rule has been extensively used for studying thermal properties of both light and heavy mesons as a reliable and well-established method [28–32].

In the present work, we calculated the temperature behavior of mass and leptonic decay constant of $D_{s0}(2317)$ meson. The knowledge of leptonic decay constants is needed to predict numerous heavy flavor electroweak transitions and to determine Standard Model parameters from the experimental data. Also leptonic decay constants play essential role in the analysis of CKM matrix, CP violation and the mixings $\overline{B_d}B_d$, $\overline{B_s}B_s$.

This paper is organized as follows. In Section 2, we calculated the annihilation and scattering parts of spectral density and give the expression for the perturbative scalar spectral function up to two-loop order α_s corrections. Also nonperturbative contributions up to the dimension six condensates are summarized. Section 3 contains our numerical analysis of the mass and leptonic decay constant using Borel transform sum rules.

2. Thermal QCD sum rule for the scalar charm strange meson

The starting point for the sum rule analysis is the two-point thermal correlator

$$\Pi\left(q^{2}\right) = i \int d^{4}x e^{iq x} \langle T(J(x)J^{+}(0)) \rangle, \qquad (1)$$

where $J(x) = (m_c - m_s) : \bar{s}(x)c(x)$: is heavy-light quark current and has the quantum numbers of the $D_{s0}(2317)$ meson, m_c and m_s are charm and strange quark masses, respectively. *s* quark mass is not neglected throughout this work. Thermal average of any operator *O* is determined by following way

$$\langle O \rangle = \operatorname{Tr} e^{-\beta H} O / \operatorname{Tr} e^{-\beta H},$$
 (2)

where H is the QCD Hamiltonian, and $\beta = 1/T$ stands for the inverse of the temperature T and traces carry out over any complete set of states. According to the basic idea of the QCD sum rule, we must calculate this correlator in terms of the physical particles (hadrons) and in quark–gluon language, and then equate both representations. First let us calculate theoretical part of the correlator Eq. (1). Up to a subtraction polynomial, which depends on the large q^2 behavior, $\Pi(q^2)$ satisfies following dispersion relation [27,29,30]

$$\Pi\left(q^{2}\right) = \int ds \frac{\rho(s)}{s+Q^{2}} + \text{subtractions}\,,\tag{3}$$

where $\rho(q) = \frac{1}{\pi} \text{Im}\Pi(q) \tanh(\frac{\beta q_0}{2})$ is spectral density. In order to calculate two-point thermal correlator in the lowest order of perturbation theory, we use quark propagator at finite temperature [33]

$$S_{11}(q) = (\gamma^{\mu}q_{\mu} + m) \left(\frac{1}{q^2 - m^2 + i\varepsilon} + 2\pi i n(\omega_q)\delta\left(q^2 - m^2\right)\right).$$
(4)

Here $n(\omega_q)$ is the Fermi distribution function, $n(\omega_q) = [\exp(\beta \omega_q) + 1]^{-1}$ and $\omega_q = \sqrt{q^2 + m^2}$. After some calculations we find that perturbative part of spectral density is given by

$$\rho_{\text{pert}}(q,T) = \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{\omega_1^2 - \mathbf{k}^2 + \mathbf{k} \cdot \mathbf{q} - \omega_1 q_0 + m_c m_s}{\omega_1 \omega_2} \\ \times [(1 - n_1 - n_2)\delta(q_0 - \omega_1 - \omega_2) + (n_1 - n_2)\delta(q_0 - \omega_1 + \omega_2)] . (5)$$

Here $\omega_1 = \sqrt{q^2 + m_c^2}$ and $\omega_2 = \sqrt{(k-q)^2 + m_s^2}$. Note that spectral density involves two pieces, one is called the annihilation term, $\rho_{a,pert}(s,T)$, which survives at T = 0. Other term is called scattering term, $\rho_{s,pert}(s,T)$, which vanishes at T = 0. As can be seen, delta function $\delta(q_0 - \omega_1 - \omega_2)$ in Eq. (5) gives the first branch cut, $q^2 \geq (m_c + m_s)^2$, which coincides with zero temperature cut and describes the standard threshold for particle decays. On the other hand, delta function $\delta(q_0 - \omega_1 + \omega_2)$ in Eq. (5) shows that an additional branch cut arises at finite temperature, $q^2 \leq (m_c - m_s)^2$, and this new branch cut corresponds to particle absorption from the medium. Therefore, delta functions $\delta(q_0 - \omega_1 - \omega_2)$ and $\delta(q_0 - \omega_1 + \omega_2)$ in Eq. (5) contribute in regions $(m_c + m_s)^2 + q^2 \leq q_0^2 \leq \infty$ and $0 \leq q_0^2 \leq q^2 + (m_c - m_s)^2$, respectively. Taking into account these contributions the annihilation and scattering parts of spectral density in the case q = 0 can be written as

$$\rho_{\text{a,pert}}(s,T) = \rho_0(s) \left[1 - n \left(\frac{\sqrt{s}}{2} \left(1 + \frac{m_c^2 - m_s^2}{s} \right) \right) -n \left(\frac{\sqrt{s}}{2} \left(1 - \frac{m_c^2 - m_s^2}{s} \right) \right) \right], \qquad (6)$$

$$\rho_{s,pert}(s,T) = \rho_0(s) \left[n \left(\frac{\sqrt{s}}{2} \left(1 + \frac{m_c^2 - m_s^2}{s} \right) \right) -n \left(-\frac{\sqrt{s}}{2} \left(1 - \frac{m_c^2 - m_s^2}{s} \right) \right) \right].$$
(7)

Here $\rho_0(s)$ is the correlation function in the lowest order of perturbation theory at zero temperature and given by

$$\rho_0(s) = \frac{3(m_c - m_s)^2}{8\pi^2 s} q^2(s) v^3(s) , \qquad (8)$$

where $q(s) = s - (m_c - m_s)^2$ and $v(s) = (1 - 4m_s m_c/q(s))^{1/2}$. The contribution of perturbative two-loop order α_s corrections to the spectral density in perturbation theory at zero temperature can be written as [21]:

$$\rho_1(s) = \frac{4\alpha_s}{3\pi} \rho_0(s) f(x) , \qquad (9)$$

where $x = m_c^2/s$, $\alpha_s = \alpha_s(m_c^2)$ and

$$f(x) = \frac{9}{4} + 2\text{Li}_2(x) + \ln x \ln(1-x) - \frac{3}{2}\ln(1/x-1) - \ln(1-x) + x \ln(1/x-1) - \frac{x}{1-x}\ln x.$$
(10)

Here $\operatorname{Li}_2(x) = -\int_0^x dt \frac{\ln(1-t)}{t}$ is dilogarithm function. Note that in α_s corrections terms the strange quark mass is set zero, though in numerical analysis, the mass of the strange quark is taken into account. The subtraction terms

in Eq. (3) are removed by using the Borel transformation, therefore we will omit these terms. The non-perturbative contribution at zero temperature to the correlator has following form

$$\Pi_{\rm np} \left(q^2 \right) = m_c \lambda \langle 0 | \bar{s}s | 0 \rangle \\
\times \left[1 - \frac{1}{2} \varepsilon (3 - \lambda) - \lambda \varepsilon^2 (1 - \lambda) + \frac{1}{2} \varepsilon^3 \left(1 + \lambda - 4\lambda^2 + 2\lambda^3 \right) \right] \\
+ \frac{1}{12\pi} \lambda \langle 0 | \alpha_{\rm s} G^2 | 0 \rangle \left[1 - 3\varepsilon \left(1 - \frac{8}{3}\lambda + 2\lambda^2 - 2\lambda(1 - \lambda) \ln(\varepsilon\lambda) \right) \right] \\
+ \frac{M_0^2}{2m_c} \langle 0 | \bar{s}s | 0 \rangle \lambda^2 (1 - \lambda) (1 - \varepsilon(2 - \lambda)) \\
- \frac{8}{27} \frac{\pi \rho}{m_c^2} \alpha_{\rm s} \langle 0 | \bar{s}s | 0 \rangle^2 \lambda^2 \left(2 - \lambda - \lambda^2 \right) ,$$
(11)

which arises in the framework of the OPE and parameterized by vacuum expectation values of quark and gluon fields in the QCD Lagrangian. In Eq. (11) $\lambda = m_c^2/(Q^2 + m_c^2)$, $\varepsilon = m_s/m_c$ and terms are organized according to their dimension. It is assumed, that the expansion (11) also remains valid, but the vacuum condensates must be replaced by their thermal expectation values [27]. For the light quark condensate at finite temperature we use the results of [34, 35] obtained in chiral perturbation theory and temperature dependence of quark condensate in a good approximation can be written as

$$\langle \bar{q}q \rangle = \langle 0|\bar{q}q|0 \rangle \left[1 - 0.4 \left(\frac{T}{T_{\rm c}}\right)^4 - 0.6 \left(\frac{T}{T_{\rm c}}\right)^8 \right], \qquad (12)$$

where T_c is critical temperature. The low temperature expansion of a gluon condensate is proportional to the trace of the energy momentum tensor [36] and can be approximated [24] as

$$\left\langle \alpha_{\rm s} G^2 \right\rangle = \left\langle 0 | \alpha_{\rm s} G^2 | 0 \right\rangle \left[1 - \left(\frac{T}{T_{\rm c}} \right)^8 \right] \,.$$
 (13)

Also, we have used for the mixed condensate the parameterization

$$g\left\langle \bar{q}\sigma_{\mu\nu}\frac{\lambda_a}{2}G_a^{\mu\nu}q\right\rangle = M_0^2\langle\bar{q}q\rangle \tag{14}$$

and deduced the value of the QCD scale Λ from the value of $\alpha_{\rm s}(M_Z) = 0.1176$.

Our next task is the calculation of the physical part of the correlator (1). According to the basic idea of quark-hadron duality assumption, the righthand side of Eq. (1) can be evaluated in a hadron-based picture. Equating OPE and hadron representations of correlation function and using quarkhadron duality the central equation of our sum-rule analysis takes the form:

$$\frac{f^2(T)m^4(T)}{Q^2 + m^2(T)} = \int_{(m_c + m_s)^2}^{s_0(T)} ds \frac{\rho_{\rm a,pert}(s, T) + \rho_1(s)}{s + Q^2} + \int_{0}^{(m_c - m_s)^2} ds \frac{\rho_{\rm s,pert}(s, T)}{s + Q^2} + \Pi_{\rm np} \left(Q^2, T\right), \quad (15)$$

where f and m are the leptonic decay constant and mass of $D_{s0}(2317)$ meson respectively. Note that in Eq. (15) the mass and leptonic decay constant were replaced by their temperature dependent values. The continuum threshold also depends on temperature; to a very good approximation its scales universally as the quark condensate [24]

$$s_0(T) = s_0 \frac{\langle \bar{q}q \rangle}{\langle 0|\bar{q}q|0 \rangle} \left(1 - \frac{(m_c + m_s)^2}{s_0}\right) + (m_c + m_s)^2, \quad (16)$$

where in the right-hand side s_0 is hadronic threshold at zero temperature: $s_0 = s(T = 0)$.

3. Numerical analysis of mass and leptonic decay constant

In this section we present our results for the temperature dependence of $D_{s0}(2317)$ meson mass and leptonic decay constant. Performing Borel transformation with respect to Q_0^2 from both sides of equation (15) and taking the derivative with respect to $1/M^2$ from both sides of obtained expression, and making some transformations we have

$$m^2(T) = \frac{B(T)}{A(T)},$$
 (17)

$$f^{2}(T) = \frac{A(T)}{m^{4}(T)} \exp\left(\frac{m^{2}(T)}{M^{2}}\right),$$
 (18)

where

$$A(T) = \int_{(m_c+m_s)^2}^{s_0(T)} ds \left(\rho_{\rm a,pert}(s) + \rho_1(s)\right) \exp\left(-\frac{s}{M^2}\right) + \int_{0}^{(m_c-m_s)^2} ds \rho_{\rm s,pert}(s) \exp\left(-\frac{s}{M^2}\right) + \Pi_{\rm np}\left(M^2, T\right), \quad (19)$$

$$\begin{aligned} \Pi_{\rm np}(M^2,T) &= m_c^3 \langle \bar{s}s \rangle e^{-\beta} \\ \times \left[1 - \frac{3}{2}\varepsilon + \frac{1}{2}\beta\varepsilon - \beta\varepsilon^2 \left(1 - \frac{1}{2}\beta \right) + \frac{1}{2}\varepsilon^3 \left(1 + \beta - 2\beta^2 + \frac{1}{3}\beta^3 \right) \right] \\ &+ \frac{1}{12} \left\langle \frac{\alpha_{\rm s}G^2}{\pi} \right\rangle m_c^2 e^{-\beta} \\ \times \left[1 - 3\varepsilon \left(1 - \frac{8}{3}\beta + \beta^2 - 2\beta(\ln(\beta\varepsilon) + \gamma - 1) + \beta^2 \left(\ln(\beta\varepsilon) + \gamma - \frac{3}{2} \right) \right) \right] \\ &+ \frac{1}{2}M_0^2 m_c \beta \langle \bar{s}s \rangle e^{-\beta} \left[1 - \frac{1}{2}\beta - 2\varepsilon \left(1 - \frac{3}{4}\beta \left(1 - \frac{1}{9}\beta \right) \right) \right] \\ &- \frac{4}{81}\pi \rho \alpha_{\rm s} \langle \bar{s}s \rangle^2 \beta e^{-\beta} \left(12 - 3\beta - \beta^2 \right) \,, \end{aligned}$$
(20)

where $\beta = m_c^2/M^2$ and $B(T) = -m_c^2 \frac{dA(T)}{d\beta}$.

For the numerical evolution of the above sum rule, we use QCD impute parameters showed in Table I. The criterion we adopt here is to fix in such a way so as to reproduce the zero temperature values of meson mass and leptonic decay constant.

TABLE I

QCD input parameters used in the analysis.

Parameters	References
m = 2317 MeV	[37]
$m_s = 120 \text{ MeV}$	[37]
$m_c = 1.47 \text{ GeV}$	[22, 37]
f = 201 MeV	[22, 37]
$\rho = 4$	[21, 31]
$\langle 0 \overline{q}q 0\rangle = -0.014 \ { m GeV}^3$	[26]
$\langle 0 \frac{1}{\pi}\alpha_{\rm s}G^2 0\rangle = 0.012 \ {\rm GeV^4}$	[26]
$\alpha_s \langle 0 \overline{q} q 0 \rangle^2 = 5.8 \times 10^{-4} \text{ GeV}^6$	[12]
$M_0^2 = 0.8 \ { m GeV^2}$	[12]
$\langle 0 \overline{s}s 0 angle = 0.8\langle 0 \overline{q}q 0 angle$	[12]

 $D_{s0}(2317)$ meson mass as a function of temperature are shown in Fig. 1, Fig. 3 and Fig. 5 at continuum threshold values $s_0 = 7.5$; 8.0; 8.5 GeV², respectively. As seen, mass decreases with increasing temperature and mesons lose approximately 10–15 percent of its mass at T = 150 MeV temperature. The results for leptonic decay constants are shown in Fig. 2, Fig. 4 and Fig. 6 at continuum threshold values $s_0 = 7.5$; 8.0; 8.5 GeV², respectively. As can be seen f decreases with increasing temperature and vanishes approximately at critical temperature. This situation may be interpreted as a signal for deconfinement and agrees with light and heavy mesons investigations [24, 25]. Numerical analysis shows that the temperature dependence of f is the same, when M^2 changes between 1.5 GeV² and 3 GeV² at fixed values of continuum threshold. Obtained results can be used for interpretation heavy ion collision experiments.



Fig. 1. Temperature dependence of meson mass at $s_0 = 7.5 \text{ GeV}^2$.



Fig. 2. Temperature dependence of leptonic decay constants at $s_0 = 7.5 \text{ GeV}^2$.



Fig. 3. Temperature dependence of meson mass at $s_0 = 8.0 \text{ GeV}^2$.



Fig. 4. Temperature dependence of leptonic decay constants at $s_0 = 8.0 \text{ GeV}^2$.



Fig. 5. Temperature dependence of meson mass at $s_0 = 8.5 \text{ GeV}^2$.



Fig. 6. Temperature dependence of leptonic decay constants at $s_0 = 8.5 \text{ GeV}^2$.

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