NUCLEAR SYMMETRY ENERGY IN RELATIVISTIC HADRONIC MODELS

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The density dependence of the symmetry energy and the correlation between parameters of the symmetry energy and the neutron skin thickness in the nucleus ^{208}Pb are investigated in relativistic hadronic models. The dependency of the symmetry energy on density is linear around saturation density. The existence of correlation between the neutron skin thickness in the nucleus ^{208}Pb and the value and the slope of the nuclear symmetry energy at saturation density is found to be dependent on the model used.

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1. Introduction

The nuclear symmetry energy plays a central role in a variety of nuclear phenomena. The stability and ground-state properties of neutron-rich nuclei depend sensitively on the nuclear symmetry energy [1,2]. The symmetry energy has a strong influence on the isoscaling properties of the fragments produced in heavy-ion reactions [3]. The nuclear symmetry energy is an important issue in astrophysics. It is a key quantity in the stability consideration in neutron star matter [4]. The proton fraction inside a neutron star depends on the nuclear symmetry energy, which has a crucial role in the thermal evolution of neutron stars [5].

Unfortunately, the density dependence of the symmetry energy and even the symmetry energy at saturation density a_4 are poorly known. An empirical determination by Green yields 23.5 MeV for a_4 [6]. The symmetry energy at normal nuclear matter density is found to lay in between 27–36 MeV in mass formula calculations, 28–38 MeV in nonrelativistic models, and 35–42 MeV in relativistic models [7]. The experimentally observed scaling parameters of fragments produced in multifragmentation reactions can be explained by a symmetry energy significantly lower than these values [3].

Analysis of the isotopic composition of particles emitted during an energetic nucleus–nucleus collision suggests preference for a stiff density dependence of the symmetry energy [8]. While heavy-ion studies favor a dependence on ρ^{γ}, with $\gamma = 0.6$–1.05 [9] and $\gamma = 0.69$ at low densities [10]. Calculations within the nonrelativistic Brueckner–Hartree–Fock approach based on a large set of modern nucleon–nucleon potentials yield symmetry energies increasing roughly linearly with density [11].

A study of neutron radii in nonrelativistic and covariant mean-field models revealed a linear correlation between the neutron skin thickness t in ^{208}Pb and parameters determining the symmetry energy, which are the value of the symmetry energy at nuclear matter saturation density a_4 and the slope of the symmetry energy at saturation density p_0 [12]. A theoretical spread of about 0.3 fm in the neutron radius of ^{208}Pb was estimated in Ref. [12], which is attributed to the poorly known density dependence of the symmetry energy. A summary of some recent experimental results on the neutron skin thickness in ^{208}Pb is given in Ref. [13], where the value of t is spread between 0.08 and 0.51 fm. More precise information on the neutron radius of ^{208}Pb might become available via a parity-violating electron scattering experiment at the Jefferson Laboratory, that promises a 1% accuracy [14].

This work calculates the nuclear symmetry energy in relativistic hadronic models, and investigates the correlation between the neutron skin thickness in ^{208}Pb on one side, and the value a_4 and the slope p_0 of the symmetry energy at saturation density on the other. Section 2 reviews the three parameter sets used, Section 3 defines the symmetry energy in nuclear matter and its parameters a_4 and p_0, Section 4 discusses the results for the value of the symmetry energy at saturation density and the density dependence of the symmetry energy, and investigates the correlation between the neutron skin thickness in ^{208}Pb and a_4 and p_0, and the final Section 5 summarizes the main conclusions.

2. Relativistic hadronic models

A standard one-boson-exchange OBE Lagrangian with electromagnetic field between protons and four mesons: the isoscalar scalar meson σ, the isoscalar vector meson ω, the isovector scalar meson δ, and the isovector vector meson ρ, is given by

$$\begin{aligned}\mathcal{L} \;=\; & \overline{\psi}\Big[i\gamma^{\mu}\partial_{\mu} - m_N - g_{\sigma}\sigma - g_{\omega}\gamma^{\mu}\omega_{\mu} - g_{\delta}\,\vec{\tau}\cdot\vec{\delta} \\ & - g_{\rho}\,\vec{\tau}\cdot\gamma^{\mu}\vec{\rho}_{\mu} - \frac{e}{2}\,(1+\tau_3)\,\gamma^{\mu}A_{\mu}\Big]\,\psi \\ & - U(\sigma) + \tfrac{1}{2}\,\partial^{\mu}\sigma\;\partial_{\mu}\sigma \\ & + \tfrac{1}{2}\,m_{\omega}^2\;\omega^{\mu}\omega_{\mu} - \tfrac{1}{4}\,\Omega^{\mu\nu}\Omega_{\mu\nu}\end{aligned}$$

$$
\begin{aligned}
&-\tfrac{1}{2}\, m_\delta^2\, \delta^2 + \tfrac{1}{2}\, \partial^\mu \vec{\delta} \cdot \partial_\mu \vec{\delta} \\
&+\tfrac{1}{2}\, m_\rho^2\, \vec{\rho}^{\,\mu} \cdot \vec{\rho}_\mu - \tfrac{1}{4}\, \vec{R}^{\,\mu\nu} \cdot \vec{R}_{\mu\nu} - \tfrac{1}{4}\, F^{\mu\nu} F_{\mu\nu}\,, \qquad (1)
\end{aligned}
$$

with

$$
\begin{aligned}
U(\sigma) &= \tfrac{1}{2}\, m_\sigma^2\, \sigma^2 + \tfrac{1}{3}\, g_2\, \sigma^3 + \tfrac{1}{4}\, g_3\, \sigma^4\,, & (2)\\
\Omega^{\mu\nu} &= \partial^\mu \omega^\nu - \partial^\nu \omega^\mu\,, & (3)\\
\vec{R}^{\,\mu\nu} &= \partial^\mu \vec{\rho}^{\,\nu} - \partial^\nu \vec{\rho}^{\,\mu}\,, & (4)\\
F^{\mu\nu} &= \partial^\mu A^\nu - \partial^\nu A^\mu\,. & (5)
\end{aligned}
$$

The baryons, protons and neutrons, are represented by Dirac spinors $\overline{\psi} = (\overline{\psi}_p, \overline{\psi}_n)$. σ, ω_μ, $\vec{\delta}$, and $\vec{\rho}_\mu$ are the fields of the different mesons. e is the proton charge and A_μ the electromagnetic field, m_N is the nucleon mass, m_i and g_i $(i = \sigma, \omega, \delta, \rho)$ are the mass and the coupling parameter of the i-meson, γ^μ are the Dirac γ matrices, $\vec{\tau}$ is the isospin vector, and τ_3 its third component, equal to $+1$ for protons and -1 for neutrons, g_2 and g_3 are the parameters of the non-linear self-interaction terms of the σ field.

The first parameter set used in this work is NL3 of Ref. [15], and its parameters are given in Table I. The NL3 set includes non-linear self-interaction terms of the σ field and no isovector scalar meson δ. The NL3 set is obtained by fitting the charge radii, the binding energies, and the available neutron radii of 10 spherical nuclei: ^{16}O, ^{40}Ca, ^{48}Ca, ^{58}Ni, ^{90}Zr, ^{116}Sn, ^{124}Sn, ^{132}Sn, ^{208}Pb, and ^{214}Pb.

TABLE I

Parameters of the NL3 set. m_i and g_i are the mass and coupling constant of the i-meson. g_2 and g_3 are the parameters of the non-linear self-interaction terms of the σ field. $m_N = 939$ MeV is the nucleon mass.

Meson i	m_i (MeV)	g_i	g_2 (fm^{-1})	g_3
σ	508.194	10.217	-10.431	-28.885
ω	782.501	12.868	—	—
ρ	763	4.474	—	—

The second parameter set used in this work is DD-ME2 of Ref. [16]. It does not include non-linear self-interaction terms of the σ field, *i.e.*, $g_2 = g_3 = 0$ in equation (2), and also no δ meson, but the coupling parameters of the three mesons σ, ω, and ρ depend on the density. Density dependent coupling parameters for the isoscalar mesons are introduced by

$$
g_i(\rho) = g_i(\rho_0)\; f_i(x)\,, \qquad i = \sigma, \omega\,, \qquad (6)
$$

where ρ_0 is the nuclear matter saturation density and

$$x = \rho/\rho_0\,, \quad (7)$$

$$f_i(x) = a_i \, \frac{1 + b_i\,(x + d_i)^2}{1 + c_i\,(x + d_i)^2}\,. \quad (8)$$

For the ρ meson coupling the density dependence has the functional form

$$g_\rho(\rho) = g_\rho(\rho_0)\exp\left[-a_\rho(x-1)\right]\,. \quad (9)$$

$g_i(\rho_0)$, a_i, b_i, c_i, and d_i are the coefficients of the density dependent function $g_i(\rho)$ ($i = \sigma, \omega$), and $g_\rho(\rho_0)$ and a_ρ are the coefficients of the density dependent function $g_\rho(\rho)$. The coefficients of the DD-ME2 set are given in Table II, and are obtained by fitting the properties of symmetric and asymmetric nuclear matter, binding energies, charge radii, and available neutron skin thicknesses of 12 spherical nuclei, which are the same 10 nuclei used in NL3, ^{204}Pb, and ^{210}Po.

TABLE II

The DD-ME2 set. $m_N = 939$ MeV and $\rho_0 = 0.152$ fm^{-3}.

Meson i	m_i (MeV)	$g_i(\rho_0)$	a_i	b_i	c_i	d_i
σ	550.1238	10.5396	1.3881	1.0943	1.7057	0.4421
ω	783	13.0189	1.3892	0.9240	1.4620	0.4775
ρ	763	3.6836	0.5647	—	—	—

The last parameter set used in this work is D(A) of Ref. [17]. It does not include non-linear self-interaction terms of the σ field, but a δ meson, and the coupling parameters of the four mesons depend on the density. Density dependent coupling parameters for the isoscalar mesons are introduced by

$$\frac{g_i(\rho)}{g_i(\rho_0)} - 1 = a_i\left(\exp\left[b_i\left(1 - \left(\frac{\rho}{\rho_0}\right)^{1/3}\right)\right] - 1\right)\,, \qquad i = \sigma, \omega\,, \quad (10)$$

where ρ_0 is the nuclear matter saturation density and a_i, b_i, and $g_i(\rho_0)$ are the coefficients of the density dependent function $g_i(\rho)$. Density dependent coupling parameters for the isovector mesons are introduced by

$$g_i(\rho) = g_i(\rho_0)\exp\left[b_i\left(1 - \frac{\rho}{\rho_0}\right)\right]\,, \qquad i = \delta, \rho\,, \quad (11)$$

where b_i and $g_i(\rho_0)$ are the coefficients of the density dependent function $g_i(\rho)$. The coefficients a_i, b_i, and $g_i(\rho_0)$ ($i = \sigma, \omega$) and b_i and $g_i(\rho_0)$

($i = \delta, \rho$) are adjusted to the outcome of the relativistic Brueckner–Hartree–Fock (RBHF) calculations of the nucleon self-energy in nuclear matter. The resulting parametrization of the RBHF potential Bonn A is called D(A) and is given in Table III.

TABLE III

The D(A) set. $m_N = 938.926$ MeV and $\rho_0 = 0.185$ fm^{-3}.

Meson i	m_i (MeV)	$g_i(\rho_0)$	a_i	b_i
σ	550	9.297	0.2941	2.217
ω	782.6	11.269	0.3451	2.113
δ	983	4.701	—	1.223
ρ	769	2.370	—	1.634

The three parameter sets NL3, DD-ME2, and D(A) are representatives of a huge variety of parameter sets used in relativistic models, but almost all follow one of the concepts introduced by these three sets, which has been successfully applied in the calculation of nuclear matter and properties of nuclei.

3. Nuclear symmetry energy

The nuclear matter equation of state EOS gives the nucleon energy e as a function of the baryon density ρ and the asymmetry parameter β

$$e = e\ (\rho, \beta)\,, \tag{12}$$

where the baryon density is the sum of the neutron and proton densities

$$\rho = \rho_n + \rho_p\,, \tag{13}$$

and the asymmetry parameter is defined by

$$\beta = \frac{\rho_n - \rho_p}{\rho}\,. \tag{14}$$

The nuclear symmetry energy $e_{\rm sym}(\rho)$ is the quantity characterizing the isospin dependence of the EOS, and is obtained by expanding the EOS in terms of the asymmetry parameter β

$$e\ (\rho, \beta) = e\ (\rho, 0) + e_{\rm sym}(\rho)\ \beta^2 + O\left(\beta^4\right)\,, \tag{15}$$

i.e., the nuclear symmetry energy is given by

$$e_{\rm sym}(\rho) = \frac{1}{2}\ \frac{\partial^2 e\ (\rho, \beta)}{\partial \beta^2}\bigg|_{\beta=0}\,. \tag{16}$$

The value of the symmetry energy at nuclear matter saturation density ρ_0 is denoted by

$$a_4 = e_{\mathrm{sym}}(\rho_0)\,, \tag{17}$$

and the parameter describing the slope of the symmetry energy at saturation density is

$$p_0 = \rho_0^2 \left. \frac{de_{\mathrm{sym}}(\rho)}{d\rho} \right|_{\rho=\rho_0} . \tag{18}$$

4. Results and discussion

Figure 1 displays the symmetry energy as a function of the density for the three sets reviewed in Section 2. In all cases the dependency of the symmetry energy on density is linear around saturation density. If one describes the functional dependence on the density by ρ^γ, the value of γ will be equal to 1 around saturation density for all sets. For the NL3 set, the value of γ changes from 1 at small densities and densities around ρ_0, to 0.84 at large densities (0.3–0.5 fm^{-3}). For the DD-ME2 set, γ changes from 0.64 at small densities to 1 at densities around ρ_0 and 0.47 at large densities. And for the D(A) set γ changes from 1.37 at small densities to 1 at densities around ρ_0 and 0.98 at large densities. The difference between NL3 and DD-ME2 results for the symmetry energy increases with density, since the ρ meson exchange contribution to the symmetry energy is proportional to g_ρ^2/m_ρ^2

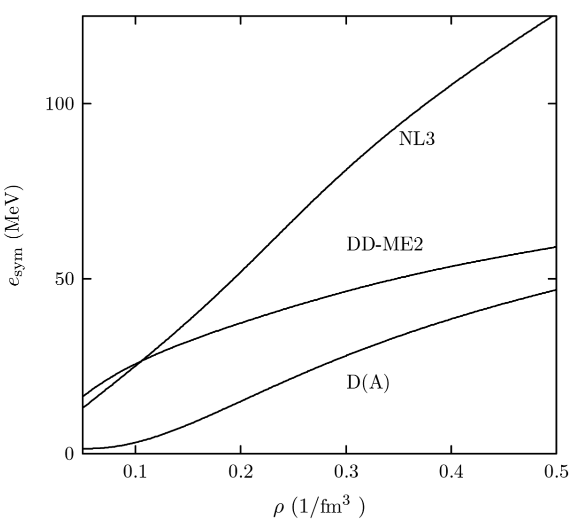

Fig. 1. Symmetry energy as a function of the density for the sets NL3, DD-ME2, and D(A).

and g_ρ decreases exponentially with the density in the case of DD-ME2, see equation (9), while g_ρ is a constant in the case of NL3. The δ meson exchange contribution to the symmetry energy is negative and proportional to $-g_\delta^2/m_\delta^2$. Therefore, the inclusion of the δ meson in the D(A) set leads to an even smaller value for the symmetry energy than in the DD-ME2 case at all densities.

Table IV lists the results for the parameters of the symmetry energy a_4 and p_0 for the three sets. It includes also the results for the value of the neutron skin thickness t in the nucleus ^{208}Pb. The results presented in Table IV confirm the existence of a correlation between the neutron skin thickness in ^{208}Pb and the value of the nuclear symmetry energy at saturation density a_4, but not with the slope of the symmetry energy at saturation density p_0. The value of p_0 obtained for the D(A) set, which produces the smallest values for a_4 and t, is not the smallest. This is due to the large value of ρ_0 in the case of D(A), which enters in the definition of p_0, see equation (18), and to the decrease of the net value of the negative δ meson contribution to the symmetry energy with increasing density, since g_δ decreases with increasing density, see equation (11), leading to a larger net value for p_0.

TABLE IV

Nuclear matter saturation density ρ_0, the parameters of the symmetry energy a_4 and p_0, and the neutron skin thickness t in the nucleus ^{208}Pb.

	NL3	DD-ME2	D(A)
ρ_0 (1/fm^3)	0.148	0.152	0.185
a_4 (MeV)	37.4	32.3	12.9
p_0 (MeV/fm^3)	5.84	2.61	4.64
t (fm)	0.279	0.19	0.129

5. Summary

The nuclear symmetry energy is calculated using three relativistic hadronic models NL3, DD-ME2, and D(A). The dependency of the symmetry energy on the density is found to be linear around saturation density. The existence of correlation between the neutron skin thickness in the nucleus ^{208}Pb and the value and the slope of the nuclear symmetry energy at saturation density is found to be dependent on the model used in the investigation. Results are explained by the density dependence of the coupling parameters and the inclusion of the δ meson.

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