THE STABILITY OF THIN-SHELL WORMHOLES WITH A PHANTOM-LIKE EQUATION OF STATE

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This paper discusses the stability to linearized radial perturbations of spherically symmetric thin-shell wormholes with a "phantom-like" equation of state for the exotic matter at the throat: $\mathcal{P} = \omega\sigma$, $\omega < 0$, where σ is the energy-density of the shell and $\mathcal{P}$ the lateral pressure. This equation is analogous to the generalized Chaplygin-gas equation of state used by E.F. Eiroa. The analysis, which differs from Eiroa's in its basic approach, is carried out for wormholes constructed from the following spacetimes: Schwarzschild, de Sitter and anti de Sitter, Reissner–Nordström, and regular charged black-hole spacetimes, followed by black holes in dilaton and generalized dilaton-axion gravity.

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1. Introduction

A powerful theoretical method for describing or mathematically constructing a class of spherically symmetric wormholes from black-hole spacetimes was proposed by Visser in 1989 [1]. This type of wormhole, constructed by the so-called cut-and-paste technique, is commonly known as a *thin-shell wormhole*, since the construction calls for grafting two black-hole spacetimes together. The junction surface is a three-dimensional thin shell. The cut-and-paste technique is now considered standard.

While there had already been a number of forerunners, the concept of a traversable wormhole was proposed by Morris and Thorne in 1988 [2]. Ten years later a renewed interest was sparked by the discovery that our Universe is undergoing an accelerated expansion [3, 4]: $\ddot{a} > 0$ in the Friedmann equation $\ddot{a}/a = -(4\pi)/3(\rho + 3p)$. (Our units are taken to be those in which $G = c = 1$.) The acceleration is caused by a negative pressure *dark energy* with equation of state (EoS) $p = \omega\rho$, $\omega < -1/3$, and $\rho > 0$.

A value of $\omega < -1/3$ is required for an accelerated expansion, while $\omega = -1$ corresponds to a cosmological constant [5]. The case $\omega < -1$ is referred to as *phantom energy* and leads to a violation of the null energy condition, a primary prerequisite for the existence of wormholes. Wormholes may also be supported by a generalized Chaplygin gas [6] whose EoS is $p = -A/\rho^\alpha$, where $A > 0$ and $0 < \alpha \leq 1$.

In a thin-shell wormhole the exotic matter is confined to the thin shell. This suggests assigning an equation of state to the exotic matter on the shell. Eiroa [7] used the above generalized Chaplygin EoS $\mathcal{P} = A/|\sigma|^\alpha$, where σ is the (negative) energy-density of the shell and $\mathcal{P}$ the lateral pressure, to perform a stability analysis for linearized radial perturbations. In this paper we will consider, analogously, the EoS $\mathcal{P} = \omega\sigma$, $\omega < 0$, which will be called a *phantom-like* equation of state. The stability analysis will be carried out for several spacetimes: Schwarzschild, de Sitter and anti de Sitter, Reissner–Nordström, and regular charged black hole spacetimes, as well as black holes in dilaton and generalized dilaton-axion gravity. The phantom-like equation of state yields explicit closed-form expressions for σ. Our approach to the stability analysis is therefore different from Eiroa's.

2. Thin-shell wormhole construction

Our starting point is the spherically symmetric metric [7]

$$ds^2 = -f(r)dt^2 + [f(r)]^{-1}dr^2 + h(r)\left(d\theta^2 + \sin^2\theta d\phi^2\right) , \tag{1}$$

where $f(r)$ and $h(r)$ are positive functions of r and $h(r)$ is increasing. (In Sections 3–6, $h(r) = r^2$.) As in Ref. [8], the construction begins with two copies of a black-hole spacetime and removing from each the four-dimensional region

$$\Omega^\pm = \{r \leq a \,|\, a > r_\mathrm{h}\} , \tag{2}$$

where $r = r_\mathrm{h}$ is the (outer) event horizon of the black hole. Now identify (in the sense of topology) the time-like hypersurfaces

$$\partial\Omega^\pm = \{r = a \,|\, a > r_\mathrm{h}\} .$$

The resulting manifold is geodesically complete and possesses two asymptotically flat regions connected by a throat. Next, we use the Lanczos equations [1, 7–15]

$$S^i{}_j = -\frac{1}{8\pi}\left(\left[K^i{}_j\right] - \delta^i{}_j[K]\right) , \tag{3}$$

where $[K_{ij}] = K^+_{ij} - K^-_{ij}$ and $[K]$ is the trace of $K^i{}_j$. In terms of the surface energy-density σ and the surface pressure $\mathcal{P}$, $S^i{}_j = \mathrm{diag}(-\sigma, \mathcal{P}, \mathcal{P})$. The Lanczos equations now yield

$$\sigma = -\frac{1}{4\pi}\left[K^{\theta}{}_{\theta}\right] \tag{4}$$

and

$$\mathcal{P} = \frac{1}{8\pi}\left([K^{\tau}{}_{\tau}] + [K^{\theta}{}_{\theta}]\right). \tag{5}$$

A dynamic analysis can be obtained by letting the radius $r = a$ be a function of time [8]. As a result,

$$\sigma = -\frac{1}{2\pi a}\sqrt{f(a) + \dot{a}^2} \tag{6}$$

and

$$\mathcal{P} = -\frac{1}{2}\sigma + \frac{1}{8\pi}\frac{2\ddot{a} + f'(a)}{\sqrt{f(a) + \dot{a}^2}}. \tag{7}$$

Since σ is negative on the shell, we are dealing with exotic matter. In fact, the weak energy condition (WEC) is trivially satisfied since the radial pressure p is zero for a thin shell. (The WEC requires the stress-energy tensor $T_{\alpha\beta}$ to obey $T_{\alpha\beta}\mu^{\alpha}\mu^{\beta} \geq 0$ for all time-like vectors and, by continuity, all null vectors.) So for the radial outgoing null vector $(1, 1, 0, 0)$, we therefore have $T_{\alpha\beta}\mu^{\alpha}\mu^{\beta} = \rho + p = \sigma + 0 < 0$.

3. Schwarzschild wormholes

For our first case, the Schwarzschild spacetime, $h(r) = r^2$ in line element (1), as noted earlier. Also, recall that the radius $r = a$ is a function of time. It is easy to check that $\mathcal{P}$ and σ obey the conservation equation

$$\frac{d}{d\tau}\left(\sigma a^2\right) + \mathcal{P}\frac{d}{d\tau}(a^2) = 0\,.$$

(In Eqs. (6) and (7), the over dot denotes the derivative with respect to τ.) The equation can be written in the form

$$\frac{d\sigma}{da} + \frac{2}{a}(\sigma + \mathcal{P}) = 0\,. \tag{8}$$

For a static configuration of radius a_0, we have $\dot{a} = 0$ and $\ddot{a} = 0$. Moreover, we will consider linearized fluctuations around a static solution characterized by the constants a_0, σ_0, and $\mathcal{P}_0$. Given the EoS $\mathcal{P} = \omega\sigma$, Eq. (8) can be solved by separation of variables to yield

$$|\sigma(a)| = |\sigma_0|\left(\frac{a_0}{a}\right)^{2(\omega+1)},$$

where $\sigma_0 = \sigma(a_0)$. So the solution is

$$\sigma(a) = \sigma_0 \left(\frac{a_0}{a}\right)^{2(\omega+1)}, \qquad \sigma_0 = \sigma(a_0)\,. \tag{9}$$

Next, we rearrange Eq. (6) to obtain the equation of motion

$$\dot{a}^2 + V(a) = 0\,.$$

Here the potential $V(a)$ is defined as

$$V(a) = f(a) - [2\pi a\sigma(a)]^2\,. \tag{10}$$

Expanding $V(a)$ around a_0, we obtain

$$V(a) = V(a_0) + V'(a_0)(a-a_0) + \tfrac{1}{2}V''(a_0)(a-a_0)^2 + O\left[(a-a_0)^3\right]. \tag{11}$$

Since we are linearizing around $a = a_0$, we require that $V(a_0) = 0$ and $V'(a_0) = 0$. The configuration is in stable equilibrium if $V''(a_0) > 0$.

Now recall that for the Schwarzschild spacetime, $f(r) = 1 - 2M/r$. It follows that

$$V(a) = 1 - \frac{2M}{a} - 4\pi^2 a^2\sigma^2 = 1 - \frac{2M}{a} - 4\pi^2 a^2\sigma_0^2\left(\frac{a_0}{a}\right)^{4+4\omega}$$

from Eq. (9). From Eq. (6) with $\dot{a} = 0$,

$$\sigma_0 = -\frac{1}{2\pi a_0}\sqrt{1 - \frac{2M}{a_0}}\,,$$

so that

$$V(a) = 1 - \frac{2M}{a} - \left(1 - \frac{2M}{a_0}\right)\frac{a_0^{2+4\omega}}{a^{2+4\omega}}\,. \tag{12}$$

The first requirement, $V(a_0) = 0$, is clearly met, but not the second. (If the exotic matter on the shell were not required to meet the extra condition in the form of an EoS, then $V'(a_0)$ would indeed be zero [8].) From

$$V'(a_0) = \frac{2M}{a_0^2} - \left(1 - \frac{2M}{a_0}\right)a_0^{2+4\omega}(-2-4\omega)a_0^{-3-4\omega} = 0$$

we obtain the condition

$$\omega = -\frac{1}{2}\frac{a_0/M - 1}{a_0/M - 2}\,. \tag{13}$$

Observe that as $a_0 \to +\infty$, $\omega \to -1/2-$, and as $a_0 \to 2M+$, $\omega \to -\infty$. At $a_0 = 3M$, $\omega = -1$. Substituting in

$$V''(a) = -\frac{4M}{a^3} - \left(1 - \frac{2M}{a_0}\right) a_0^{2+4\omega}(2+4\omega)(3+4\omega)a^{-4-4\omega}$$

and simplifying, we obtain the intermediate result

$$V''(a_0) = \frac{2}{a_0^2}\left(-\frac{2}{a_0/M} + \frac{1}{a_0/M}\frac{a_0/M-4}{a_0/M-2}\right) > 0\,. \tag{14}$$

Since the Schwarzschild black hole has an event horizon at $r = 2M$, $a_0/M - 2 > 0$, and we conclude that the inequality $V''(a_0) > 0$ can only be satisfied if

$$a_0 < 0\,.$$

As a result, there are no stable solutions.

To allow a comparison to some of the other cases, let us choose (arbitrarily) $a_0/M = 5$, as a result of which $\omega = -2/3$, and plot $V(a)$ against a/M, as shown in Fig. 1.

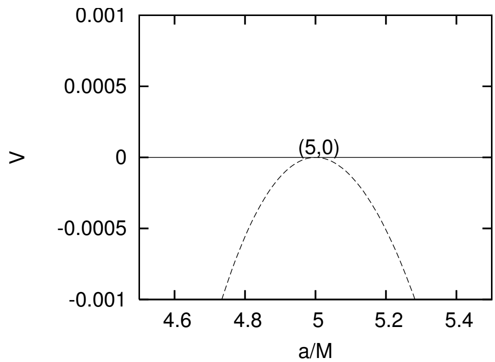

Fig. 1. The wormhole is unstable.

The more general analysis in Ref. [8] depends on the parameter $\beta^2(\sigma) = \partial\mathcal{P}/\partial\sigma$, where β is usually interpreted as the speed of sound, so that $0 < \beta^2 \le 1$. There are no stable solutions in this range. However, as discussed in Ref. [8], since we are dealing with exotic matter, this assumption may be questioned, that is, β^2 may be just a convenient parameter. In that case, some stable configurations may not be out of question. Our additional assumption, the EoS $\mathcal{P} = \omega\sigma$ on the shell, eliminates this possibility.

4. Wormholes with a cosmological constant

4.1. Schwarzschild–de Sitter spacetimes

In the presence of a cosmological constant, $f(r) = 1-(2M)/r-(1/3)\Lambda r^2$. For the de Sitter case, $\Lambda > 0$. To keep $f(r)$ from becoming negative, we must have $\Lambda M^2 \leq 1/9$. This condition results in two event horizons, where the inner horizon is between $2M$ and $3M$. (See Ref. [7] for details.) We therefore assume that a is greater than the outer horizon. Proceeding as in Sec. 3,

$$V(a) = 1 - \frac{2M}{a} - \frac{1}{3}\Lambda a^2 - \left(1 - \frac{2M}{a_0} - \frac{1}{3}\Lambda a_0^2\right)\left(\frac{a_0}{a}\right)^{2+4\omega} . \tag{15}$$

Observe that $V(a_0) = 0$. As before, we have to determine the condition on ω so that $V'(a_0) = 0$:

$$\omega = -\frac{1}{2}\,\frac{1 - 1/(a_0/M) - (2/3)\Lambda M^2(a_0/M)^2}{1 - 2/(a_0/M) - (1/3)\Lambda M^2(a_0/M)^2} . \tag{16}$$

(As in the Schwarzschild case, as $a_0 \to +\infty$, $\omega \to -1/2-$, and $\omega \to -\infty$ as a_0 approaches the outer event horizon.) Substituting in $V''(a_0)$ and simplifying, we get

$$V''(a_0) = \frac{2}{a_0^2}\,\frac{-1/(a_0/M) + 3\Lambda M^2(a_0/M) - (2/3)\Lambda M^2(a_0/M)^2}{1 - 2/(a_0/M) - (1/3)\Lambda M^2(a_0/M)^2} > 0 . \tag{17}$$

The form of $V''(a_0)$ forces us to consider two cases, a positive and negative denominator.

If the denominator is positive, then

$$\Lambda M^2 > \frac{1}{(a_0/M)[3(a_0/M) - (2/3)(a_0/M)^2]} . \tag{18}$$

This inequality implies that $a_0/M < 4.5$ to keep the right side positive. It is easy to show analytically that $\Lambda M^2 > 1/9$; in fact, (3, 1/9) is a minimum. We may also plot ΛM^2 against a/M, as shown in Fig. 2. So for this case, the condition $V''(a_0) > 0$ cannot be met (since we must have $\Lambda M^2 \leq 1/9$), and we get only unstable solutions. Plotting $V(a)$ around $a_0/M = 5$ yields a graph that is very similar to the graph in Fig. 1.

For the second case,

$$1 - \frac{2}{a_0/M} - \frac{1}{3}\Lambda M^2\left(\frac{a_0}{M}\right)^2 < 0 \tag{19}$$

in inequality (17) we obtain

$$\Lambda M^2\left[3\left(\frac{a_0}{M}\right) - \frac{2}{3}\left(\frac{a_0}{M}\right)^2\right] < \frac{1}{a_0/M} . \tag{20}$$

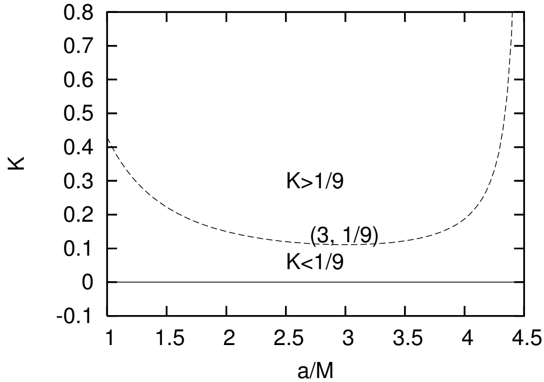

Fig. 2. $K = \Lambda M^2$ is plotted against a/M.

If $a_0/M > 4.5$, then the left side is negative, and the condition is automatically satisfied. If $a_0/M < 4.5$, then, according to Fig. 2, $\Lambda M^2 < 1/9$, the region below the graph. So we conclude that in the second case, the wormholes are stable.

For comparison, let us choose $a_0/M = 5$ again and $\Lambda M^2 = 0.11 < 1/9$, resulting in $\omega = -1.63$. The plot of $V(a)$ against a/M is shown in Fig. 3.

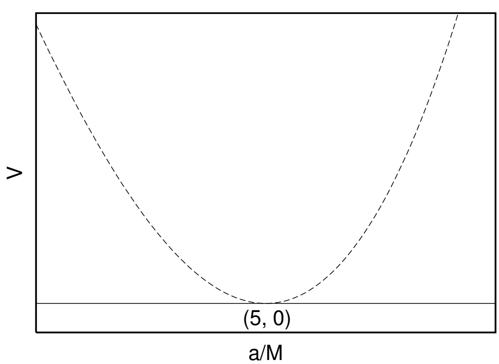

Fig. 3. The wormhole is stable.

In summary, in the Schwarzschild–de Sitter case, the thin-shell wormholes are stable if, and only if,

$$1 - \frac{2}{a_0/M} - \frac{1}{3}\Lambda M^2 \left(\frac{a_0}{M}\right)^2 < 0\,.$$

4.2. Schwarzschild–anti de Sitter spacetimes

To study the case $\Lambda < 0$, we return to inequality (17) and consider first a negative denominator

$$1 - \frac{2}{a_0/M} - \frac{1}{3}\Lambda M^2 \left(\frac{a_0}{M}\right)^2 < 0 \,.$$

Solving for ΛM^2, we obtain

$$\Lambda M^2 > \frac{3 - 6/(a_0/M)}{(a_0/M)^2} \,.$$

Since $a_0/M > 2$, we conclude that $\Lambda M^2 > 0$, so that this case cannot arise.

Reversing the sense of the inequality, we have from inequality (17)

$$\Lambda M^2 \left[3\left(\frac{a_0}{M}\right) - \frac{2}{3}\left(\frac{a_0}{M}\right)^2\right] > \frac{1}{a_0/M} \,.$$

Then the second factor on the left must be negative, which implies that $a_0/M > 4.5$.

So the wormhole is stable whenever

$$(1) \quad \Lambda M^2 < \frac{1}{(a_0/M)[3(a_0/M) - (2/3)(a_0/M)^2]}$$

and

$$(2) \quad a_0/M > 4.5 \,.$$

5. Reissner–Nordström wormholes

If the starting point is a Reissner–Nordström spacetime, then

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} \,, \tag{21}$$

where M and Q are the mass and charge, respectively, of the black hole. For $0 < |Q| < M$, this black hole has two event horizons at $r = M \pm \sqrt{M^2 - Q^2}$. As usual, we require that $r = a$ is larger than the outer horizon.

Here we have

$$V(a) = 1 - \frac{2M}{a} + \frac{Q^2}{a^2} - \left(1 - \frac{2M}{a_0} + \frac{Q^2}{a_0^2}\right)\left(\frac{a_0}{a}\right)^{2+4\omega} \,. \tag{22}$$

Once again, $V(a_0) = 0$. From $V'(a_0) = 0$ we obtain

$$\omega = -\frac{1}{2}\,\frac{(a_0/M)^2 - a_0/M}{(a_0/M)^2 - 2(a_0/M) + Q^2/M^2} \,. \tag{23}$$

Substituting into $V''(a_0)$ and simplifying, yields the following inequality

$$V''(a_0) = \frac{2}{a_0^2}\frac{-a_0/M - (Q^2/M^2)[1/(a_0/M)] + 2Q^2/M^2}{(a_0/M)^2 - 2(a_0/M) + Q^2/M^2} > 0\,. \tag{24}$$

Since $a_0/M > 2$, the denominator is positive. Solving for Q^2/M^2, leads to

$$\frac{|Q|}{M} > \frac{a_0/M}{\sqrt{2(a_0/M) - 1}}\,, \tag{25}$$

which exceeds unity. To meet this condition, $|Q|$ would have to exceed M.

So to obtain a stable solution, we will have to tolerate a naked singularity at $r = 0$, but since $a_0 > 0$, the naked singularity is removed from the wormhole spacetime.

6. Wormholes from regular charged black holes

Thin-shell wormholes from regular charged black holes, due to Ayon-Beato and García [16], are discussed in Ref. [17]. For this black hole

$$f(r) = 1 - \frac{2M}{r} + \frac{2M}{r}\tanh\left(\frac{Q^2}{2Mr}\right). \tag{26}$$

Again, M and Q are the mass and charge, respectively. It is shown in Ref. [16] that the black hole has two event horizons whenever $|Q| < 1.05\,M$. Consider next

$$\begin{aligned} V(a) &= 1 - \frac{2M}{a} + \frac{2M}{a}\tanh\left(\frac{Q^2}{2Ma}\right) \\ &\quad - \left[1 - \frac{2M}{a_0} + \frac{2M}{a_0}\tanh\left(\frac{Q^2}{2Ma_0}\right)\right]\left(\frac{a_0}{a}\right)^{2+4\omega}. \end{aligned} \tag{27}$$

As before, $V(a_0) = 0$, and from $V'(a_0) = 0$, we get

$$\omega = \frac{1}{2}\left[-1 + \frac{g(a_0)}{a_0/M - 2 + 2\,\tanh\left[Q^2/(2Ma_0)\right]}\right], \tag{28}$$

where

$$g(a_0) = -1 + \tanh\left(\frac{Q^2}{2Ma_0}\right) + \frac{Q^2/M^2}{2a_0/M}\,\mathrm{sech}^2\left(\frac{Q^2}{2Ma_0}\right).$$

Based on the graphical output, we get only unstable solutions. For example, choosing $a_0/M = 5$ again for comparison and letting $|Q|/M = 0.9$, we get $\omega = -0.63$. The resulting graph, shown in Fig. 4, resembles Fig. 1. Other choices of the parameters lead to similar results.

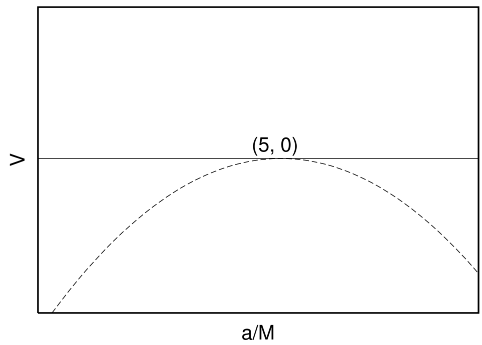

Fig. 4. The wormhole is unstable.

7. Wormholes from black holes in dilaton and dilaton-axion gravity

Of the remaining thin-shell wormholes, based on dilaton and dilaton-axion black holes, respectively, we will consider in detail only the latter, which is the more complicated of the two.

The dilaton-axion black-hole solution, inspired by low-energy string theory, was discovered by Sur, *et al.*, [18] and is also discussed in Ref. [19]. We need to list certain parameters in order to define $V(a)$. As in the Reissner–Nordström wormhole, there are two event horizons, denoted by r_- and r_+, respectively. Returning now to line element (1), we can list both $f(r)$ and $h(r)$ [18]

$$f(r) = \frac{(r-r_-)(r-r_+)}{(r-r_0)^{2-2n}(r+r_0)^{2n}},$$

$$h(r) = \frac{(r+r_0)^{2n}}{(r-r_0)^{2n-2}}.$$

Since $h(r)$ is no longer equal to r^2 in line element (1), Eq. (6) becomes

$$\sigma = -\frac{1}{4\pi}\frac{h'(a)}{h(a)}\sqrt{f(a)+\dot{a}^2} \tag{29}$$

and the conservation equation (8) has to be replaced by [7]

$$\frac{d}{d\tau}(\sigma\mathcal{A}) + \mathcal{P}\frac{d\mathcal{A}}{d\tau} = \left\{[h'(a)]^2 - 2h(a)h''(a)\right\}\frac{\dot{a}\sqrt{f(a)+\dot{a}^2}}{2h(a)}, \tag{30}$$

where $\mathcal{A} = 4\pi h(a)$ is the area of the throat by Eq. (1). The prime and dot denote, respectively, the derivatives with respect to a and τ. Substituting Eq. (29) on the right-hand side, we get

$$\frac{d}{d\tau}[4\pi h(a)\sigma] + \mathcal{P}\frac{d}{d\tau}[4\pi h(a)] = -\left\{[h'(a)]^2 - 2h(a)h''(a)\right\}\frac{\dot{a}(4\pi\sigma)}{2h'(a)},$$

whence

$$\frac{d}{da}[\sigma h(a)] + \mathcal{P}\frac{d}{da}[h(a)] = -\left\{[h'(a)]^2 - 2h(a)h''(a)\right\}\frac{\sigma}{2h'(a)}.$$

Our final form is

$$h(a)\sigma' + h'(a)(\sigma + \mathcal{P}) + \left\{[h'(a)]^2 - 2h(a)h''(a)\right\}\frac{\sigma}{2h'(a)}. \tag{31}$$

Making use of $\mathcal{P} = \omega\sigma$, this equation can be solved by separation of variables

$$\sigma(a) = \sigma_0\left[\frac{h(a_0)}{h(a)}\right]^{3/2+\omega}\left[\frac{h'(a_0)}{h'(a)}\right]^{-1}. \tag{32}$$

(Here we used the fact that $h'(a) > 0$.) It is shown in Ref. [17] that

$$\sigma_0 = -\frac{4\left[a_0 + (1-2n)r_0\right](a_0 - r_-)(a_0 - r_+)}{D(a_0 - r_0)(a_0 + r_0)}, \tag{33}$$

where

$$D = 8\pi(a_0 - r_0)^{1-n}(a_0 + r_0)^n\sqrt{(a_0 - r_-)(a_0 - r_+)}. \tag{34}$$

Using the equation of motion $\dot{a}^2 + V(a) = 0$ once again, we get from Eq. (29),

$$V(a) = f(a) - \left[4\pi\frac{h(a)}{h'(a)}\sigma(a)\right]^2. \tag{35}$$

Eq. (32) now yields

$$V(a) = \frac{(a - r_-)(a - r_+)}{(a - r_0)^{2-2n}(a + r_0)^{2n}} - \left[4\pi\frac{h(a)}{h'(a_0)}\sigma_0\right]^2\left[\frac{h(a_0)}{h'(a)}\right]^{3+2\omega}. \tag{36}$$

While it is easy enough to check that $V(a_0) = 0$, it is no longer convenient to compute ω as a function of the various parameters. Plotting $V(a)$ against a instead of a/M, we can determine ω by trial and error: $V(a)$ must be tangent to the a-axis at $a = a_0$, where $V(a_0) = 0$ automatically. For example, if $a_0 = 5$, $r_0 = 1$, $r_- = 2$, $r_+ = 2.05$, and $n = 0.8$, then $\omega = -0.915$. If $a_0 = 5$, $r_0 = 1$, $r_- = 2$, $r_+ = 3$, and $n = 0.8$, then $\omega = -1.132$. Reducing n to 0.6 produces $\omega = -0.84$ in the first case and $\omega = -1.041$ in the second.

In all cases the graphs are concave down at $a_0 = 5$ and look similar to the graph in Fig. 1. Based on the graphical output, there do not appear to be any stable solutions.

For the dilaton case we have [20]

$$V(a) = \left(1-\frac{A}{a}\right)\left(1-\frac{B}{a}\right)^{(1-b^2)/(1+b^2)} - \left[4\pi\frac{h(a)}{h'(a)}\sigma_0\right]^2 \left[\frac{h(a_0)}{h(a)}\right]^{3+2\omega}, \quad (37)$$

where $h(a) = a^2(1-B/a)^{2b^2/(1+b^2)}$ for various constants. Once again, one can readily check that $V(a_0) = 0$.

As in the dilaton-axion case, ω can be found by trial and error. For example, if $a_0 = 5$, $b = 0.5$, $A = 2$, and $B = 1$, then $\omega = -0.693$; if $a_0 = 6$, $b = 0.8$, $A = 4$, and $B = 2$, then $\omega = -0.94$, *etc.* The resulting graphs are similar to those in the dilaton-axion case.

8. Conclusion

This paper discusses the stability to linearized radial perturbations of spherically symmetric thin-shell wormholes with the equation of state $\mathcal{P} = \omega\sigma$, $\omega < 0$, for the exotic matter at the throat. This EoS is referred to as phantom-like. Various spacetimes were considered.

It was found that the wormholes are unstable if constructed from Schwarzschild spacetimes, as well as from black holes in dilaton and dilaton-axion gravity. For the Reissner–Nordström case, stable solutions exist only if

$$\frac{|Q|}{M} > \frac{a_0/M}{\sqrt{2(a_0/M)-1}},$$

leading to a naked singularity. For the Schwarzschild–de Sitter case, the wormholes are stable if, and only if

$$1 - \frac{2}{a_0/M} - \frac{1}{3}\Lambda M^2\left(\frac{a_0}{M}\right)^2 < 0.$$

In the Schwarzschild–anti de Sitter case, the configurations are stable whenever

$$(1) \qquad \Lambda M^2 < \frac{1}{(a_0/M)[3(a_0/M)-(2/3)(a_0/M)^2]}$$

and

$$(2) \qquad a_0/M > 4.5.$$

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