ROTATIONAL BANDS IN Fm ISOTOPES WITHIN THE LUBLIN STRASBOURG DROP AND YUKAWA FOLDED MODEL*

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An investigation of heavy and superheavy nuclei requires a proper model to reproduce masses and rotational energies. We obtain a very good agreement with experimental data with the Yukawa-folded (YF) single particle potential and the Lublin Strasbourg Drop (LSD). Using the Strutinsky method we add shell and pairing energy corrections to the macroscopic energy. The pairing corrections are evaluated within the BCS theory. The equilibrium deformations of Fm isotopes are determined. Ground-state masses and rotational states obtained using the cranking moments of inertia are compared to the experimental data.

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1. Introduction

The agreement of the experimental and theoretical rotational states of nuclei is a good test of the model giving equilibrium deformations, masses and moments of inertia of nuclei. Since years several papers have been devoted to calculate these quantities [1,2,3,4,5,7,6] giving quite satisfactory results for many nuclei. The aim of present paper is to find the optimal pairing strengths reproducing masses and rotational states of nuclei. The present work concerns the even–even Fm isotopes and quotes the results of [6] for ^{254}No. Here the potential energies were evaluated using the macroscopic–microscopic [8] method with the LSD model [10], the YF mean field [9], the Strutinsky shell correction [11] and the BCS pairing theory [12] in order to calculate the rotational states with cranking moments of inertia [13].

The calculations were performed for the following fermium isotopes: ^{248}Fm–^{256}Fm on a two-dimensional deformation "Modified Funny Hills" (MFH) [14] grid on the elongation (c) and neck (h) parameters plane. The

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equilibrium deformations were found by minimising the total energy on the (c,h) plane and the theoretical ground-state masses were compared to the experimental ones. The rotational energies of Fm isotopes for $L/\hbar = 2,4,6,8$ agree well with the experimental data when a pairing strengths [15] $G = G_0/N_q^{2/3}\hbar\omega_0\,(N_q = N,Z)$ is fitted to the energies of 2^+ states. In our calculations we have used the pairing window consisting of $2\sqrt{15N_q}$ levels closest to the Fermi energy [16]. The pairing strength influences not only the moments of inertia, but also the potential energy and its minimum. The calculation demands many steps to get:

I. Single particle levels on the deformation grid.

II. Total potential energy with the starting pairing strength on the same grid.

III. Equilibrium deformation by minimisation of this energy.

IV. Moments of inertia in the ground state.

V. Rotational energies.

If the rotational energies do not agree with the data we change the pairing strength and come back to point II repeating the calculation up to the best reproduction of the known rotational bands. We were changing G_0 value around the starting one, diminishing the step to the 0.0001 MeV to get the experimental E_{2+} states with the accuracy 0.1 keV. For the last pairing strengths a prediction of the E_{L+} rotational states for all of the investigated nuclei is calculated and the masses are compared to the experimental ones.

2. Nuclear energy

In the macroscopic–microscopic method [8] the total energy of a nucleus at a given deformation can be calculated as a sum of the macroscopic energy and the corrections due to shell and pairing effects for protons and neutrons

$$E = E_{\rm LSD} + E_{\rm shell} + E_{\rm pair}\,. \tag{1}$$

The shell corrections are obtained by subtracting the average single-particle (s.p.) energy sum from the corresponding quantum result

$$E_{\rm shell} = \sum_k e_k - \tilde{E}\,, \tag{2}$$

where e_k are the s.p. levels and $\tilde{E}$ is the average energy smoothed by Strutinsky procedure [11].

$$\tilde{E} = 2 \int_{-\infty}^{\tilde{\lambda}} e\bar{\rho}(e)de\,, \tag{3}$$

where the smoothed density distribution is

$$\bar{\rho}(e) = \frac{1}{\gamma} \sum_k j_6\left(\frac{e-e_k}{\gamma}\right)\,, \tag{4}$$

where the smearing function with the $6^{\rm th}$ order correctional polynomial is

$$j_6(u) = \frac{1}{\sqrt{\pi}} e^{-u^2} \left(\frac{35}{16} - \frac{35}{8}u^2 + \frac{7}{4}u^4 - \frac{1}{6}u^6\right)\,. \tag{5}$$

The Fermi level $\tilde{\lambda}$ in the smoothed system is found from the condition

$$\mathcal{N} = 2 \int_{-\infty}^{\tilde{\lambda}} \rho(e)de\,. \tag{6}$$

The pairing corrections are determined as the difference between the BCS [12] energy and the single-particle energy sum minus the average pairing energy [17]

$$E_{\rm pair} = E_{\rm BCS} - \sum_k e_k - \langle E_{\rm pair}\rangle\,. \tag{7}$$

In the BCS approximation the ground-state energy of a system with even number of particles and the monopole pairing forces is given by

$$E_{\rm BCS} = \sum_{k>0} 2e_k v_k^2 - \frac{\Delta^2}{G} - G\sum_{k>0} v_k^4\,, \tag{8}$$

where v_k are the occupation factors and Δ the pairing gap of BCS theory [17].

As the macroscopic part of energy the LSD model was used [10]

$$\begin{aligned} E_{\rm LSD} &= -b_{\rm vol}\left(1-\kappa_{\rm vol}I^2\right)A + b_{\rm surf}\left(1-\kappa_{\rm surf}I^2\right)A^{2/3} + b_{\rm cur}\left(1-\kappa_{\rm cur}I^2\right)A^{1/3} \\ &\quad + \frac{3}{5}e^2\frac{Z^2}{r_0^{\rm ch}A^{1/3}} - C_4\frac{Z^2}{A} - 10\exp\left(-42|I|/10\right)\,, \end{aligned}$$

where the parameters are

$$\begin{aligned} b_{\rm vol} &= 15.4920 \text{ MeV}, & \kappa_{\rm vol} &= 1.8601, \\ b_{\rm surf} &= 16.9707 \text{ MeV}, & \kappa_{\rm surf} &= 2.2938, \\ b_{\rm cur} &= 3.8602 \text{ MeV}, & \kappa_{\rm cur} &= -2.3764, \\ r_0^{\rm ch} &= 1.21725 \text{ fm}, & & \\ C_4 &= 0.91810 \text{ MeV}, & I &= (N-Z)/A\,. \end{aligned}$$

The mass formula reads

$$M_{\rm th} = ZM_{\rm H} + NM_n - 0.00001433Z^{2.39} + E_{\rm LSD}\,, \tag{9}$$

where $M_{\rm H}$ is the hydrogen atom mass, M_n neutron mass and the third term stays for the electron interactions.

3. Moment of inertia

For low angular velocities ω the eigenproblem of rotating single-particle Hamiltonian can be solved in the second order perturbation theory (equivalent to the cranking approximation [13]) and one obtains for the rotational energy the following equation

$$E_L = \frac{L(L+1)}{2\mathcal{J}}\hbar^2\,, \tag{10}$$

where

$$\mathcal{J} = 2\hbar^2 \sum_\mu \sum_\nu \frac{|\langle\nu|\hat{j}_x|\mu\rangle|^2}{E_\mu + E_\nu}(u_\mu v_\nu - u_\nu v_\mu)^2 \tag{11}$$

is the cranking moment of inertia. E_μ are the quasiparticle energies u_ν unoccupation factors of BCS theory, $\hat{j}_x$ is the intrinsic spin operator.

4. Results

The calculations were performed for the series of nuclei: ^{248}Fm–^{256}Fm on the MFH [14] deformation grid with $c = 0.8$ up to 1.6 with step length 0.05 and $h = -0.4$ to 0.4 with step length 0.1. The plateau condition of the shell corrections is well fulfilled with $\gamma = 1.2\hbar\omega_0$, where $\hbar\omega_0 = 41/A^{1/3}$ MeV.

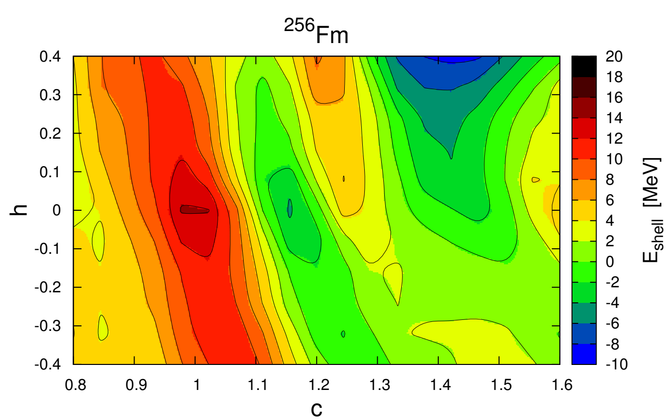

Fig. 1. Shell correction $E_{\rm shell}$ obtained with the Yukawa-folded mean field for ^{256}Fm as function of elongation c and neck parameter h.

We show in Fig. 1 the map of the shell-correction energy ($\mathrm{E_{shell}}$) for ^{256}Fm. A deep minimum at $c_{\mathrm{eq}} = 1.15$, $h_{\mathrm{eq}} = 0$ is observed. The deformation energy $\mathrm{E_{def}} = E(c,h) - E(1,0)$ of ^{256}Fm is plotted in Fig. 2. Two pronounced minima corresponding to the ground and the shape-isomeric states are visible.

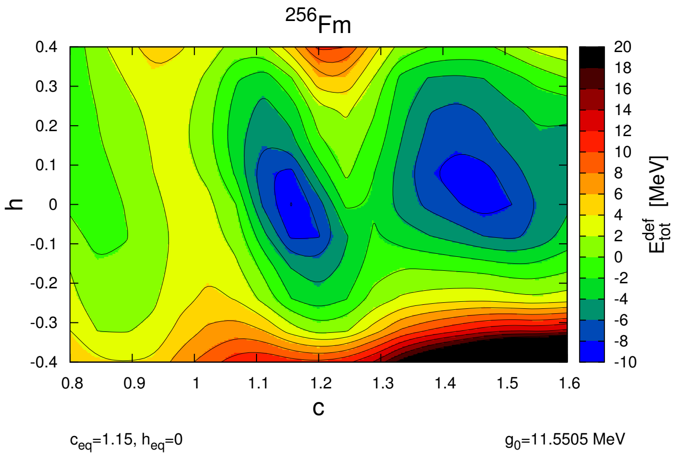

Fig. 2. Deformation energy $\mathrm{E_{def}}$ for ^{256}Fm as function of elongation c and neck parameter h obtained with the Yukawa-folded mean field.

In Fig. 3 we present the differences of theoretical and experimental [19] masses for Fm and No isotopes. Except of ^{256}Fm this deviation never exceeds 0.3 MeV.

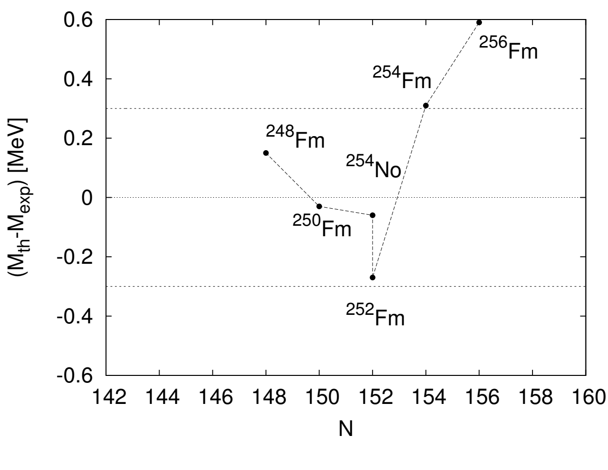

Fig. 3. Difference between theoretical and experimental masses $\mathrm{M_{th}} - \mathrm{M_{exp}}$ for Fm and No isotopes as function of the neutron number N. The dotted lines represent the 0.3 MeV limits.

To obtain the rotational energies E_{L^+} the moments of inertia $\mathcal{J}$ were calculated microscopically within the cranking model [13] using the YF s.p. potential. In Fig. 4 the cranking moment of inertia of ^{256}Fm is plotted in the (c, h) plane.

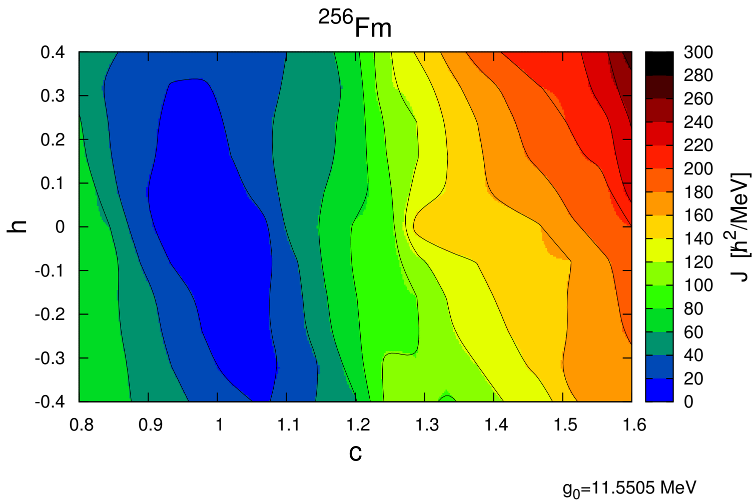

Fig. 4. Cranking moment of inertia $\mathcal{J}$ of ^{256}Fm as function of elongation c and neck parameter h.

The theoretical estimates of the rotational energies E_{L^+} of ^{256}Fm isotopes for $L/\hbar = 2, 4, 6, 8$ are compared with the experimental data [20] in Fig. 5.

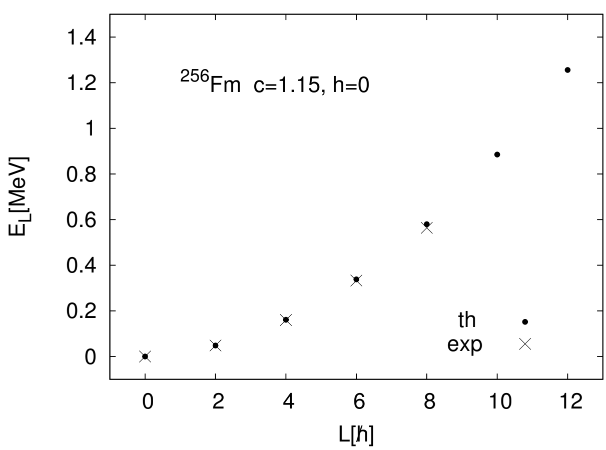

Fig. 5. Theoretical and experimental rotational energies E_L for ^{256}Fm as functions of angular momentum L.

One obtains an almost perfect agreement when the pairing strengths $g_0 = G_0\hbar\omega = 11.5505$ MeV. The rotational energies E_{L^+} of ^{254}Fm isotopes for $L/\hbar = 2, 4$ are compared with the experimental data [20] in Fig. 6. The pairing strengths $g_0 = 12.2752$ MeV was found here.

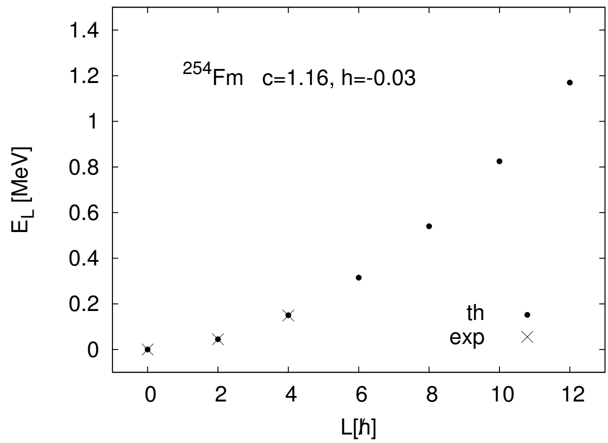

Fig. 6. Theoretical and experimental rotational energies E_{L^+} for ^{254}Fm as functions of angular momentum L.

In Fig. 7 the rotational levels scheme for ^{250}Fm is shown. The theoretical results are obtained with the pairing strength $g_0 = 12.6064$ MeV. The levels up to $L/\hbar = 14$ are well reproduced. The higher spins demand more accurate calculations which should take into account the change with rotation of the equilibrium deformation and the pairing field. The experimental data are taken from Ref. [21].

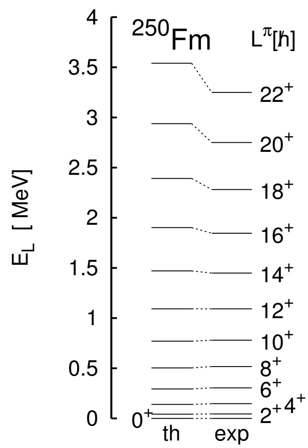

Fig. 7. Theoretical and experimental rotational energies E_{L^+} for ^{250}Fm.

Fig. 8 gives E_{2+} rotational states for Fm isotopes. Below the pairing strengths found for corresponding nuclei are drawn. Unfortunately, because of the shortage of points, no satisfactory isospin dependent function, reproducing this line was found. The more detailed calculations are in progress now.

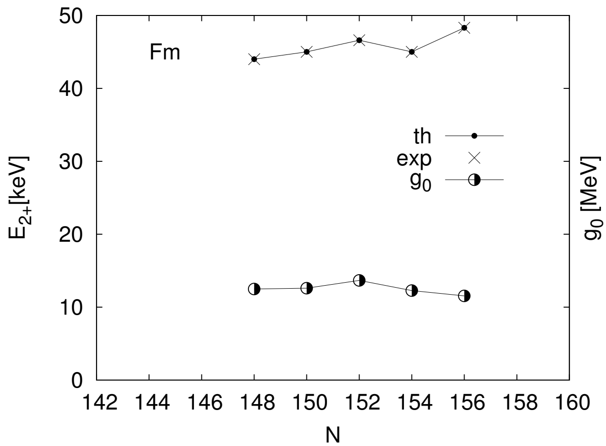

Fig. 8. Rotational energies E_{2+} for Fm isotopes as functions of neutron number N. The corresponding pairing strengths parameters g_0 are denoted below.

5. Conclusions

The following conclusions can be drawn from our calculation:

— Yukawa-folded mean field potential describes well the shell structure of heavy nuclei.

— Strutinsky shell correction and BCS pairing energy with pairing strength adjusted to the rotational states give the proper equilibrium deformations and masses of nuclei.

— Rotational model with the cranking moments of inertia reproduces very well the ground-state rotational band in the even–even isotopes.

More results would be required for finding the isotopic dependence of pairing strength in this region of nuclei. Such calculations are in progress now.

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