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SATURATION AND SCALING OF MULTIPLICITY, MEAN $p_{\rm T}$ AND $p_{\rm T}$ DISTRIBUTIONS FROM 200 GeV $\leq \sqrt{s} \leq 7$ TeV

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The multiplicity, average transverse momentum, and charged particle transverse momentum distributions have recently been measured in LHC experiments. The multiplicity and average transverse momentum grow with beam energy. Such growth is expected in the theory of the Color Glass Condensate, a theory that incorporates the physics of saturation into the evolution of the gluon distribution. We show that the energy dependence of the $p\overline{p}$ data and the LHC data for pp scattering at $\sqrt{s} \geq 200$ GeV may be simply described using a minimal amount of model input. Such a description uses parameters consistent with the Color Glass Condensate descriptions of HERA and RHIC experimental data.

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1. Introduction

The first LHC data have been released on total charged particle multiplicity as a function of energy, average transverse momenta of charged particles as a function of energy [1–5] and transverse momentum of charged particles as a function of charged particle multiplicity [6]. The generic features of the experimental data are that the charged particle pseudo-rapidity densities rise as a power of energy, $dN/d\eta \sim E^{0.23}$, and that the average transverse momentum rises with both energy and charged particle multiplicity. This behavior has a natural explanation within the theory of saturation and the Color Glass Condensate [7–12]. Within this theory, the total multiplicity of produced particles is computable [13, 14]. Assuming local parton hadron duality [15], the initially produced gluon multiplicity is proportional to the charged particle multiplicity. The pseudo-rapidity multiplicity density can be expressed in terms of the saturation momentum as [16–21]:

$$\frac{1}{\sigma}\frac{dN_{\rm ch}}{d\eta}=\frac{\rm const.}{\alpha_{\rm S}(Q_{\rm sat})}Q_{\rm sat}^2\,. \tag{1}$$

The strong coupling constant is evaluated at the saturation momentum scale.

The saturation momentum scale is proportional to the transverse gluon density. Evolution equations that build in the effects of high gluon density have been derived in Refs. [22–33]. The dependence of the saturation momentum on fractional gluon momentum may be determined from such considerations [34]. It has been shown that the generic features of such a description of saturation can describe the HERA data on inclusive and diffractive deep inelastic scattering [35–38], and correctly predicts observed scaling properties of experimental data [39, 40]. The results of this analysis relevant for our purposes is that the saturation momentum at x values appropriate for the LHC scales with x as $Q_{\rm sat}^2 \sim 1/x^{\lambda}$, where $\lambda \sim 0.2$–0.3.

There have been detailed saturation based predictions and descriptions of the recent results from the LHC [21,41–43]. The goal of this paper is not to improve upon the descriptions provided in these works. Our goal is to show that the simplest generic features of saturation based descriptions are adequate to quantitatively describe data on the dependence particle multiplicities on energy and the dependence of such average transverse momentum upon multiplicity and beam energy. We will also argue that there is an approximate geometric scaling of transverse momenta distributions measured at LHC energies.

2. Color Glass Condensate description of the LHC data

To reduce the Color Glass Condensate description to its simplest possible form, we will assume that the density of produced charged particles per unit pseudo-rapidity scales as

$$\frac{dN_{\rm ch}}{d\eta}=\kappa Q_{\rm sat}^2=AE^{\lambda}\,. \tag{2}$$

Here the energy is measured in units of TeV.

The parameters λ and A can be determined by a fit to the LHC data. In Fig. 1, an excellent fit is shown to the data that also includes lower energy data for proton–anti-proton scattering. (This agreement with the $p\overline{p}$ data is a little surprising since there should be some small difference between pp and $p\overline{p}$ scattering at the energies of interest.)

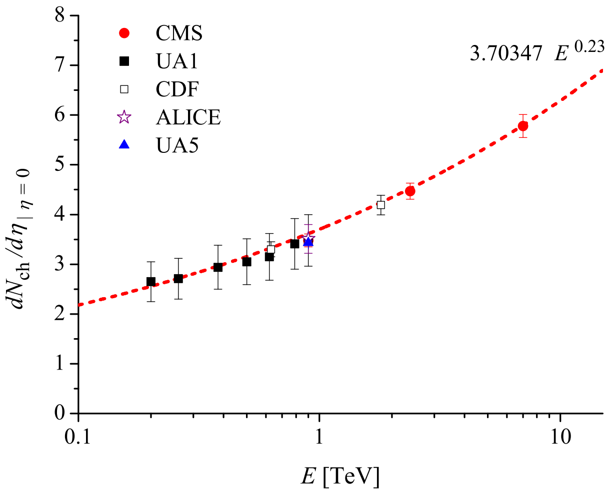

Fig. 1. Charged particle rapidity density as a function of energy compared to a power law 3.7043 $E^{0.23}$. Data from the LHC [1–5], UA(1) [44], UA(5) [45] and CDF [46, 47].

On dimensional grounds, at very high energy, we expect the average transverse momentum will be proportional to the saturation momentum. Since this term does not entirely dominate the contribution to the transverse momentum at accessible energies, we add a constant, so that the result has a reasonable low limit at lower energy. In this case, there is a small difference seen in the data between pp and $p\overline{p}$ scattering. The power law growth should be, however, universal. We fit the average p_T with the following form

$$\langle p_T \rangle = B + CE^{\lambda/2} \,. \tag{3}$$

The results of such a fit are shown in Fig. 2. We see that indeed the power of the energy and its coefficient (within errors which we do not quote here) are identical both for pp and $p\overline{p}$ data. Similar form of the average $\langle p_T \rangle$ has been recently postulated in Ref. [48] with higher power of E equal to 0.414. However, the recent 7 TeV CMS point is far below their curve.

If the saturation momentum is the only scale that controls p_T distributions, on dimensional grounds, these distributions should have a geometrical scaling

$$\frac{1}{\sigma}\frac{dN_{ch}}{d\eta d^2 p_T} = F\left(\frac{p_T}{Q_{sat}(p_T/\sqrt{s})}\right) \,. \tag{4}$$

This means that there is a universal function of the scaling variable

$$\tau = \frac{p_T^2}{Q_{sat}^2(p_T/\sqrt{s})} = \frac{p_T^2}{1\,\text{GeV}^2}\left(\frac{p_T}{\sqrt{s}\times 10^{-3}}\right)^{\lambda} \tag{5}$$

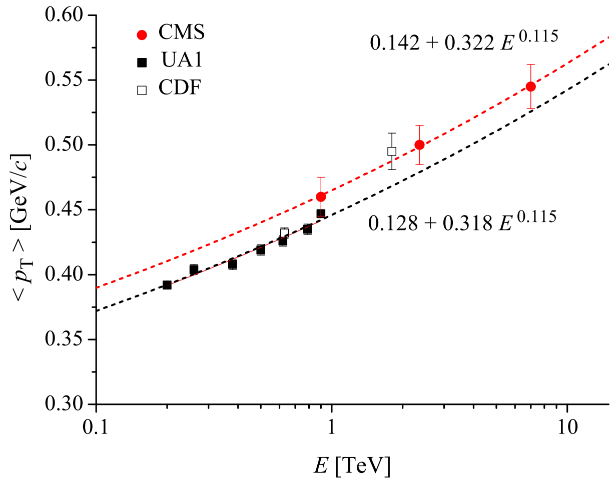

Fig. 2. Average transverse momentum as a function of energy compared to a power law $B+CE^{0.115}$. Data from the LHC [1–5], UA(1) [44], UA(5) [45] and CDF [46,47].

that describes the data at different energies (where $p_{\rm T}$ and $\sqrt{s}$ are in GeV). This is, of course, limited in the range of $p_{\rm T}$, so that one is not probing quark and gluon distributions outside the saturation region. By a rescaling of variables, one can check if the data from CMS fall on a universal scaling curve. Since the data points of the CMS $p_{\rm T}$ distributions are not yet publicly accessible we shall use throughout this paper an analytical parametrization in terms of Tsallis fit [49] as given in Refs. [4, 5]:

$$\frac{dN_{\rm ch}}{d\eta d^2 p_{\rm T}} = C\frac{p}{E}\frac{dN_{\rm ch}}{d\eta}\left(1+\frac{E_T}{nT}\right)^{-n}\,, \tag{6}$$

where $E_{\rm T}=\sqrt{m_\pi^2+p_{\rm T}^2}-m_\pi$ and

$\sqrt{s}$ [TeV]	T [GeV]	n
0.9	0.130	7.7
2.36	0.140	6.7
7.0	0.145	6.6

Up to about 4–6 GeV, the limit of the available data, such a scaling relation is satisfied, as shown in Fig. 3.

The scaling plot in Fig. 3 has been obtained by using power λ in Eq. (5) fixed from the DIS data at HERA. It is interesting to see whether this is also the optimal power for hadron–hadron scattering. To this end we compute

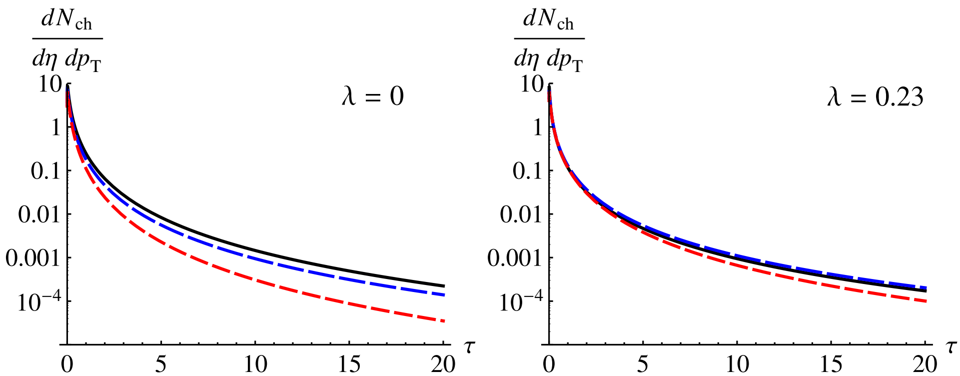

Fig. 3. The CMS data for transverse momentum distributions on the left as functions of p_T^2 (note that for $\lambda = 0$ scaling variable $\tau = p_T^2/(1\ \text{GeV}^2)$). On the right, the same p_T distribution rescaled in terms of the scaling variable $\tau = p_T^2/Q_{\text{sat}}^2(p_T/\sqrt{s})$.

the mean deviation of the scaled p_T distributions for different energies

$$\sigma^2_{E_1-E_2} = \int_0^{\tau_{\max}} \left(\left.\frac{dN_{\text{ch}}}{d\eta d\tau}\right|_{E_1} - \left.\frac{dN_{\text{ch}}}{d\eta d\tau}\right|_{E_2} \right)^2 d\tau \tag{7}$$

and normalizing them to the sum

$$s_{E_1-E_2} = \int_0^{\tau_{\max}} \left(\left.\frac{dN_{\text{ch}}}{d\eta d\tau}\right|_{E_1} + \left.\frac{dN_{\text{ch}}}{d\eta d\tau}\right|_{E_2} \right) d\tau \tag{8}$$

we define quantities

$$\varDelta_{E_1-E_2} = \frac{\sigma_{E_1-E_2}}{s_{E_1-E_2}} \tag{9}$$

that are plotted in Fig. 4. We see that the minima obtained with the Tsallis fit (6) are rather shallow and include the optimal value of λ obtained from DIS, although the preferred value would be slightly bigger. We checked, however, that the higher value of λ is incompatible with the energy dependence of charged multiplicity shown in Fig. 1. We find this agreement (note that we use Tsallis parametrization instead of real data) as a strong support of the applicability of geometric scaling to hadron–hadron scattering.

In the ATLAS experiment, the average p_T of events with various multiplicities was computed. There was a transverse momentum cutoff of $p_T \geq 500$ MeV. We expect as in Eq. (3) , that the average transverse momentum will be

$$\langle p_T \rangle = C + D\sqrt{N_{\text{ch}}}\,. \tag{10}$$

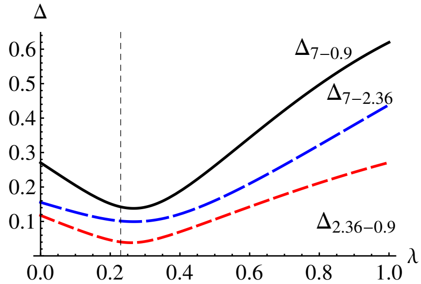

Fig. 4. Normalized square deviations between scaled $p_{\rm T}$ distributions for different CMS energies as function of saturation parameter λ. Optimal DIS $\lambda = 0.23$ is marked by a thin vertical line.

Since the CMS and ATLAS cuts are different we simply show in Fig. 5 that such functional form provides a good description of the experimental data. We have presented two fits: one to the whole region of available multiplicities and the second one for $N_{\rm ch} > 15$. The latter choice is dictated by the slight change of the curvature of the data around $N_{\rm ch} \sim 10$.

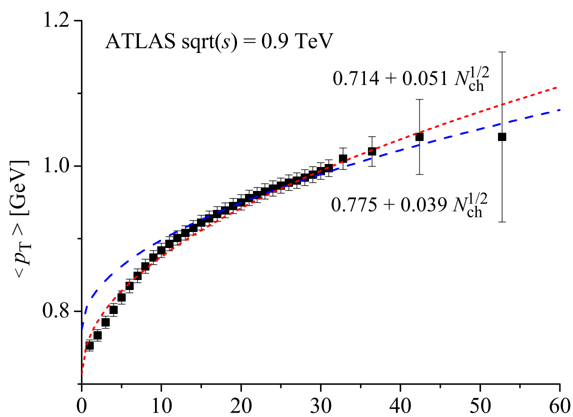

Fig. 5. ATLAS data compared to the square root fit of Eq. (10). Short-dashed (red) curve corresponds to the fit over the whole range of multiplicities, whereas long-dashed (blue) fit is restricted to $N_{\rm ch} > 15$.

3. Predictions for higher energy

Using the scaling analysis in this paper we can make predictions for the multiplicity per unit rapidity, average transverse momentum and transverse momentum distributions at higher LHC energies. Our predictions for the multiplicity per unit rapidity are shown, in fact, in Fig. 1. In order to estimate roughly the error of that fit we simply propagated the experimental error of the 7 TeV point with the help of Eq. (2) obtaining $dN_{ch}/d\eta|_{\eta=0} = 6.29 \pm 0.25$ and 6.80 ± 0.27 for $\sqrt{s} = 10$ and 14 TeV, respectively. In a similar way we have estimated average $\langle p_T \rangle = 0.562 \pm 0.017$ and 0.579 ± 0.017. By minimizing Δ_{10-7} and Δ_{14-7} with respect to parameters T and n of the Tsallis formula (6) we have obtained p_T distributions at $\sqrt{s} = 10$ and 14 TeV that are shown in Fig. 6. The corresponding Tsallis parameters read: $T_{10} = 0.153$, $n_{10} = 6.6$ for $\sqrt{s} = 10$ TeV and $T_{14} = 0.162$, $n_{14} = 6.7$ for 14 TeV.

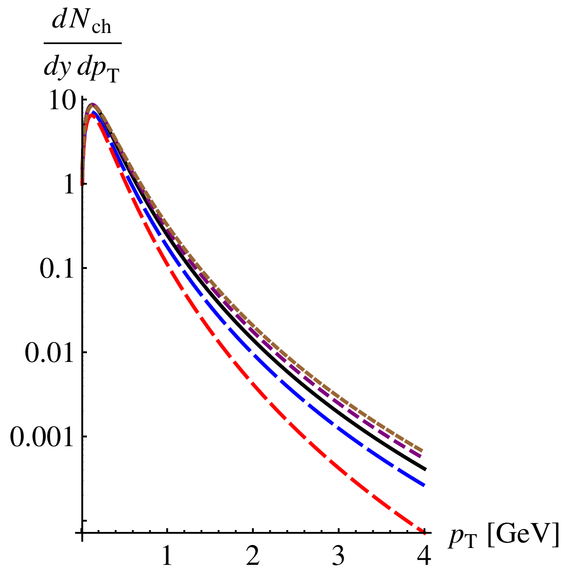

Fig. 6. CMS particle spectra in terms of Tsallis parametrization of Refs. [4, 5] for different energies. Solid line and long- and short-dashed lines below correspond to $\sqrt{s} = 7$, 2.36 and 0.9 TeV, respectively. Dashed and short-dashed lines above the solid line correspond to our predictions for $\sqrt{s} = 10$ and 14 TeV, respectively.

The scaling behavior we see in *pp* collisions can be used to estimate initial state effects for the heavy ion collisions. Such effects might be very important for measuring jet quenching effects, once the A dependence of the saturation momentum is established at LHC energy. Note that the asymptotic behavior of the Tsallis fit for high energies is controlled by the parameter n^2T/p_T. If T scales as $A^{1/3}$, then the asymptotic limit is obtained only at very high transverse momentum values, suggesting that saturation effects can influence transverse momentum distributions out to very high large values. This may influence experimental studies of jet quenching as an attempt to extract properties of the Quark Gluon Plasma.

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